# Supplementary materials for Significance tests for analyzing gene expression data with small sample sizes

**Insha Ullah, Sudhir Paul, Zhenjie Hong & You-Gan Wang**

# 1. TABLES OF FALSE POSITIVE RATE FOR THE NORMALLY DISTRIBUTED DATA

TABLE 1

*Empirical false positive rate of the Welch's t-test and the MCT at the nominal levels* $\alpha = .01, .05, .10$. *The samples of size* $m = n = 4, 5, 8, 10, 25$ *are drawn from normal populations; the variance ratios* $\rho = \sigma_x^2/\sigma_y^2$ *varying from* $2^{-8}$ *to* $2^8$ *and* $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; *results based on 10,000 simulated samples.*

| | | | $\alpha = .01$ | | $\alpha = .05$ | | $\alpha = .10$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 1/256 | 0.011 | 0.013 | 0.052 | 0.053 | 0.107 | 0.109 |
| | 1/64 | 1/64 | 0.015 | 0.019 | 0.054 | 0.057 | 0.100 | 0.102 |
| | 1/16 | 1/16 | 0.014 | 0.016 | 0.055 | 0.060 | 0.106 | 0.110 |
| | 1/4 | 1/4 | 0.010 | 0.011 | 0.043 | 0.046 | 0.096 | 0.100 |
| 4 | 1 | 1 | 0.008 | 0.009 | 0.042 | 0.045 | 0.089 | 0.091 |
| | 4 | 4 | 0.010 | 0.013 | 0.045 | 0.048 | 0.099 | 0.103 |
| | 16 | 16 | 0.014 | 0.018 | 0.057 | 0.060 | 0.106 | 0.110 |
| | 64 | 64 | 0.014 | 0.018 | 0.055 | 0.060 | 0.103 | 0.106 |
| | 256 | 256 | 0.010 | 0.013 | 0.048 | 0.049 | 0.103 | 0.105 |
| | 1/256 | 1/256 | 0.010 | 0.011 | 0.051 | 0.052 | 0.103 | 0.103 |
| | 1/64 | 1/64 | 0.013 | 0.014 | 0.050 | 0.052 | 0.099 | 0.101 |
| | 1/16 | 1/16 | 0.014 | 0.016 | 0.054 | 0.056 | 0.102 | 0.104 |
| | 1/4 | 1/4 | 0.009 | 0.010 | 0.051 | 0.054 | 0.102 | 0.104 |
| 5 | 1 | 1 | 0.009 | 0.010 | 0.046 | 0.047 | 0.093 | 0.095 |
| | 4 | 4 | 0.012 | 0.014 | 0.049 | 0.051 | 0.097 | 0.099 |
| | 16 | 16 | 0.012 | 0.013 | 0.054 | 0.056 | 0.107 | 0.110 |
| | 64 | 64 | 0.013 | 0.015 | 0.052 | 0.054 | 0.106 | 0.107 |
| | 256 | 256 | 0.011 | 0.012 | 0.050 | 0.050 | 0.096 | 0.097 |
| | 1/256 | 1/256 | 0.009 | 0.009 | 0.053 | 0.052 | 0.101 | 0.100 |
| | 1/64 | 1/64 | 0.011 | 0.011 | 0.054 | 0.054 | 0.097 | 0.098 |
| | 1/16 | 1/16 | 0.010 | 0.011 | 0.050 | 0.050 | 0.104 | 0.104 |
| | 1/4 | 1/4 | 0.010 | 0.010 | 0.051 | 0.052 | 0.102 | 0.103 |
| 8 | 1 | 1 | 0.009 | 0.009 | 0.050 | 0.050 | 0.104 | 0.104 |
| | 4 | 4 | 0.010 | 0.010 | 0.053 | 0.054 | 0.096 | 0.097 |
| | 16 | 16 | 0.010 | 0.011 | 0.055 | 0.055 | 0.104 | 0.104 |
| | 64 | 64 | 0.010 | 0.011 | 0.051 | 0.051 | 0.101 | 0.102 |
| | 256 | 256 | 0.010 | 0.010 | 0.049 | 0.049 | 0.108 | 0.108 |
| | 1/256 | 1/256 | 0.012 | 0.012 | 0.051 | 0.051 | 0.101 | 0.101 |
| | 1/64 | 1/64 | 0.009 | 0.010 | 0.049 | 0.048 | 0.101 | 0.102 |
| | 1/16 | 1/16 | 0.010 | 0.010 | 0.055 | 0.055 | 0.100 | 0.101 |
| | 1/4 | 1/4 | 0.010 | 0.010 | 0.050 | 0.051 | 0.095 | 0.096 |
| 10 | 1 | 1 | 0.008 | 0.008 | 0.052 | 0.051 | 0.098 | 0.099 |
| | 4 | 4 | 0.011 | 0.011 | 0.047 | 0.047 | 0.099 | 0.100 |
| | 16 | 16 | 0.013 | 0.014 | 0.052 | 0.053 | 0.102 | 0.103 |
| | 64 | 64 | 0.009 | 0.009 | 0.046 | 0.047 | 0.103 | 0.103 |
| | 256 | 256 | 0.010 | 0.010 | 0.048 | 0.048 | 0.096 | 0.096 |
| | 1/256 | 1/256 | 0.010 | 0.010 | 0.047 | 0.047 | 0.102 | 0.102 |
| | 1/64 | 1/64 | 0.010 | 0.009 | 0.050 | 0.050 | 0.100 | 0.100 |
| | 1/16 | 1/16 | 0.011 | 0.011 | 0.051 | 0.051 | 0.102 | 0.102 |
| | 1/4 | 1/4 | 0.011 | 0.010 | 0.048 | 0.048 | 0.102 | 0.103 |
| 25 | 1 | 1 | 0.010 | 0.010 | 0.047 | 0.048 | 0.102 | 0.102 |
| | 4 | 4 | 0.012 | 0.012 | 0.052 | 0.052 | 0.098 | 0.099 |
| | 16 | 16 | 0.010 | 0.010 | 0.052 | 0.053 | 0.101 | 0.100 |
| | 64 | 64 | 0.010 | 0.010 | 0.049 | 0.049 | 0.100 | 0.100 |
| | 256 | 256 | 0.010 | 0.010 | 0.049 | 0.050 | 0.097 | 0.098 |

TABLE 2

*Empirical false positive rate of the Welch's t-test and the MCT at the nominal levels* $\alpha = .01, .05, .10$. *The samples are drawn from normal populations;* $(m, n) = (4, 5), (4, 8), (4, 10), (4, 25)$*; the variance ratios* $\rho = \sigma_x^2/\sigma_y^2$ *varying from* $2^{-8}$ *to* $2^8$ *and* $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$*; results based on 10,000 simulated samples.*

| | | | $\alpha = .01$ | | $\alpha = .05$ | | $\alpha = .10$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 4/1280 | 0.012 | 0.013 | 0.053 | 0.054 | 0.108 | 0.108 |
| | 1/64 | 4/320 | 0.012 | 0.014 | 0.054 | 0.056 | 0.101 | 0.103 |
| | 1/16 | 4/80 | 0.011 | 0.012 | 0.049 | 0.051 | 0.103 | 0.105 |
| | 1/4 | 4/20 | 0.008 | 0.008 | 0.048 | 0.051 | 0.094 | 0.096 |
| 5 | 1 | 4/5 | 0.007 | 0.008 | 0.044 | 0.046 | 0.095 | 0.098 |
| | 4 | 16/5 | 0.012 | 0.013 | 0.051 | 0.054 | 0.102 | 0.106 |
| | 16 | 64/5 | 0.014 | 0.018 | 0.057 | 0.062 | 0.101 | 0.107 |
| | 64 | 256/5 | 0.015 | 0.018 | 0.052 | 0.055 | 0.098 | 0.101 |
| | 256 | 1024/5 | 0.012 | 0.015 | 0.052 | 0.053 | 0.099 | 0.101 |
| | 1/256 | 4/2048 | 0.011 | 0.011 | 0.050 | 0.051 | 0.102 | 0.102 |
| | 1/64 | 4/512 | 0.010 | 0.010 | 0.051 | 0.051 | 0.099 | 0.099 |
| | 1/16 | 4/128 | 0.010 | 0.010 | 0.048 | 0.049 | 0.101 | 0.102 |
| | 1/4 | 4/32 | 0.008 | 0.009 | 0.048 | 0.048 | 0.095 | 0.095 |
| 8 | 1 | 4/8 | 0.010 | 0.011 | 0.050 | 0.052 | 0.097 | 0.100 |
| | 4 | 16/8 | 0.017 | 0.021 | 0.057 | 0.062 | 0.106 | 0.111 |
| | 16 | 64/8 | 0.018 | 0.021 | 0.055 | 0.061 | 0.111 | 0.116 |
| | 64 | 256/8 | 0.013 | 0.018 | 0.051 | 0.055 | 0.101 | 0.103 |
| | 256 | 1024/8 | 0.011 | 0.013 | 0.050 | 0.052 | 0.100 | 0.101 |
| | 1/256 | 4/2560 | 0.010 | 0.010 | 0.052 | 0.052 | 0.100 | 0.101 |
| | 1/64 | 4/640 | 0.010 | 0.010 | 0.050 | 0.050 | 0.098 | 0.098 |
| | 1/16 | 4/160 | 0.008 | 0.009 | 0.049 | 0.050 | 0.098 | 0.099 |
| | 1/4 | 4/40 | 0.009 | 0.009 | 0.043 | 0.043 | 0.099 | 0.101 |
| 10 | 1 | 4/10 | 0.013 | 0.015 | 0.057 | 0.060 | 0.106 | 0.110 |
| | 4 | 16/10 | 0.018 | 0.023 | 0.061 | 0.068 | 0.109 | 0.116 |
| | 16 | 64/10 | 0.016 | 0.023 | 0.053 | 0.059 | 0.102 | 0.107 |
| | 64 | 256/10 | 0.013 | 0.017 | 0.051 | 0.056 | 0.098 | 0.100 |
| | 256 | 1024/10 | 0.010 | 0.012 | 0.053 | 0.054 | 0.094 | 0.096 |
| | 1/256 | 4/6400 | 0.009 | 0.009 | 0.048 | 0.049 | 0.099 | 0.099 |
| | 1/64 | 4/1600 | 0.010 | 0.009 | 0.052 | 0.052 | 0.102 | 0.102 |
| | 1/16 | 4/400 | 0.010 | 0.010 | 0.048 | 0.049 | 0.098 | 0.099 |
| | 1/4 | 4/100 | 0.013 | 0.015 | 0.057 | 0.060 | 0.108 | 0.111 |
| 25 | 1 | 4/25 | 0.022 | 0.026 | 0.062 | 0.069 | 0.108 | 0.115 |
| | 4 | 16/25 | 0.020 | 0.026 | 0.058 | 0.066 | 0.106 | 0.112 |
| | 16 | 64/25 | 0.015 | 0.020 | 0.055 | 0.060 | 0.105 | 0.109 |
| | 64 | 256/25 | 0.014 | 0.016 | 0.052 | 0.054 | 0.102 | 0.103 |
| | 256 | 1024/25 | 0.011 | 0.013 | 0.052 | 0.053 | 0.097 | 0.098 |

