Opioid use disorder among Ohio’s Medicaid population: Prevalence estimates from nineteen counties using a multiplier method

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Contents
1 Web Appendix 1: Statistical Model ........................................... 2
2 Web Appendix 2: Model Selection ........................................... 5
Web Table 1 ........................................................................... 8

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1 Web Appendix 1: Statistical Model

Repeating some information from the main text with notation added, each stratum represented in the analytic dataset identified with index $i$ was characterized by the following: (1) stratifying characteristics county, sex, and age group; (2) the total size $N_i$ of the stratum population; (3) the number $n_i$ with observed OUD; (4) the cumulative person-years $m_i$ not on MOUD (number of months divided by 12); (5) the number of opioid-related deaths $d_i^{(o)}$ that occurred while off MOUD treatment among those with an observed OUD; and finally, (6), the number of opioid-related deaths $d_i^{(u)}$ among those with an unknown OUD status. Superscript parenthetical roman characters $o$ and $u$ are labels for observed and unobserved OUD status, respectively.

Expressions (1-4) present a generic form of the model more concretely. It is generic in expressions (2) and (3) in the sense that data $w_i$ and $x_i$ that characterize strata and the associated parameters $\beta$ and $\gamma$ along with the random effects $\theta$ and $\zeta$ are flexible in how they are specified. The final specification was determined based on optimization of a complexity-penalized relative fit criterion.
\( d_i^{(o)} \sim \text{Poisson}(\lambda_i m_i) \)  \hspace{1cm} (1)

\[ \lambda_i = \exp(\beta' w_i + \theta_{g(i)}) \]  \hspace{1cm} (2)

\[ n_i + \eta_i \sim \text{TBin}_{m_i}^N \left( N_i, \logit^{-1}(\gamma' x_i + \zeta_{h(i)}) \right) \]  \hspace{1cm} (3)

\[ d_i^{(u)} \sim \text{Poisson}(\lambda_i \eta_i) \]  \hspace{1cm} (4)

Expressions (1) and (2) define the Poisson model of the number of opioid-related deaths \( d_i^{(o)} \) among the people in stratum \( i \) with an observed OUD diagnosis only if the death occurred while off treatment. Conceptually, this model is used to estimate \( \beta \) (expression (2)), and therefore \( \lambda_i \), the stratum-specific opioid-related death rate for people with OUD while untreated in stratum \( i \). The observed variable \( m_i \) is the number of person-years spent untreated among the entire group for whom an OUD diagnosis was observed. The product \( \lambda_i m_i \) is the expectation for the number of deaths in stratum \( i \) among those with observed OUD.

Expression (2) defines the flexibility of the death rate \( \lambda_i \) as a function of a vector of stratum characteristics \( w_i \) along with its associated parameter vector \( \beta \) and possibly also random effects \( \theta_{g(i)} \). The subscript \( g(i) \) defines a mapping of a stratum index \( i \) to some grouping of indices to allow for sharing of a random effect across similar strata (e.g., those within a single county). The choices of characteristics represented in \( w_i \) and the grouping map \( g(i) \) constituted the process of model selection, described later.
values are exponentiated to map the sum such that it is non-negative.

Expression (3) defines the binomial model that is used, along with the component described by expression (4), to estimate the number of individuals $\eta_i$ without an OUD diagnosis but who truly have OUD in stratum $i$. Because the left hand side of the binomial is a sum of the observed number of people who have an OUD diagnosis $n_i$—a fixed lower boundary of the total—and the number with OUD but who are undiagnosed $\eta_i$, we ensure the binomial does not go below the observed number by using a truncated binomial where the lower boundary is set to $n_i$. The upper boundary is the total number of people in the stratum $N_i$. The proportion parameter for the binomial is, like in the death model above, a function of a vector of stratum characteristics $x_i$ along with its associated parameter vector $\gamma$ and possibly also random effects $\zeta_{h(i)}$. The subscript $h(i)$, like $g(i)$ above, defines a separate mapping of a stratum index $i$ to some grouping of indices to facilitate random effect sharing across similar strata. Choices related to $x_i$ and random effect sharing are part of the model selection process. The inverse logit transformation of the linear predictor implies a logit link function for this model.

Finally, expression (4) defines the model of opioid-related deaths $d_i^{(u)}$ among the group with unobserved OUD status, and therefore those who may have a latent OUD. The rate parameter for the Poisson distribution is the product of the stratum-specific death rate $\lambda_i$ and the unknown $\eta_i$, the number of people with latent OUD. Since $\lambda_i$ is estimated in a different part of the model and $d_i^{(u)}$ is observed, this part of the model is responsible for estimating $\eta_i$.

When the model included random effects, they were modeled with a Gaussian dis-
tribution with a zero mean and unknown variance. We placed weakly informative priors on all model parameters as defined below.

\[ \beta \sim N(0, 5) \]
\[ \gamma \sim N(0, 5) \]
\[ \theta_{g(i)} \sim N(0, \tau_{\theta}) \]
\[ \tau_{\theta} \sim \text{uniform}(0, 10) \]
\[ \zeta_{h(i)} \sim N(0, \tau_{\zeta}) \]
\[ \tau_{\zeta} \sim \text{uniform}(0, 10) \]

Our prior distribution choices are not completely uninformative. However, a sensitivity check using very low information priors (e.g., \( \beta \sim N(0, 100) \) and \( \tau_{\theta} \sim \text{uniform}(0, 100) \)) did not quantitatively change the results. Thus, it was clear that prior choices did not drive the results.