TABLE 3

*Empirical false positive rate of the Welch's t-test and the MCT at the nominal levels* $\alpha = .01, .05, .10$. *The samples are drawn from normal populations;* $(m, n) = (5, 4), (5, 8), (5, 10), (5, 25)$*; the variance ratios* $\rho = \sigma_x^2/\sigma_y^2$ *varying from* $2^{-8}$ *to* $2^8$ *and* $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$*; results based on 10,000 simulated samples.*

| | | | $\alpha = .01$ | | $\alpha = .05$ | | $\alpha = .10$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 5/1024 | 0.012 | 0.015 | 0.052 | 0.054 | 0.102 | 0.103 |
| | 1/64 | 5/256 | 0.012 | 0.016 | 0.055 | 0.060 | 0.102 | 0.104 |
| | 1/16 | 5/64 | 0.014 | 0.017 | 0.060 | 0.064 | 0.111 | 0.115 |
| | 1/4 | 5/16 | 0.013 | 0.014 | 0.054 | 0.058 | 0.102 | 0.104 |
| 4 | 1 | 5/4 | 0.008 | 0.008 | 0.045 | 0.047 | 0.093 | 0.095 |
| | 4 | 20/4 | 0.008 | 0.009 | 0.045 | 0.048 | 0.094 | 0.096 |
| | 16 | 80/4 | 0.012 | 0.013 | 0.050 | 0.052 | 0.105 | 0.106 |
| | 64 | 320/4 | 0.011 | 0.013 | 0.055 | 0.057 | 0.099 | 0.101 |
| | 256 | 1280/4 | 0.013 | 0.014 | 0.050 | 0.050 | 0.104 | 0.105 |
| | 1/256 | 5/2048 | 0.010 | 0.010 | 0.051 | 0.051 | 0.102 | 0.103 |
| | 1/64 | 5/512 | 0.011 | 0.012 | 0.050 | 0.050 | 0.098 | 0.098 |
| | 1/16 | 5/128 | 0.011 | 0.011 | 0.048 | 0.047 | 0.096 | 0.098 |
| | 1/4 | 5/32 | 0.008 | 0.009 | 0.050 | 0.051 | 0.097 | 0.097 |
| 8 | 1 | 5/8 | 0.009 | 0.010 | 0.046 | 0.047 | 0.096 | 0.099 |
| | 4 | 20/8 | 0.013 | 0.015 | 0.057 | 0.059 | 0.104 | 0.106 |
| | 16 | 80/8 | 0.014 | 0.016 | 0.056 | 0.059 | 0.108 | 0.111 |
| | 64 | 320/8 | 0.011 | 0.013 | 0.048 | 0.050 | 0.109 | 0.110 |
| | 256 | 1280/8 | 0.012 | 0.013 | 0.053 | 0.054 | 0.097 | 0.097 |
| | 1/256 | 5/2560 | 0.012 | 0.013 | 0.052 | 0.052 | 0.100 | 0.099 |
| | 1/64 | 5/640 | 0.010 | 0.010 | 0.053 | 0.053 | 0.102 | 0.104 |
| | 1/16 | 5/160 | 0.012 | 0.012 | 0.051 | 0.052 | 0.095 | 0.096 |
| | 1/4 | 5/40 | 0.007 | 0.007 | 0.047 | 0.047 | 0.099 | 0.099 |
| 10 | 1 | 5/10 | 0.009 | 0.011 | 0.053 | 0.055 | 0.102 | 0.105 |
| | 4 | 20/10 | 0.015 | 0.017 | 0.058 | 0.061 | 0.103 | 0.107 |
| | 16 | 80/10 | 0.015 | 0.019 | 0.055 | 0.060 | 0.102 | 0.104 |
| | 64 | 320/10 | 0.014 | 0.016 | 0.046 | 0.049 | 0.098 | 0.099 |
| | 256 | 1280/10 | 0.010 | 0.011 | 0.048 | 0.048 | 0.102 | 0.103 |
| | 1/256 | 5/6400 | 0.008 | 0.008 | 0.054 | 0.054 | 0.101 | 0.101 |
| | 1/64 | 5/1600 | 0.008 | 0.007 | 0.050 | 0.050 | 0.097 | 0.097 |
| | 1/16 | 5/400 | 0.008 | 0.008 | 0.052 | 0.052 | 0.094 | 0.095 |
| | 1/4 | 5/100 | 0.010 | 0.011 | 0.054 | 0.055 | 0.098 | 0.100 |
| 25 | 1 | 5/25 | 0.018 | 0.020 | 0.058 | 0.062 | 0.113 | 0.116 |
| | 4 | 20/25 | 0.013 | 0.017 | 0.056 | 0.061 | 0.105 | 0.108 |
| | 16 | 80/25 | 0.010 | 0.013 | 0.053 | 0.055 | 0.106 | 0.108 |
| | 64 | 320/25 | 0.011 | 0.013 | 0.054 | 0.055 | 0.098 | 0.100 |
| | 256 | 1280/25 | 0.011 | 0.012 | 0.051 | 0.051 | 0.105 | 0.105 |

TABLE 4

*Empirical false positive rate of the Welch's t-test and the MCT at the nominal levels* $\alpha = .01, .05, .10$. *The samples are drawn from normal populations;* $(m, n) = (8, 4), (8, 5), (8, 10), (8, 25)$*; the variance ratios* $\rho = \sigma_x^2/\sigma_y^2$ *varying from* $2^{-8}$ *to* $2^8$ *and* $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$*; results based on 10,000 simulated samples.*

| | | | $\alpha = .01$ | | $\alpha = .05$ | | $\alpha = .10$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 8/1024 | 0.012 | 0.013 | 0.052 | 0.053 | 0.102 | 0.103 |
| | 1/64 | 8/256 | 0.012 | 0.017 | 0.051 | 0.056 | 0.100 | 0.104 |
| | 1/16 | 8/64 | 0.016 | 0.021 | 0.053 | 0.059 | 0.107 | 0.111 |
| | 1/4 | 8/16 | 0.017 | 0.021 | 0.060 | 0.065 | 0.108 | 0.114 |
| 4 | 1 | 8/4 | 0.010 | 0.011 | 0.054 | 0.057 | 0.098 | 0.100 |
| | 4 | 32/4 | 0.008 | 0.009 | 0.043 | 0.044 | 0.096 | 0.098 |
| | 16 | 128/4 | 0.010 | 0.010 | 0.046 | 0.046 | 0.102 | 0.102 |
| | 64 | 512/4 | 0.009 | 0.010 | 0.047 | 0.048 | 0.105 | 0.106 |
| | 256 | 2048/4 | 0.012 | 0.012 | 0.052 | 0.052 | 0.101 | 0.101 |
| | 1/256 | 8/1280 | 0.010 | 0.011 | 0.054 | 0.053 | 0.100 | 0.101 |
| | 1/64 | 8/320 | 0.012 | 0.014 | 0.053 | 0.055 | 0.097 | 0.099 |
| | 1/16 | 8/80 | 0.014 | 0.017 | 0.053 | 0.056 | 0.103 | 0.105 |
| | 1/4 | 8/20 | 0.014 | 0.016 | 0.050 | 0.052 | 0.108 | 0.111 |
| 5 | 1 | 8/5 | 0.009 | 0.010 | 0.050 | 0.051 | 0.100 | 0.102 |
| | 4 | 32/5 | 0.008 | 0.008 | 0.048 | 0.048 | 0.096 | 0.097 |
| | 16 | 128/5 | 0.010 | 0.010 | 0.052 | 0.053 | 0.100 | 0.100 |
| | 64 | 512/5 | 0.010 | 0.010 | 0.049 | 0.050 | 0.097 | 0.098 |
| | 256 | 2048/5 | 0.011 | 0.011 | 0.050 | 0.050 | 0.098 | 0.098 |
| | 1/256 | 8/2560 | 0.011 | 0.012 | 0.050 | 0.050 | 0.101 | 0.100 |
| | 1/64 | 8/640 | 0.010 | 0.010 | 0.055 | 0.055 | 0.103 | 0.103 |
| | 1/16 | 8/160 | 0.010 | 0.010 | 0.050 | 0.051 | 0.099 | 0.100 |
| | 1/4 | 8/40 | 0.012 | 0.012 | 0.052 | 0.052 | 0.096 | 0.097 |
| 10 | 1 | 8/10 | 0.008 | 0.008 | 0.047 | 0.047 | 0.102 | 0.103 |
| | 4 | 32/10 | 0.012 | 0.012 | 0.051 | 0.052 | 0.105 | 0.105 |
| | 16 | 128/10 | 0.010 | 0.011 | 0.047 | 0.049 | 0.097 | 0.098 |
| | 64 | 512/10 | 0.010 | 0.010 | 0.050 | 0.050 | 0.104 | 0.104 |
| | 256 | 2048/10 | 0.009 | 0.010 | 0.049 | 0.049 | 0.100 | 0.099 |
| | 1/256 | 8/6400 | 0.010 | 0.010 | 0.051 | 0.051 | 0.108 | 0.108 |
| | 1/64 | 8/1600 | 0.013 | 0.012 | 0.049 | 0.049 | 0.096 | 0.095 |
| | 1/16 | 8/400 | 0.010 | 0.010 | 0.051 | 0.052 | 0.103 | 0.103 |
| | 1/4 | 8/100 | 0.011 | 0.011 | 0.046 | 0.047 | 0.095 | 0.095 |
| 25 | 1 | 8/25 | 0.011 | 0.012 | 0.051 | 0.053 | 0.099 | 0.099 |
| | 4 | 32/25 | 0.012 | 0.013 | 0.052 | 0.053 | 0.103 | 0.104 |
| | 16 | 128/25 | 0.011 | 0.012 | 0.050 | 0.051 | 0.096 | 0.096 |
| | 64 | 512/25 | 0.011 | 0.011 | 0.050 | 0.050 | 0.102 | 0.102 |
| | 256 | 2048/25 | 0.011 | 0.011 | 0.051 | 0.051 | 0.103 | 0.104 |

TABLE 5

*Empirical false positive rate of the Welch's t-test and the MCT at the nominal levels* $\alpha = .01, .05, .10$. *The samples are drawn from normal populations;* $(m, n) = (10, 4), (10, 5), (10, 8), (10, 25)$*; the variance ratios* $\rho = \sigma_x^2/\sigma_y^2$ *varying from* $2^{-8}$ *to* $2^8$ *and* $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$*; results based on 10,000 simulated samples.*

| | | | $\alpha = .01$ | | $\alpha = .05$ | | $\alpha = .10$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 10/1024 | 0.013 | 0.015 | 0.050 | 0.052 | 0.106 | 0.106 |
| | 1/64 | 10/256 | 0.011 | 0.015 | 0.051 | 0.056 | 0.096 | 0.099 |
| | 1/16 | 10/64 | 0.018 | 0.024 | 0.055 | 0.063 | 0.110 | 0.115 |
| | 1/4 | 10/16 | 0.021 | 0.025 | 0.059 | 0.065 | 0.109 | 0.116 |
| 4 | 1 | 10/4 | 0.015 | 0.017 | 0.053 | 0.056 | 0.103 | 0.106 |
| | 4 | 40/4 | 0.009 | 0.010 | 0.045 | 0.046 | 0.091 | 0.092 |
| | 16 | 160/4 | 0.010 | 0.010 | 0.050 | 0.051 | 0.096 | 0.096 |
| | 64 | 640/4 | 0.010 | 0.010 | 0.051 | 0.051 | 0.100 | 0.100 |
| | 256 | 2560/4 | 0.010 | 0.011 | 0.053 | 0.053 | 0.100 | 0.099 |
| | 1/256 | 10/1280 | 0.009 | 0.010 | 0.051 | 0.052 | 0.100 | 0.100 |
| | 1/64 | 10/320 | 0.012 | 0.014 | 0.049 | 0.051 | 0.098 | 0.099 |
| | 1/16 | 10/80 | 0.014 | 0.017 | 0.052 | 0.055 | 0.094 | 0.096 |
| | 1/4 | 10/20 | 0.013 | 0.015 | 0.061 | 0.065 | 0.100 | 0.103 |
| 5 | 1 | 10/5 | 0.011 | 0.012 | 0.049 | 0.050 | 0.102 | 0.104 |
| | 4 | 40/5 | 0.009 | 0.009 | 0.046 | 0.047 | 0.098 | 0.100 |
| | 16 | 160/5 | 0.011 | 0.011 | 0.048 | 0.049 | 0.099 | 0.100 |
| | 64 | 640/5 | 0.010 | 0.010 | 0.048 | 0.049 | 0.100 | 0.102 |
| | 256 | 2560/5 | 0.009 | 0.010 | 0.052 | 0.052 | 0.100 | 0.100 |
| | 1/256 | 10/2048 | 0.010 | 0.011 | 0.053 | 0.053 | 0.100 | 0.099 |
| | 1/64 | 10/512 | 0.011 | 0.011 | 0.049 | 0.049 | 0.101 | 0.102 |
| | 1/16 | 10/128 | 0.010 | 0.010 | 0.047 | 0.048 | 0.104 | 0.105 |
| | 1/4 | 10/32 | 0.012 | 0.012 | 0.054 | 0.054 | 0.104 | 0.104 |
| 8 | 1 | 10/8 | 0.009 | 0.009 | 0.048 | 0.049 | 0.102 | 0.102 |
| | 4 | 40/10 | 0.012 | 0.013 | 0.049 | 0.049 | 0.104 | 0.104 |
| | 16 | 160/8 | 0.012 | 0.012 | 0.050 | 0.051 | 0.106 | 0.107 |
| | 64 | 640/8 | 0.010 | 0.011 | 0.049 | 0.050 | 0.106 | 0.107 |
| | 256 | 2560/8 | 0.008 | 0.008 | 0.050 | 0.050 | 0.097 | 0.097 |
| | 1/256 | 10/6400 | 0.011 | 0.011 | 0.050 | 0.051 | 0.095 | 0.095 |
| | 1/64 | 10/1600 | 0.011 | 0.011 | 0.049 | 0.049 | 0.096 | 0.096 |
| | 1/16 | 10/400 | 0.010 | 0.011 | 0.048 | 0.049 | 0.098 | 0.099 |
| | 1/4 | 10/100 | 0.010 | 0.010 | 0.048 | 0.048 | 0.099 | 0.099 |
| 25 | 1 | 10/25 | 0.012 | 0.013 | 0.053 | 0.054 | 0.101 | 0.102 |
| | 4 | 40/25 | 0.011 | 0.012 | 0.055 | 0.056 | 0.099 | 0.099 |
| | 16 | 160/25 | 0.010 | 0.011 | 0.049 | 0.049 | 0.099 | 0.100 |
| | 64 | 640/25 | 0.010 | 0.010 | 0.052 | 0.052 | 0.096 | 0.095 |
| | 256 | 2560/25 | 0.011 | 0.012 | 0.047 | 0.047 | 0.100 | 0.100 |

TABLE 6

*Empirical false positive rate of the Welch's t-test and the MCT at the nominal levels* $\alpha = .01, .05, .10$. *The samples are drawn from normal populations;* $(m, n) = (25, 4), (25, 5), (25, 8), (25, 10)$*; the variance ratios* $\rho = \sigma_x^2/\sigma_y^2$ *varying from* $2^{-8}$ *to* $2^8$ *and* $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$*; results based on 10,000 simulated samples.*

| | | | $\alpha = .01$ | | $\alpha = .05$ | | $\alpha = .10$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 25/1024 | 0.011 | 0.012 | 0.049 | 0.050 | 0.102 | 0.102 |
| | 1/64 | 25/256 | 0.011 | 0.014 | 0.050 | 0.052 | 0.101 | 0.102 |
| | 1/16 | 25/64 | 0.014 | 0.020 | 0.056 | 0.061 | 0.104 | 0.108 |
| | 1/4 | 25/16 | 0.021 | 0.028 | 0.060 | 0.066 | 0.104 | 0.111 |
| 4 | 1 | 25/4 | 0.019 | 0.026 | 0.059 | 0.066 | 0.106 | 0.112 |
| | 4 | 100/4 | 0.014 | 0.015 | 0.051 | 0.054 | 0.102 | 0.106 |
| | 16 | 400/4 | 0.012 | 0.013 | 0.049 | 0.049 | 0.100 | 0.102 |
| | 64 | 1600/4 | 0.011 | 0.011 | 0.045 | 0.046 | 0.104 | 0.104 |
| | 256 | 6400/4 | 0.010 | 0.010 | 0.053 | 0.053 | 0.102 | 0.101 |
| | 1/256 | 25/1280 | 0.009 | 0.010 | 0.048 | 0.048 | 0.098 | 0.098 |
| | 1/64 | 25/320 | 0.012 | 0.012 | 0.052 | 0.053 | 0.101 | 0.101 |
| | 1/16 | 25/80 | 0.014 | 0.017 | 0.054 | 0.056 | 0.100 | 0.102 |
| | 1/4 | 25/20 | 0.016 | 0.019 | 0.056 | 0.060 | 0.104 | 0.107 |
| 5 | 1 | 25/5 | 0.016 | 0.019 | 0.055 | 0.060 | 0.109 | 0.114 |
| | 4 | 100/5 | 0.011 | 0.012 | 0.050 | 0.052 | 0.102 | 0.103 |
| | 16 | 400/5 | 0.010 | 0.010 | 0.049 | 0.049 | 0.102 | 0.103 |
| | 64 | 1600/5 | 0.010 | 0.009 | 0.053 | 0.053 | 0.098 | 0.098 |
| | 256 | 6400/5 | 0.010 | 0.010 | 0.050 | 0.050 | 0.096 | 0.097 |
| | 1/256 | 25/2048 | 0.013 | 0.013 | 0.048 | 0.047 | 0.100 | 0.100 |
| | 1/64 | 25/512 | 0.011 | 0.011 | 0.046 | 0.047 | 0.097 | 0.097 |
| | 1/16 | 25/128 | 0.010 | 0.011 | 0.050 | 0.050 | 0.100 | 0.101 |
| | 1/4 | 25/32 | 0.011 | 0.012 | 0.055 | 0.056 | 0.093 | 0.094 |
| 8 | 1 | 25/8 | 0.012 | 0.013 | 0.055 | 0.056 | 0.101 | 0.103 |
| | 4 | 100/10 | 0.010 | 0.010 | 0.049 | 0.050 | 0.102 | 0.102 |
| | 16 | 400/8 | 0.009 | 0.009 | 0.048 | 0.048 | 0.096 | 0.096 |
| | 64 | 1600/8 | 0.009 | 0.010 | 0.050 | 0.051 | 0.101 | 0.102 |
| | 256 | 6400/8 | 0.009 | 0.009 | 0.052 | 0.052 | 0.103 | 0.103 |
| | 1/256 | 25/2560 | 0.009 | 0.009 | 0.051 | 0.051 | 0.102 | 0.102 |
| | 1/64 | 25/640 | 0.010 | 0.010 | 0.052 | 0.052 | 0.097 | 0.097 |
| | 1/16 | 25/160 | 0.009 | 0.009 | 0.048 | 0.048 | 0.101 | 0.102 |
| | 1/4 | 25/40 | 0.012 | 0.012 | 0.050 | 0.052 | 0.103 | 0.104 |
| 10 | 1 | 25/10 | 0.014 | 0.014 | 0.048 | 0.050 | 0.098 | 0.098 |
| | 4 | 100/10 | 0.010 | 0.010 | 0.050 | 0.050 | 0.105 | 0.105 |
| | 16 | 400/10 | 0.008 | 0.009 | 0.050 | 0.050 | 0.098 | 0.098 |
| | 64 | 1600/10 | 0.010 | 0.010 | 0.049 | 0.049 | 0.102 | 0.102 |
| | 256 | 6400/10 | 0.009 | 0.009 | 0.050 | 0.050 | 0.106 | 0.107 |

## 2. POWER TABLES FOR THE NORMALLY DISTRIBUTED DATA

TABLE 7

*Empirical power of the Welch's t-test and the MCT at the nominal levels $\alpha = .05$. The samples of size $m = n = 4, 5, 8, 10, 25$ are drawn from normal populations; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; the deviation from equality of means is obtained by $\delta = \eta\sqrt{\sigma_x^2/m + \sigma_y^2/n}$ with $\eta = 1, 2, 3$; results based on 10,000 simulated samples.*