2 Web Appendix 2: Model Selection

In this section we describe a simplified version of the process by which we selected our final model. It does not detail all models that were attempted or the checks and exploratory analyses that were used to make decisions during the process. It does give
a broad sense of how the process evolved.

Web Table 1 shows a subset of all attempted models ordered approximately in the order that we fit them. A description of each model is provided along with the model number and the LOOIC value associated with that model. We set a restriction that all models would have the same specification in both the death rate and prevalence components of the model. This was a choice to keep the space of possible models manageable in our context where fitting a model was time-intensive. But it was by no means a technically necessary restriction. The objective was to find the model with the lowest (most negative) LOOIC (relative fit metric) that also has an acceptable absolute fit to the data based on a $\chi^2$ test using a Mahalanobis distance between observed and predicted death count frequencies as the test statistic.

Model 1, referred to as the base model, is the simplest model on which all other models were constructed. It included the stratum sex and age group as fixed effects and no random effects. Its LOOIC was -149.0. We next tried including the other three observed county-level variables individually in models 2–4. Model 2, which included the rurality measure, showed the best improvement (LOOIC: -158.7). Model 3 using the proportion of the population that were persons of color (POC, hereafter) was a lesser improvement (LOOIC: -156.5). And model 4 that included the proportion of the population that has an income below 150% of the federal poverty level (poverty, hereafter) was barely an improvement over model 1 (LOOIC: -149.6). Model 5 included all three of these fixed effects together and it performed slightly worse (LOOIC: -158.4) than the model that added a rurality fixed effect alone—model 2.
We then explored whether random effects would benefit the model further. Model 6 included county random effects and was a substantial improvement (LOOIC: -174.5) over the previously-leading model 2. Models 7 and 8 built on model 6 by adding all three additional fixed effects or only rurality (as it was the most effective in earlier models), respectively. Both models performed similarly and slightly worse than the model with county random effects and no fixed effects beyond the base sex and age (i.e., model 6).

Because random effects seemed a valuable direction at this point in our process, we also explored giving each stratum its own independent parameter with an overarching hierarchical prior. We tried three different regularizing hierarchical priors—a Gaussian prior (model 9, LOOIC: -163.5), a horseshoe prior (model 10, LOOIC: -150.3), and a LASSO prior (model 11, LOOIC: -150.6). None of the three performed better than the less-complex model 6.

It appears as though increases in model complexity beyond that in model 6 did not provide a sufficient return in fit to the data to warrant preferring the additional complexity. We therefore confirmed absolute fit of model 6 to the data (as reported in the main manuscript) and selected it for use in calculating prevalence. We also calculated prevalence for all models in table 1 as a check that prevalence does not depend too substantially on model specification among models that fit reasonably well. In fact, among those models that had acceptable absolute fit, the difference between the minimum (13.6%) and maximum (14.2%) prevalence estimates was 0.6 percentage points.
Web Table 1: Comparison of a subset of models fit during the model selection process.

<table>
<thead>
<tr>
<th>Model Number</th>
<th>Model Descriptiona</th>
<th>LOOIC</th>
<th>Absolute Fit Test p values (p_o, p_u)</th>
<th>Overall Prevalence</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Base</td>
<td>-149.0</td>
<td>&lt; 0.001, 0.003</td>
<td>19.5</td>
</tr>
<tr>
<td>2</td>
<td>Base plus rurality FE</td>
<td>-158.7</td>
<td>&lt; 0.001, &lt; 0.001</td>
<td>16.7</td>
</tr>
<tr>
<td>3</td>
<td>Base plus POC FE</td>
<td>-156.5</td>
<td>&lt; 0.001, &lt; 0.001</td>
<td>15.6</td>
</tr>
<tr>
<td>4</td>
<td>Base plus poverty FE</td>
<td>-149.6</td>
<td>&lt; 0.001, 0.007</td>
<td>16.7</td>
</tr>
<tr>
<td>5</td>
<td>Base plus rurality, POC, poverty FEs</td>
<td>-158.4</td>
<td>0.001, &lt; 0.001</td>
<td>15.7</td>
</tr>
<tr>
<td>6</td>
<td>Base plus county REs</td>
<td>-174.5</td>
<td>0.231, 0.697</td>
<td>13.6</td>
</tr>
<tr>
<td>7</td>
<td>Base plus county REs, and rurality, POC, poverty FEs</td>
<td>-173.2</td>
<td>0.136, 0.613</td>
<td>13.9</td>
</tr>
<tr>
<td>8</td>
<td>Base plus county REs and rurality FE</td>
<td>-173.3</td>
<td>0.186, 0.626</td>
<td>14.0</td>
</tr>
<tr>
<td>9</td>
<td>One parameter per stratum: Gaussian hierarchical prior</td>
<td>-163.5</td>
<td>0.072, 0.828</td>
<td>14.2</td>
</tr>
<tr>
<td>10</td>
<td>One parameter per stratum: Horseshoe hierarchical prior</td>
<td>-150.3</td>
<td>&lt; 0.001, 0.023</td>
<td>17.7</td>
</tr>
<tr>
<td>11</td>
<td>One parameter per stratum: LASSO hierarchical prior</td>
<td>-150.6</td>
<td>&lt; 0.001, 0.025</td>
<td>17.6</td>
</tr>
</tbody>
</table>

Abbreviations. $p_o, p_u$: p-values reflecting fit test results for the observed-OUD ($o$) and unknown-OUD status ($u$) groups, respectively; Base: the model with age and sex fixed effects; FE: fixed effects; RE: random effects; POC: percent of county population that are persons of color.

aAll descriptions apply to death rate and prevalence models.