| | | | $\eta = 1$ | | $\eta = 2$ | | $\eta = 3$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 1/256 | 0.112 | 0.116 | 0.292 | 0.296 | 0.542 | 0.547 |
| | 1/64 | 1/64 | 0.116 | 0.123 | 0.297 | 0.309 | 0.560 | 0.571 |
| | 1/16 | 1/16 | 0.125 | 0.135 | 0.320 | 0.336 | 0.580 | 0.598 |
| | 1/4 | 1/4 | 0.123 | 0.129 | 0.350 | 0.366 | 0.625 | 0.646 |
| 4 | 1 | 1 | 0.111 | 0.117 | 0.334 | 0.347 | 0.659 | 0.673 |
| | 4 | 4 | 0.122 | 0.128 | 0.336 | 0.349 | 0.629 | 0.650 |
| | 16 | 16 | 0.131 | 0.138 | 0.315 | 0.334 | 0.576 | 0.599 |
| | 64 | 64 | 0.125 | 0.131 | 0.305 | 0.316 | 0.548 | 0.562 |
| | 256 | 256 | 0.112 | 0.115 | 0.289 | 0.294 | 0.539 | 0.542 |
| | 1/256 | 1/256 | 0.124 | 0.125 | 0.344 | 0.346 | 0.618 | 0.618 |
| | 1/64 | 1/64 | 0.130 | 0.134 | 0.346 | 0.350 | 0.626 | 0.632 |
| | 1/16 | 1/16 | 0.132 | 0.138 | 0.350 | 0.360 | 0.650 | 0.660 |
| | 1/4 | 1/4 | 0.140 | 0.144 | 0.387 | 0.396 | 0.697 | 0.707 |
| 5 | 1 | 1 | 0.126 | 0.130 | 0.396 | 0.402 | 0.724 | 0.732 |
| | 4 | 4 | 0.136 | 0.140 | 0.377 | 0.387 | 0.689 | 0.700 |
| | 16 | 16 | 0.135 | 0.140 | 0.365 | 0.376 | 0.648 | 0.657 |
| | 64 | 64 | 0.122 | 0.124 | 0.345 | 0.350 | 0.624 | 0.628 |
| | 256 | 256 | 0.125 | 0.127 | 0.346 | 0.347 | 0.619 | 0.623 |
| | 1/256 | 1/256 | 0.136 | 0.135 | 0.404 | 0.405 | 0.728 | 0.728 |
| | 1/64 | 1/64 | 0.144 | 0.145 | 0.414 | 0.414 | 0.741 | 0.742 |
| | 1/16 | 1/16 | 0.147 | 0.149 | 0.420 | 0.424 | 0.746 | 0.747 |
| | 1/4 | 1/4 | 0.155 | 0.156 | 0.439 | 0.443 | 0.772 | 0.776 |
| 8 | 1 | 1 | 0.149 | 0.150 | 0.464 | 0.464 | 0.795 | 0.796 |
| | 4 | 4 | 0.148 | 0.151 | 0.438 | 0.439 | 0.771 | 0.773 |
| | 16 | 16 | 0.140 | 0.143 | 0.422 | 0.424 | 0.747 | 0.750 |
| | 64 | 64 | 0.143 | 0.144 | 0.416 | 0.417 | 0.740 | 0.739 |
| | 256 | 256 | 0.142 | 0.142 | 0.401 | 0.401 | 0.731 | 0.731 |
| | 1/256 | 1/256 | 0.143 | 0.143 | 0.427 | 0.426 | 0.752 | 0.752 |
| | 1/64 | 1/64 | 0.151 | 0.152 | 0.426 | 0.428 | 0.758 | 0.759 |
| | 1/16 | 1/16 | 0.148 | 0.150 | 0.435 | 0.437 | 0.771 | 0.772 |
| | 1/4 | 1/4 | 0.154 | 0.157 | 0.462 | 0.463 | 0.797 | 0.799 |
| 10 | 1 | 1 | 0.157 | 0.157 | 0.459 | 0.460 | 0.799 | 0.800 |
| | 4 | 4 | 0.154 | 0.155 | 0.460 | 0.461 | 0.789 | 0.790 |
| | 16 | 16 | 0.150 | 0.151 | 0.447 | 0.448 | 0.775 | 0.776 |
| | 64 | 64 | 0.152 | 0.154 | 0.446 | 0.446 | 0.767 | 0.767 |
| | 256 | 256 | 0.148 | 0.148 | 0.427 | 0.427 | 0.764 | 0.764 |
| | 1/256 | 1/256 | 0.159 | 0.158 | 0.484 | 0.483 | 0.823 | 0.822 |
| | 1/64 | 1/64 | 0.161 | 0.161 | 0.482 | 0.481 | 0.822 | 0.821 |
| | 1/16 | 1/16 | 0.160 | 0.160 | 0.488 | 0.487 | 0.827 | 0.827 |
| | 1/4 | 1/4 | 0.162 | 0.163 | 0.496 | 0.496 | 0.824 | 0.825 |
| 25 | 1 | 1 | 0.166 | 0.166 | 0.507 | 0.506 | 0.831 | 0.831 |
| | 4 | 4 | 0.165 | 0.166 | 0.498 | 0.499 | 0.840 | 0.841 |
| | 16 | 16 | 0.163 | 0.163 | 0.483 | 0.483 | 0.820 | 0.820 |
| | 64 | 64 | 0.163 | 0.164 | 0.488 | 0.488 | 0.815 | 0.815 |
| | 256 | 256 | 0.164 | 0.164 | 0.490 | 0.491 | 0.820 | 0.820 |

TABLE 8

*Empirical power of the Welch's t-test and the MCT at the nominal levels $\alpha = .05$. The samples are drawn from normal populations; $(m, n) = (4, 5), (4, 8), (4, 10), (4, 25)$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; the deviation from equality of means is obtained by $\delta = \eta\sqrt{\sigma_x^2/m + \sigma_y^2/n}$ with $\eta = 1, 2, 3$; results based on 10,000 simulated samples.*

| | | | $\eta = 1$ | | $\eta = 2$ | | $\eta = 3$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 4/1280 | 0.121 | 0.123 | 0.343 | 0.345 | 0.623 | 0.625 |
| | 1/64 | 4/320 | 0.126 | 0.129 | 0.352 | 0.358 | 0.632 | 0.637 |
| | 1/16 | 4/80 | 0.137 | 0.142 | 0.357 | 0.367 | 0.654 | 0.664 |
| | 1/4 | 4/20 | 0.133 | 0.137 | 0.365 | 0.372 | 0.680 | 0.690 |
| 5 | 1 | 4/5 | 0.122 | 0.128 | 0.378 | 0.389 | 0.690 | 0.702 |
| | 4 | 16/5 | 0.134 | 0.143 | 0.353 | 0.373 | 0.622 | 0.645 |
| | 16 | 64/5 | 0.123 | 0.132 | 0.325 | 0.343 | 0.568 | 0.590 |
| | 64 | 256/5 | 0.122 | 0.128 | 0.296 | 0.306 | 0.545 | 0.558 |
| | 256 | 1024/5 | 0.117 | 0.120 | 0.285 | 0.288 | 0.531 | 0.535 |
| | 1/256 | 4/2048 | 0.142 | 0.142 | 0.409 | 0.410 | 0.730 | 0.731 |
| | 1/64 | 4/512 | 0.142 | 0.142 | 0.418 | 0.421 | 0.743 | 0.743 |
| | 1/16 | 4/128 | 0.143 | 0.144 | 0.426 | 0.427 | 0.756 | 0.758 |
| | 1/4 | 4/32 | 0.136 | 0.138 | 0.418 | 0.422 | 0.755 | 0.760 |
| 8 | 1 | 4/8 | 0.142 | 0.148 | 0.381 | 0.399 | 0.695 | 0.717 |
| | 4 | 16/8 | 0.138 | 0.150 | 0.341 | 0.366 | 0.611 | 0.644 |
| | 16 | 64/8 | 0.129 | 0.140 | 0.316 | 0.335 | 0.559 | 0.580 |
| | 64 | 256/8 | 0.112 | 0.119 | 0.297 | 0.306 | 0.544 | 0.551 |
| | 256 | 1024/8 | 0.115 | 0.117 | 0.286 | 0.290 | 0.532 | 0.536 |
| | 1/256 | 4/2560 | 0.144 | 0.144 | 0.434 | 0.433 | 0.761 | 0.760 |
| | 1/64 | 4/640 | 0.152 | 0.153 | 0.428 | 0.430 | 0.767 | 0.768 |
| | 1/16 | 4/160 | 0.144 | 0.144 | 0.440 | 0.441 | 0.770 | 0.771 |
| | 1/4 | 4/40 | 0.136 | 0.140 | 0.424 | 0.432 | 0.760 | 0.765 |
| 10 | 1 | 4/10 | 0.140 | 0.148 | 0.399 | 0.421 | 0.682 | 0.706 |
| | 4 | 16/10 | 0.137 | 0.151 | 0.338 | 0.364 | 0.594 | 0.628 |
| | 16 | 64/10 | 0.124 | 0.134 | 0.313 | 0.333 | 0.545 | 0.564 |
| | 64 | 256/10 | 0.119 | 0.124 | 0.293 | 0.301 | 0.542 | 0.550 |
| | 256 | 1024/10 | 0.109 | 0.112 | 0.295 | 0.299 | 0.536 | 0.539 |
| | 1/256 | 4/6400 | 0.156 | 0.156 | 0.494 | 0.494 | 0.824 | 0.824 |
| | 1/64 | 4/1600 | 0.158 | 0.158 | 0.488 | 0.488 | 0.814 | 0.814 |
| | 1/16 | 4/400 | 0.161 | 0.164 | 0.468 | 0.472 | 0.797 | 0.803 |
| | 1/4 | 4/100 | 0.155 | 0.162 | 0.424 | 0.446 | 0.732 | 0.754 |
| 25 | 1 | 4/25 | 0.146 | 0.161 | 0.367 | 0.398 | 0.623 | 0.664 |
| | 4 | 16/25 | 0.128 | 0.140 | 0.322 | 0.348 | 0.566 | 0.595 |
| | 16 | 64/25 | 0.120 | 0.128 | 0.303 | 0.314 | 0.545 | 0.556 |
| | 64 | 256/25 | 0.112 | 0.114 | 0.281 | 0.286 | 0.544 | 0.548 |
| | 256 | 1024/25 | 0.115 | 0.115 | 0.286 | 0.286 | 0.540 | 0.542 |

TABLE 9

*Empirical power of the Welch's t-test and the MCT at the nominal levels $\alpha = .05$. The samples are drawn from normal populations; $(m, n) = (5, 4), (5, 8), (5, 10), (5, 25)$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; the deviation from equality of means is obtained by $\delta = \eta\sqrt{\sigma_x^2/m + \sigma_y^2/n}$ with $\eta = 1, 2, 3$; results based on 10,000 simulated samples.*

| | | | $\eta = 1$ | | $\eta = 2$ | | $\eta = 3$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 5/1024 | 0.108 | 0.113 | 0.289 | 0.292 | 0.538 | 0.542 |
| | 1/64 | 5/256 | 0.123 | 0.129 | 0.300 | 0.310 | 0.546 | 0.558 |
| | 1/16 | 5/64 | 0.129 | 0.139 | 0.313 | 0.333 | 0.566 | 0.587 |
| | 1/4 | 5/16 | 0.131 | 0.140 | 0.350 | 0.371 | 0.623 | 0.648 |
| 4 | 1 | 5/4 | 0.131 | 0.135 | 0.368 | 0.378 | 0.688 | 0.700 |
| | 4 | 20/4 | 0.122 | 0.126 | 0.373 | 0.382 | 0.688 | 0.697 |
| | 16 | 80/4 | 0.125 | 0.129 | 0.359 | 0.369 | 0.644 | 0.656 |
| | 64 | 320/4 | 0.125 | 0.129 | 0.338 | 0.345 | 0.627 | 0.632 |
| | 256 | 1280/4 | 0.120 | 0.122 | 0.331 | 0.334 | 0.625 | 0.627 |
| | 1/256 | 5/2048 | 0.138 | 0.139 | 0.417 | 0.418 | 0.734 | 0.734 |
| | 1/64 | 5/512 | 0.138 | 0.139 | 0.416 | 0.421 | 0.742 | 0.742 |
| | 1/16 | 5/128 | 0.141 | 0.142 | 0.429 | 0.431 | 0.743 | 0.747 |
| | 1/4 | 5/32 | 0.146 | 0.148 | 0.436 | 0.438 | 0.765 | 0.767 |
| 8 | 1 | 5/8 | 0.143 | 0.147 | 0.425 | 0.433 | 0.739 | 0.746 |
| | 4 | 20/8 | 0.135 | 0.142 | 0.384 | 0.398 | 0.674 | 0.691 |
| | 16 | 80/8 | 0.132 | 0.138 | 0.354 | 0.365 | 0.641 | 0.651 |
| | 64 | 320/8 | 0.128 | 0.131 | 0.344 | 0.350 | 0.625 | 0.629 |
| | 256 | 1280/8 | 0.126 | 0.127 | 0.338 | 0.339 | 0.619 | 0.621 |
| | 1/256 | 5/2560 | 0.151 | 0.151 | 0.436 | 0.436 | 0.770 | 0.771 |
| | 1/64 | 5/640 | 0.148 | 0.147 | 0.438 | 0.439 | 0.763 | 0.764 |
| | 1/16 | 5/160 | 0.150 | 0.150 | 0.440 | 0.442 | 0.780 | 0.781 |
| | 1/4 | 5/40 | 0.146 | 0.146 | 0.445 | 0.448 | 0.784 | 0.785 |
| 10 | 1 | 5/10 | 0.148 | 0.152 | 0.417 | 0.427 | 0.733 | 0.745 |
| | 4 | 20/10 | 0.135 | 0.142 | 0.380 | 0.394 | 0.680 | 0.695 |
| | 16 | 80/10 | 0.130 | 0.137 | 0.356 | 0.366 | 0.632 | 0.642 |
| | 64 | 320/10 | 0.126 | 0.129 | 0.339 | 0.343 | 0.632 | 0.634 |
| | 256 | 1280/10 | 0.126 | 0.127 | 0.328 | 0.328 | 0.626 | 0.627 |
| | 1/256 | 5/6400 | 0.163 | 0.163 | 0.494 | 0.493 | 0.816 | 0.815 |
| | 1/64 | 5/1600 | 0.160 | 0.161 | 0.486 | 0.484 | 0.820 | 0.820 |
| | 1/16 | 5/400 | 0.156 | 0.157 | 0.482 | 0.484 | 0.815 | 0.817 |
| | 1/4 | 5/100 | 0.157 | 0.162 | 0.444 | 0.456 | 0.770 | 0.781 |
| 25 | 1 | 5/25 | 0.146 | 0.156 | 0.400 | 0.418 | 0.701 | 0.720 |
| | 4 | 20/25 | 0.136 | 0.145 | 0.356 | 0.367 | 0.646 | 0.658 |
| | 16 | 80/25 | 0.130 | 0.134 | 0.347 | 0.355 | 0.624 | 0.629 |
| | 64 | 320/25 | 0.118 | 0.120 | 0.344 | 0.347 | 0.618 | 0.619 |
| | 256 | 1280/25 | 0.113 | 0.113 | 0.333 | 0.334 | 0.613 | 0.614 |

TABLE 10

*Empirical power of the Welch's t-test and the MCT at the nominal levels $\alpha = .05$. The samples are drawn from normal populations; $(m, n) = (8, 4), (8, 5), (8, 10), (8, 25)$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; the deviation from equality of means is obtained by $\delta = \eta\sqrt{\sigma_x^2/m + \sigma_y^2/n}$ with $\eta = 1, 2, 3$; results based on 10,000 simulated samples.*

| | | | $\eta = 1$ | | $\eta = 2$ | | $\eta = 3$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 8/1024 | 0.110 | 0.113 | 0.296 | 0.297 | 0.530 | 0.533 |
| | 1/64 | 8/256 | 0.115 | 0.122 | 0.299 | 0.307 | 0.542 | 0.549 |
| | 1/16 | 8/64 | 0.125 | 0.137 | 0.308 | 0.326 | 0.562 | 0.583 |
| | 1/4 | 8/16 | 0.138 | 0.148 | 0.346 | 0.374 | 0.602 | 0.633 |
| 4 | 1 | 8/4 | 0.138 | 0.144 | 0.388 | 0.406 | 0.685 | 0.706 |
| | 4 | 32/4 | 0.141 | 0.143 | 0.417 | 0.423 | 0.757 | 0.760 |
| | 16 | 128/4 | 0.145 | 0.146 | 0.430 | 0.431 | 0.748 | 0.750 |
| | 64 | 512/4 | 0.142 | 0.144 | 0.410 | 0.412 | 0.733 | 0.735 |
| | 256 | 2048/4 | 0.138 | 0.139 | 0.407 | 0.408 | 0.736 | 0.736 |
| | 1/256 | 8/1280 | 0.126 | 0.125 | 0.345 | 0.346 | 0.622 | 0.621 |
| | 1/64 | 8/320 | 0.122 | 0.125 | 0.346 | 0.349 | 0.627 | 0.631 |
| | 1/16 | 8/80 | 0.130 | 0.136 | 0.361 | 0.370 | 0.637 | 0.647 |
| | 1/4 | 8/20 | 0.144 | 0.150 | 0.380 | 0.392 | 0.673 | 0.687 |
| 5 | 1 | 8/5 | 0.134 | 0.138 | 0.404 | 0.413 | 0.746 | 0.754 |
| | 4 | 32/5 | 0.143 | 0.144 | 0.436 | 0.439 | 0.769 | 0.772 |
| | 16 | 128/5 | 0.137 | 0.139 | 0.433 | 0.435 | 0.751 | 0.753 |
| | 64 | 512/5 | 0.144 | 0.145 | 0.421 | 0.423 | 0.734 | 0.736 |
| | 256 | 2048/5 | 0.142 | 0.142 | 0.418 | 0.418 | 0.731 | 0.732 |
| | 1/256 | 8/2560 | 0.143 | 0.143 | 0.433 | 0.433 | 0.755 | 0.754 |
| | 1/64 | 8/640 | 0.148 | 0.148 | 0.435 | 0.437 | 0.759 | 0.758 |
| | 1/16 | 8/160 | 0.152 | 0.153 | 0.440 | 0.441 | 0.772 | 0.775 |
| | 1/4 | 8/40 | 0.149 | 0.150 | 0.450 | 0.450 | 0.789 | 0.789 |
| 10 | 1 | 8/10 | 0.148 | 0.149 | 0.454 | 0.457 | 0.796 | 0.797 |
| | 4 | 32/10 | 0.156 | 0.159 | 0.438 | 0.442 | 0.775 | 0.779 |
| | 16 | 128/10 | 0.152 | 0.154 | 0.426 | 0.429 | 0.750 | 0.752 |
| | 64 | 512/10 | 0.146 | 0.145 | 0.412 | 0.412 | 0.734 | 0.735 |
| | 256 | 2048/10 | 0.140 | 0.141 | 0.418 | 0.419 | 0.736 | 0.736 |
| | 1/256 | 8/6400 | 0.161 | 0.161 | 0.487 | 0.488 | 0.824 | 0.824 |
| | 1/64 | 8/1600 | 0.161 | 0.161 | 0.483 | 0.483 | 0.818 | 0.816 |
| | 1/16 | 8/400 | 0.159 | 0.159 | 0.491 | 0.491 | 0.823 | 0.823 |
| | 1/4 | 8/100 | 0.159 | 0.159 | 0.479 | 0.481 | 0.822 | 0.822 |
| 25 | 1 | 8/25 | 0.156 | 0.160 | 0.455 | 0.460 | 0.781 | 0.788 |
| | 4 | 32/25 | 0.141 | 0.145 | 0.429 | 0.434 | 0.752 | 0.756 |
| | 16 | 128/25 | 0.136 | 0.137 | 0.403 | 0.405 | 0.732 | 0.733 |
| | 64 | 512/25 | 0.138 | 0.137 | 0.405 | 0.406 | 0.732 | 0.732 |
| | 256 | 2048/25 | 0.138 | 0.138 | 0.409 | 0.408 | 0.734 | 0.734 |

TABLE 11

*Empirical power of the Welch's t-test and the MCT at the nominal levels $\alpha = .05$. The samples are drawn from normal populations; $(m, n) = (10, 4), (10, 5), (10, 8), (10, 25)$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; the deviation from equality of means is obtained by $\delta = \eta\sqrt{\sigma_x^2/m + \sigma_y^2/n}$ with $\eta = 1, 2, 3$; results based on 10,000 simulated samples.*

| | | | $\eta = 1$ | | $\eta = 2$ | | $\eta = 3$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 10/1024 | 0.113 | 0.116 | 0.288 | 0.290 | 0.529 | 0.532 |
| | 1/64 | 10/256 | 0.114 | 0.118 | 0.290 | 0.298 | 0.531 | 0.540 |
| | 1/16 | 10/64 | 0.119 | 0.130 | 0.306 | 0.323 | 0.554 | 0.572 |
| | 1/4 | 10/16 | 0.135 | 0.147 | 0.344 | 0.372 | 0.592 | 0.624 |
| 4 | 1 | 10/4 | 0.142 | 0.150 | 0.395 | 0.416 | 0.685 | 0.711 |
| | 4 | 40/4 | 0.142 | 0.146 | 0.425 | 0.432 | 0.768 | 0.776 |
| | 16 | 160/4 | 0.149 | 0.148 | 0.438 | 0.441 | 0.778 | 0.778 |
| | 64 | 640/4 | 0.146 | 0.146 | 0.432 | 0.433 | 0.771 | 0.770 |
| | 256 | 2560/4 | 0.143 | 0.143 | 0.431 | 0.430 | 0.760 | 0.760 |
| | 1/256 | 10/1280 | 0.124 | 0.125 | 0.334 | 0.332 | 0.618 | 0.619 |
| | 1/64 | 10/320 | 0.124 | 0.127 | 0.336 | 0.341 | 0.624 | 0.628 |
| | 1/16 | 10/80 | 0.132 | 0.138 | 0.347 | 0.358 | 0.641 | 0.648 |
| | 1/4 | 10/20 | 0.141 | 0.148 | 0.390 | 0.406 | 0.678 | 0.695 |
| 5 | 1 | 10/5 | 0.145 | 0.148 | 0.422 | 0.431 | 0.748 | 0.758 |
| | 4 | 40/5 | 0.144 | 0.145 | 0.435 | 0.438 | 0.780 | 0.782 |
| | 16 | 160/5 | 0.148 | 0.148 | 0.444 | 0.445 | 0.778 | 0.779 |
| | 64 | 640/5 | 0.149 | 0.150 | 0.437 | 0.439 | 0.766 | 0.767 |
| | 256 | 2560/5 | 0.144 | 0.144 | 0.428 | 0.428 | 0.766 | 0.766 |
| | 1/256 | 10/2048 | 0.141 | 0.141 | 0.397 | 0.398 | 0.737 | 0.735 |
| | 1/64 | 10/512 | 0.145 | 0.146 | 0.413 | 0.415 | 0.736 | 0.737 |
| | 1/16 | 10/128 | 0.144 | 0.146 | 0.432 | 0.436 | 0.745 | 0.746 |
| | 1/4 | 10/32 | 0.146 | 0.149 | 0.442 | 0.446 | 0.767 | 0.771 |
| 8 | 1 | 10/8 | 0.157 | 0.158 | 0.463 | 0.465 | 0.801 | 0.802 |
| | 4 | 40/10 | 0.156 | 0.157 | 0.459 | 0.461 | 0.790 | 0.792 |
| | 16 | 160/8 | 0.153 | 0.153 | 0.438 | 0.442 | 0.770 | 0.771 |
| | 64 | 640/8 | 0.147 | 0.148 | 0.433 | 0.434 | 0.756 | 0.757 |
| | 256 | 2560/8 | 0.145 | 0.145 | 0.434 | 0.434 | 0.757 | 0.757 |
| | 1/256 | 10/6400 | 0.160 | 0.159 | 0.482 | 0.483 | 0.823 | 0.821 |
| | 1/64 | 10/1600 | 0.167 | 0.168 | 0.485 | 0.484 | 0.823 | 0.823 |
| | 1/16 | 10/400 | 0.165 | 0.165 | 0.492 | 0.490 | 0.829 | 0.828 |
| | 1/4 | 10/100 | 0.160 | 0.160 | 0.482 | 0.482 | 0.828 | 0.828 |
| 25 | 1 | 10/25 | 0.159 | 0.161 | 0.473 | 0.473 | 0.802 | 0.805 |
| | 4 | 40/25 | 0.151 | 0.153 | 0.451 | 0.454 | 0.780 | 0.782 |
| | 16 | 160/25 | 0.145 | 0.145 | 0.438 | 0.439 | 0.767 | 0.768 |
| | 64 | 640/25 | 0.148 | 0.148 | 0.423 | 0.423 | 0.757 | 0.757 |
| | 256 | 2560/25 | 0.145 | 0.144 | 0.433 | 0.434 | 0.769 | 0.768 |

TABLE 12

*Empirical power of the Welch's t-test and the MCT at the nominal levels $\alpha = .05$. The samples are drawn from normal populations; $(m, n) = (25, 4), (25, 5), (25, 8), (25, 10)$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; the deviation from equality of means is obtained by $\delta = \eta\sqrt{\sigma_x^2/m + \sigma_y^2/n}$ with $\eta = 1, 2, 3$; results based on 10,000 simulated samples.*

| | | | $\eta = 1$ | | $\eta = 2$ | | $\eta = 3$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 25/1024 | 0.111 | 0.112 | 0.281 | 0.283 | 0.538 | 0.540 |
| | 1/64 | 25/256 | 0.115 | 0.117 | 0.293 | 0.297 | 0.535 | 0.540 |
| | 1/16 | 25/64 | 0.117 | 0.125 | 0.299 | 0.311 | 0.536 | 0.548 |
| | 1/4 | 25/16 | 0.131 | 0.148 | 0.317 | 0.340 | 0.571 | 0.598 |
| 4 | 1 | 25/4 | 0.144 | 0.159 | 0.367 | 0.397 | 0.630 | 0.665 |
| | 4 | 100/4 | 0.158 | 0.168 | 0.435 | 0.455 | 0.720 | 0.744 |
| | 16 | 400/4 | 0.160 | 0.161 | 0.472 | 0.477 | 0.810 | 0.814 |
| | 64 | 1600/4 | 0.162 | 0.162 | 0.484 | 0.485 | 0.823 | 0.823 |
| | 256 | 6400/4 | 0.158 | 0.159 | 0.486 | 0.486 | 0.826 | 0.826 |
| | 1/256 | 25/1280 | 0.121 | 0.121 | 0.338 | 0.338 | 0.615 | 0.614 |
| | 1/64 | 25/320 | 0.122 | 0.124 | 0.336 | 0.339 | 0.617 | 0.619 |
| | 1/16 | 25/80 | 0.129 | 0.133 | 0.338 | 0.346 | 0.628 | 0.635 |
| | 1/4 | 25/20 | 0.134 | 0.142 | 0.348 | 0.360 | 0.643 | 0.658 |
| 5 | 1 | 25/5 | 0.153 | 0.160 | 0.403 | 0.422 | 0.695 | 0.714 |
| | 4 | 100/5 | 0.151 | 0.156 | 0.447 | 0.456 | 0.783 | 0.794 |
| | 16 | 400/5 | 0.153 | 0.155 | 0.477 | 0.478 | 0.820 | 0.821 |
| | 64 | 1600/5 | 0.164 | 0.164 | 0.483 | 0.484 | 0.822 | 0.823 |
| | 256 | 6400/5 | 0.162 | 0.161 | 0.489 | 0.489 | 0.830 | 0.829 |
| | 1/256 | 25/2048 | 0.130 | 0.131 | 0.400 | 0.401 | 0.733 | 0.733 |
| | 1/64 | 25/512 | 0.140 | 0.140 | 0.420 | 0.421 | 0.730 | 0.729 |
| | 1/16 | 25/128 | 0.142 | 0.143 | 0.409 | 0.412 | 0.733 | 0.734 |
| | 1/4 | 25/32 | 0.149 | 0.151 | 0.424 | 0.428 | 0.748 | 0.752 |
| 8 | 1 | 25/8 | 0.154 | 0.155 | 0.459 | 0.467 | 0.782 | 0.786 |
| | 4 | 100/10 | 0.161 | 0.162 | 0.486 | 0.489 | 0.818 | 0.821 |
| | 16 | 400/8 | 0.162 | 0.162 | 0.481 | 0.480 | 0.823 | 0.822 |
| | 64 | 1600/8 | 0.160 | 0.160 | 0.486 | 0.485 | 0.822 | 0.822 |
| | 256 | 6400/8 | 0.163 | 0.163 | 0.481 | 0.480 | 0.827 | 0.827 |
| | 1/256 | 25/2560 | 0.145 | 0.147 | 0.429 | 0.428 | 0.768 | 0.769 |
| | 1/64 | 25/640 | 0.147 | 0.147 | 0.441 | 0.442 | 0.761 | 0.762 |
| | 1/16 | 25/160 | 0.150 | 0.151 | 0.445 | 0.447 | 0.764 | 0.765 |
| | 1/4 | 25/40 | 0.156 | 0.158 | 0.441 | 0.444 | 0.772 | 0.774 |
| 10 | 1 | 25/10 | 0.162 | 0.164 | 0.474 | 0.477 | 0.797 | 0.799 |
| | 4 | 100/10 | 0.156 | 0.156 | 0.495 | 0.495 | 0.832 | 0.832 |
| | 16 | 400/10 | 0.172 | 0.172 | 0.486 | 0.486 | 0.828 | 0.828 |
| | 64 | 1600/10 | 0.159 | 0.158 | 0.490 | 0.490 | 0.827 | 0.827 |
| | 256 | 6400/10 | 0.156 | 0.157 | 0.486 | 0.488 | 0.829 | 0.828 |

## 3. TABLES OF FALSE POSITIVE RATE FOR DATA SIMULATED FROM A $t$-DISTRIBUTION

TABLE 13

*Empirical size of the Welch's t-test and the MCT at the nominal levels $\alpha = .01, .05, .10$. The samples are drawn from a t-distribution with 5 degrees of freedom; $m = n = 4, 5, 8, 10, 25$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; results based on 10,000 simulated samples.*

| | | | $\alpha = .01$ | | $\alpha = .05$ | | $\alpha = .10$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 1/256 | 0.008 | 0.012 | 0.048 | 0.050 | 0.092 | 0.094 |
| | 1/64 | 1/64 | 0.010 | 0.013 | 0.044 | 0.048 | 0.092 | 0.096 |
| | 1/16 | 1/16 | 0.010 | 0.012 | 0.048 | 0.052 | 0.093 | 0.098 |
| | 1/4 | 1/4 | 0.006 | 0.008 | 0.041 | 0.044 | 0.085 | 0.089 |
| 4 | 1 | 1 | 0.006 | 0.007 | 0.035 | 0.038 | 0.080 | 0.082 |
| | 4 | 4 | 0.006 | 0.007 | 0.037 | 0.040 | 0.086 | 0.089 |
| | 16 | 16 | 0.011 | 0.013 | 0.045 | 0.050 | 0.092 | 0.095 |
| | 64 | 64 | 0.012 | 0.014 | 0.046 | 0.049 | 0.092 | 0.094 |
| | 256 | 256 | 0.010 | 0.012 | 0.042 | 0.044 | 0.087 | 0.088 |
| | 1/256 | 1/256 | 0.008 | 0.010 | 0.041 | 0.043 | 0.092 | 0.094 |
| | 1/64 | 1/64 | 0.010 | 0.011 | 0.046 | 0.047 | 0.093 | 0.095 |
| | 1/16 | 1/16 | 0.010 | 0.012 | 0.045 | 0.047 | 0.096 | 0.098 |
| | 1/4 | 1/4 | 0.006 | 0.006 | 0.046 | 0.048 | 0.092 | 0.093 |
| 5 | 1 | 1 | 0.007 | 0.007 | 0.036 | 0.038 | 0.090 | 0.091 |
| | 4 | 4 | 0.007 | 0.008 | 0.042 | 0.044 | 0.090 | 0.090 |
| | 16 | 16 | 0.009 | 0.010 | 0.047 | 0.050 | 0.099 | 0.100 |
| | 64 | 64 | 0.009 | 0.011 | 0.046 | 0.048 | 0.094 | 0.096 |
| | 256 | 256 | 0.009 | 0.010 | 0.042 | 0.043 | 0.093 | 0.094 |
| | 1/256 | 1/256 | 0.007 | 0.008 | 0.044 | 0.044 | 0.096 | 0.097 |
| | 1/64 | 1/64 | 0.009 | 0.010 | 0.045 | 0.046 | 0.095 | 0.096 |
| | 1/16 | 1/16 | 0.010 | 0.010 | 0.049 | 0.050 | 0.100 | 0.100 |
| | 1/4 | 1/4 | 0.009 | 0.009 | 0.046 | 0.048 | 0.094 | 0.095 |
| 8 | 1 | 1 | 0.008 | 0.008 | 0.045 | 0.045 | 0.095 | 0.096 |
| | 4 | 4 | 0.008 | 0.008 | 0.046 | 0.046 | 0.102 | 0.103 |
| | 16 | 16 | 0.009 | 0.010 | 0.045 | 0.045 | 0.096 | 0.097 |
| | 64 | 64 | 0.009 | 0.010 | 0.044 | 0.044 | 0.097 | 0.098 |
| | 256 | 256 | 0.008 | 0.008 | 0.044 | 0.045 | 0.095 | 0.096 |
| | 1/256 | 1/256 | 0.008 | 0.008 | 0.048 | 0.047 | 0.097 | 0.097 |
| | 1/64 | 1/64 | 0.008 | 0.009 | 0.042 | 0.042 | 0.095 | 0.095 |
| | 1/16 | 1/16 | 0.009 | 0.009 | 0.045 | 0.045 | 0.097 | 0.098 |
| | 1/4 | 1/4 | 0.008 | 0.008 | 0.043 | 0.044 | 0.094 | 0.094 |
| 10 | 1 | 1 | 0.008 | 0.008 | 0.043 | 0.044 | 0.099 | 0.100 |
| | 4 | 4 | 0.009 | 0.010 | 0.043 | 0.044 | 0.093 | 0.094 |
| | 16 | 16 | 0.009 | 0.010 | 0.049 | 0.049 | 0.094 | 0.095 |
| | 64 | 64 | 0.008 | 0.008 | 0.045 | 0.046 | 0.099 | 0.100 |
| | 256 | 256 | 0.008 | 0.008 | 0.045 | 0.045 | 0.094 | 0.094 |
| | 1/256 | 1/256 | 0.011 | 0.010 | 0.046 | 0.048 | 0.095 | 0.095 |
| | 1/64 | 1/64 | 0.010 | 0.010 | 0.052 | 0.052 | 0.092 | 0.093 |
| | 1/16 | 1/16 | 0.008 | 0.008 | 0.046 | 0.047 | 0.099 | 0.099 |
| | 1/4 | 1/4 | 0.008 | 0.008 | 0.047 | 0.048 | 0.099 | 0.099 |
| 25 | 1 | 1 | 0.009 | 0.009 | 0.047 | 0.047 | 0.099 | 0.099 |
| | 4 | 4 | 0.008 | 0.009 | 0.049 | 0.050 | 0.097 | 0.098 |
| | 16 | 16 | 0.010 | 0.010 | 0.049 | 0.049 | 0.094 | 0.094 |
| | 64 | 64 | 0.009 | 0.009 | 0.048 | 0.047 | 0.100 | 0.100 |
| | 256 | 256 | 0.007 | 0.006 | 0.049 | 0.049 | 0.103 | 0.103 |

TABLE 14

*Empirical size of the Welch's t-test and the MCT at the nominal levels $\alpha = .01, .05, .10$. The samples are drawn from a t-distribution with 5 degrees of freedom; $(m, n) = (8, 4), (8, 5), (8, 10), (8, 25)$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; results based on 10,000 simulated samples.*

| | | | $\alpha = .01$ | | $\alpha = .05$ | | $\alpha = .10$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 8/1024 | 0.000 | 0.000 | 0.002 | 0.003 | 0.009 | 0.009 |
| | 1/64 | 8/256 | 0.000 | 0.000 | 0.002 | 0.003 | 0.012 | 0.012 |
| | 1/16 | 8/64 | 0.000 | 0.000 | 0.004 | 0.004 | 0.012 | 0.013 |
| | 1/4 | 8/16 | 0.001 | 0.001 | 0.005 | 0.006 | 0.019 | 0.022 |
| 4 | 1 | 8/4 | 0.001 | 0.001 | 0.012 | 0.013 | 0.033 | 0.035 |
| | 4 | 32/4 | 0.003 | 0.003 | 0.024 | 0.024 | 0.055 | 0.055 |
| | 16 | 128/4 | 0.004 | 0.004 | 0.038 | 0.038 | 0.088 | 0.088 |
| | 64 | 512/4 | 0.008 | 0.008 | 0.041 | 0.042 | 0.094 | 0.093 |
| | 256 | 2048/4 | 0.008 | 0.008 | 0.044 | 0.045 | 0.094 | 0.094 |
| | 1/256 | 8/1280 | 0.000 | 0.000 | 0.008 | 0.008 | 0.025 | 0.025 |
| | 1/64 | 8/320 | 0.001 | 0.001 | 0.007 | 0.008 | 0.024 | 0.025 |
| | 1/16 | 8/80 | 0.002 | 0.002 | 0.009 | 0.010 | 0.034 | 0.035 |
| | 1/4 | 8/20 | 0.001 | 0.002 | 0.012 | 0.014 | 0.037 | 0.039 |
| 5 | 1 | 8/5 | 0.002 | 0.002 | 0.019 | 0.020 | 0.048 | 0.048 |
| | 4 | 32/5 | 0.004 | 0.004 | 0.031 | 0.032 | 0.073 | 0.073 |
| | 16 | 128/5 | 0.006 | 0.006 | 0.038 | 0.039 | 0.086 | 0.086 |
| | 64 | 512/5 | 0.008 | 0.008 | 0.046 | 0.046 | 0.095 | 0.096 |
| | 256 | 2048/5 | 0.008 | 0.008 | 0.046 | 0.045 | 0.093 | 0.093 |
| | 1/256 | 8/2560 | 0.019 | 0.018 | 0.075 | 0.075 | 0.143 | 0.143 |
| | 1/64 | 8/640 | 0.019 | 0.020 | 0.075 | 0.075 | 0.134 | 0.134 |
| | 1/16 | 8/160 | 0.017 | 0.018 | 0.077 | 0.078 | 0.141 | 0.143 |
| | 1/4 | 8/40 | 0.017 | 0.018 | 0.070 | 0.070 | 0.134 | 0.133 |
| 10 | 1 | 8/10 | 0.009 | 0.010 | 0.056 | 0.057 | 0.117 | 0.118 |
| | 4 | 32/10 | 0.010 | 0.010 | 0.048 | 0.049 | 0.105 | 0.106 |
| | 16 | 128/10 | 0.009 | 0.010 | 0.047 | 0.048 | 0.095 | 0.096 |
| | 64 | 512/10 | 0.007 | 0.007 | 0.046 | 0.046 | 0.096 | 0.096 |
| | 256 | 2048/10 | 0.007 | 0.007 | 0.047 | 0.046 | 0.090 | 0.091 |
| | 1/256 | 8/6400 | 0.151 | 0.150 | 0.289 | 0.288 | 0.367 | 0.366 |
| | 1/64 | 8/1600 | 0.145 | 0.145 | 0.281 | 0.281 | 0.355 | 0.355 |
| | 1/16 | 8/400 | 0.116 | 0.115 | 0.245 | 0.246 | 0.337 | 0.337 |
| | 1/4 | 8/100 | 0.076 | 0.077 | 0.177 | 0.178 | 0.273 | 0.273 |
| 25 | 1 | 8/25 | 0.034 | 0.036 | 0.105 | 0.107 | 0.181 | 0.182 |
| | 4 | 32/25 | 0.018 | 0.019 | 0.065 | 0.065 | 0.125 | 0.126 |
| | 16 | 128/25 | 0.010 | 0.010 | 0.051 | 0.051 | 0.104 | 0.104 |
| | 64 | 512/25 | 0.008 | 0.008 | 0.045 | 0.045 | 0.099 | 0.099 |
| | 256 | 2048/25 | 0.009 | 0.009 | 0.045 | 0.045 | 0.094 | 0.094 |

## 4. POWER TABLES FOR THE DATA SIMULATED FROM $t$-DISTRIBUTION

TABLE 15

*Empirical power of the Welch's t-test and the MCT at the nominal levels $\alpha = .05$. The samples are drawn from a t-distribution with 5 degrees of freedom; $m = n = 4, 5, 8, 10, 25$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; the deviation from equality of means is obtained by $\delta = \eta\sqrt{\sigma_x^2/m + \sigma_y^2/n}$ with $\eta = 1, 2, 3$; results based on 10,000 simulated samples.*

| | | | $\eta = 1$ | | $\eta = 2$ | | $\eta = 3$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 1/256 | 0.094 | 0.097 | 0.245 | 0.250 | 0.452 | 0.456 |
| | 1/64 | 1/64 | 0.096 | 0.101 | 0.246 | 0.258 | 0.459 | 0.470 |
| | 1/16 | 1/16 | 0.105 | 0.112 | 0.268 | 0.285 | 0.477 | 0.495 |
| | 1/4 | 1/4 | 0.095 | 0.102 | 0.273 | 0.287 | 0.501 | 0.519 |
| 4 | 1 | 1 | 0.088 | 0.093 | 0.265 | 0.277 | 0.505 | 0.520 |
| | 4 | 4 | 0.096 | 0.100 | 0.279 | 0.291 | 0.505 | 0.522 |
| | 16 | 16 | 0.106 | 0.114 | 0.268 | 0.281 | 0.467 | 0.487 |
| | 64 | 64 | 0.103 | 0.108 | 0.261 | 0.271 | 0.453 | 0.466 |
| | 256 | 256 | 0.096 | 0.100 | 0.242 | 0.246 | 0.440 | 0.446 |
| | 1/256 | 1/256 | 0.106 | 0.106 | 0.275 | 0.277 | 0.506 | 0.507 |
| | 1/64 | 1/64 | 0.104 | 0.107 | 0.277 | 0.284 | 0.508 | 0.513 |
| | 1/16 | 1/16 | 0.111 | 0.116 | 0.290 | 0.298 | 0.532 | 0.542 |
| | 1/4 | 1/4 | 0.112 | 0.115 | 0.296 | 0.304 | 0.552 | 0.561 |
| 5 | 1 | 1 | 0.097 | 0.101 | 0.298 | 0.304 | 0.564 | 0.572 |
| | 4 | 4 | 0.106 | 0.110 | 0.290 | 0.296 | 0.547 | 0.557 |
| | 16 | 16 | 0.103 | 0.108 | 0.292 | 0.300 | 0.525 | 0.535 |
| | 64 | 64 | 0.104 | 0.108 | 0.277 | 0.282 | 0.509 | 0.516 |
| | 256 | 256 | 0.099 | 0.100 | 0.269 | 0.273 | 0.504 | 0.506 |
| | 1/256 | 1/256 | 0.116 | 0.116 | 0.314 | 0.314 | 0.593 | 0.592 |
| | 1/64 | 1/64 | 0.117 | 0.118 | 0.318 | 0.319 | 0.583 | 0.585 |
| | 1/16 | 1/16 | 0.116 | 0.118 | 0.332 | 0.336 | 0.593 | 0.595 |
| | 1/4 | 1/4 | 0.114 | 0.116 | 0.328 | 0.330 | 0.610 | 0.614 |
| 8 | 1 | 1 | 0.114 | 0.115 | 0.328 | 0.329 | 0.609 | 0.610 |
| | 4 | 4 | 0.106 | 0.108 | 0.320 | 0.323 | 0.601 | 0.602 |
| | 16 | 16 | 0.119 | 0.120 | 0.329 | 0.331 | 0.590 | 0.593 |
| | 64 | 64 | 0.112 | 0.114 | 0.316 | 0.317 | 0.584 | 0.586 |
| | 256 | 256 | 0.113 | 0.114 | 0.323 | 0.323 | 0.578 | 0.577 |
| | 1/256 | 1/256 | 0.116 | 0.118 | 0.333 | 0.334 | 0.602 | 0.601 |
| | 1/64 | 1/64 | 0.115 | 0.116 | 0.326 | 0.326 | 0.595 | 0.595 |
| | 1/16 | 1/16 | 0.122 | 0.122 | 0.324 | 0.324 | 0.607 | 0.608 |
| | 1/4 | 1/4 | 0.122 | 0.123 | 0.334 | 0.336 | 0.615 | 0.618 |
| 10 | 1 | 1 | 0.120 | 0.121 | 0.338 | 0.339 | 0.616 | 0.618 |
| | 4 | 4 | 0.112 | 0.113 | 0.332 | 0.333 | 0.621 | 0.622 |
| | 16 | 16 | 0.127 | 0.128 | 0.328 | 0.329 | 0.609 | 0.609 |
| | 64 | 64 | 0.119 | 0.118 | 0.330 | 0.330 | 0.608 | 0.608 |
| | 256 | 256 | 0.122 | 0.122 | 0.329 | 0.330 | 0.606 | 0.603 |
| | 1/256 | 1/256 | 0.118 | 0.118 | 0.347 | 0.347 | 0.633 | 0.633 |
| | 1/64 | 1/64 | 0.121 | 0.120 | 0.348 | 0.348 | 0.620 | 0.620 |
| | 1/16 | 1/16 | 0.120 | 0.121 | 0.348 | 0.348 | 0.637 | 0.638 |
| | 1/4 | 1/4 | 0.130 | 0.130 | 0.345 | 0.346 | 0.639 | 0.638 |
| 25 | 1 | 1 | 0.127 | 0.127 | 0.348 | 0.348 | 0.643 | 0.644 |
| | 4 | 4 | 0.123 | 0.123 | 0.359 | 0.360 | 0.637 | 0.637 |
| | 16 | 16 | 0.123 | 0.124 | 0.352 | 0.352 | 0.631 | 0.631 |
| | 64 | 64 | 0.126 | 0.126 | 0.350 | 0.351 | 0.638 | 0.638 |
| | 256 | 256 | 0.122 | 0.122 | 0.346 | 0.346 | 0.637 | 0.638 |

TABLE 16

*Empirical power of the Welch's t-test and the MCT at the nominal levels $\alpha = .05$. The samples are drawn from a t-distribution with 5 degrees of freedom; $(m, n) = (8, 4), (8, 5), (8, 10), (8, 25)$; the variance ratios $\rho = \sigma_x^2/\sigma_y^2$ varying from $2^{-8}$ to $2^8$ and $\tau = (\sigma_x^2/m)/(\sigma_y^2/n)$; the deviation from equality of means is obtained by $\delta = \eta\sqrt{\sigma_x^2/m + \sigma_y^2/n}$ with $\eta = 1, 2, 3$; results based on 10,000 simulated samples.*

| | | | $\eta = 1$ | | $\eta = 2$ | | $\eta = 3$ | |
|---|---|---|---|---|---|---|---|---|
| $n$ | $\rho$ | $\tau$ | Welch | MCT | Welch | MCT | Welch | MCT |
| | 1/256 | 8/1024 | 0.018 | 0.020 | 0.114 | 0.117 | 0.343 | 0.345 |
| | 1/64 | 8/256 | 0.018 | 0.020 | 0.119 | 0.128 | 0.343 | 0.353 |
| | 1/16 | 8/64 | 0.024 | 0.029 | 0.136 | 0.154 | 0.368 | 0.397 |
| | 1/4 | 8/16 | 0.035 | 0.041 | 0.167 | 0.190 | 0.429 | 0.463 |
| 4 | 1 | 8/4 | 0.054 | 0.059 | 0.223 | 0.240 | 0.510 | 0.536 |
| | 4 | 32/4 | 0.081 | 0.083 | 0.286 | 0.292 | 0.576 | 0.584 |
| | 16 | 128/4 | 0.111 | 0.113 | 0.323 | 0.326 | 0.589 | 0.589 |
| | 64 | 512/4 | 0.114 | 0.115 | 0.314 | 0.315 | 0.588 | 0.590 |
| | 256 | 2048/4 | 0.115 | 0.117 | 0.309 | 0.310 | 0.586 | 0.586 |
| | 1/256 | 8/1280 | 0.043 | 0.044 | 0.187 | 0.188 | 0.455 | 0.457 |
| | 1/64 | 8/320 | 0.045 | 0.047 | 0.195 | 0.200 | 0.453 | 0.460 |
| | 1/16 | 8/80 | 0.051 | 0.055 | 0.211 | 0.224 | 0.473 | 0.486 |
| | 1/4 | 8/20 | 0.056 | 0.061 | 0.237 | 0.249 | 0.517 | 0.536 |
| 5 | 1 | 8/5 | 0.078 | 0.080 | 0.269 | 0.278 | 0.563 | 0.573 |
| | 4 | 32/5 | 0.094 | 0.097 | 0.304 | 0.304 | 0.589 | 0.591 |
| | 16 | 128/5 | 0.104 | 0.106 | 0.331 | 0.332 | 0.593 | 0.597 |
| | 64 | 512/5 | 0.110 | 0.111 | 0.310 | 0.311 | 0.592 | 0.591 |
| | 256 | 2048/5 | 0.114 | 0.113 | 0.319 | 0.319 | 0.575 | 0.576 |
| | 1/256 | 8/2560 | 0.157 | 0.156 | 0.366 | 0.365 | 0.601 | 0.600 |
| | 1/64 | 8/640 | 0.155 | 0.157 | 0.357 | 0.358 | 0.610 | 0.611 |
| | 1/16 | 8/160 | 0.154 | 0.155 | 0.368 | 0.368 | 0.608 | 0.610 |
| | 1/4 | 8/40 | 0.145 | 0.146 | 0.354 | 0.355 | 0.615 | 0.617 |
| 10 | 1 | 8/10 | 0.126 | 0.127 | 0.340 | 0.340 | 0.609 | 0.610 |
| | 4 | 32/10 | 0.123 | 0.124 | 0.333 | 0.336 | 0.614 | 0.617 |
| | 16 | 128/10 | 0.121 | 0.122 | 0.329 | 0.332 | 0.582 | 0.584 |
| | 64 | 512/10 | 0.119 | 0.120 | 0.322 | 0.324 | 0.582 | 0.583 |
| | 256 | 2048/10 | 0.116 | 0.115 | 0.324 | 0.325 | 0.587 | 0.588 |
| | 1/256 | 8/6400 | 0.338 | 0.336 | 0.454 | 0.453 | 0.612 | 0.613 |
| | 1/64 | 8/1600 | 0.328 | 0.328 | 0.466 | 0.466 | 0.629 | 0.629 |
| | 1/16 | 8/400 | 0.306 | 0.308 | 0.444 | 0.444 | 0.619 | 0.619 |
| | 1/4 | 8/100 | 0.242 | 0.242 | 0.404 | 0.405 | 0.606 | 0.608 |
| 25 | 1 | 8/25 | 0.176 | 0.178 | 0.360 | 0.364 | 0.593 | 0.597 |
| | 4 | 32/25 | 0.136 | 0.137 | 0.333 | 0.338 | 0.591 | 0.594 |
| | 16 | 128/25 | 0.124 | 0.125 | 0.312 | 0.315 | 0.581 | 0.584 |
| | 64 | 512/25 | 0.112 | 0.113 | 0.325 | 0.326 | 0.588 | 0.588 |
| | 256 | 2048/25 | 0.116 | 0.117 | 0.314 | 0.314 | 0.580 | 0.578 |

## 5. SUPPLEMENTARY MATERIALS FOR REAL CHILDHOOD ACUTE LYMPHOBLASTIC LEUKEMIA GENE EXPRESSION STUDY

Sparse principal component analysis (sPCA) was recently introduced as a powerful tool that, together with dimension reduction, also helps in the selection of important variables (Shen & Huang, 2008). The sPCA allows for the number of non-zero loadings that one wishes to keep in each principal component (PC) to be controlled. In this case, the second PC reflects the within-group variation (see Figure 1). Therefore, we let the loadings of all 6,307 genes as non-zero in the second PC (since this PC is not of our interest). The first component reflects the between-group variation and we forced the loadings of all the genes to be equal to zero except 500, 300, 200 and 100 (see corresponding PCA plots in Figure 1).

Interestingly, most of the 72 additional genes were selected among top 500, 300, 200 and 100 identified as important by the sPCA (to differentiate between the two groups) and are ranked, respectively, from one to four stars in Table 17.

We also used sparse PLS Discriminant Analysis (sPLS-DA) (Boitard et al., 2011) to identify differentially expressed genes. Similar to the sPCA, sPLS-DA also allowed us to specify the number of non-zero loadings for a particular component. The component-1 of sPLS-DA represent the between-group variation (see Figure 1) and we specified 500, 300, 200 and 100 of the loadings of component-1 to be non-zero. Since component-2 represents a within-group variation, we allowed all 6,307 genes to have non-zero loadings. Many of the 72 additional genes were selected as important by sPLS-DA to differentiate the two ALL types. We ranked these important genes in Table 18 from one to four stars depending on whether they were included in top 500, 300, 200 and 100, respectively.

Note that we performed sPCA and sPLS-DA using spca() and splsda() functions implemented in R package mixOmics (Rohart et al., 2017).

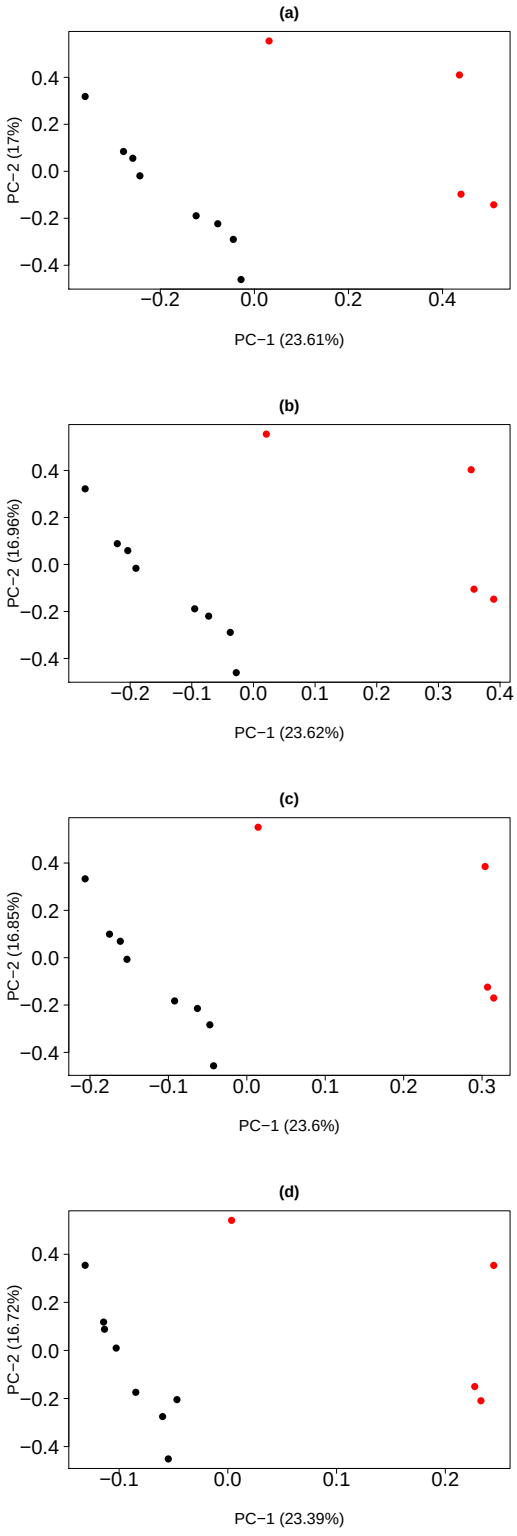


FIG 1. *Sparse PCA plot. The Sparse PCA algorithm is forced to select top; (a) 500, (b) 300, (c) 200, and (d) 100 differentially expressed genes for the second component that represents variation between the two groups and all 6307 genes for first component that represent with-in group variation. The red dots represent subtype BCR-ABL and the black dots represent subtype E2A-rearranged (EP).*

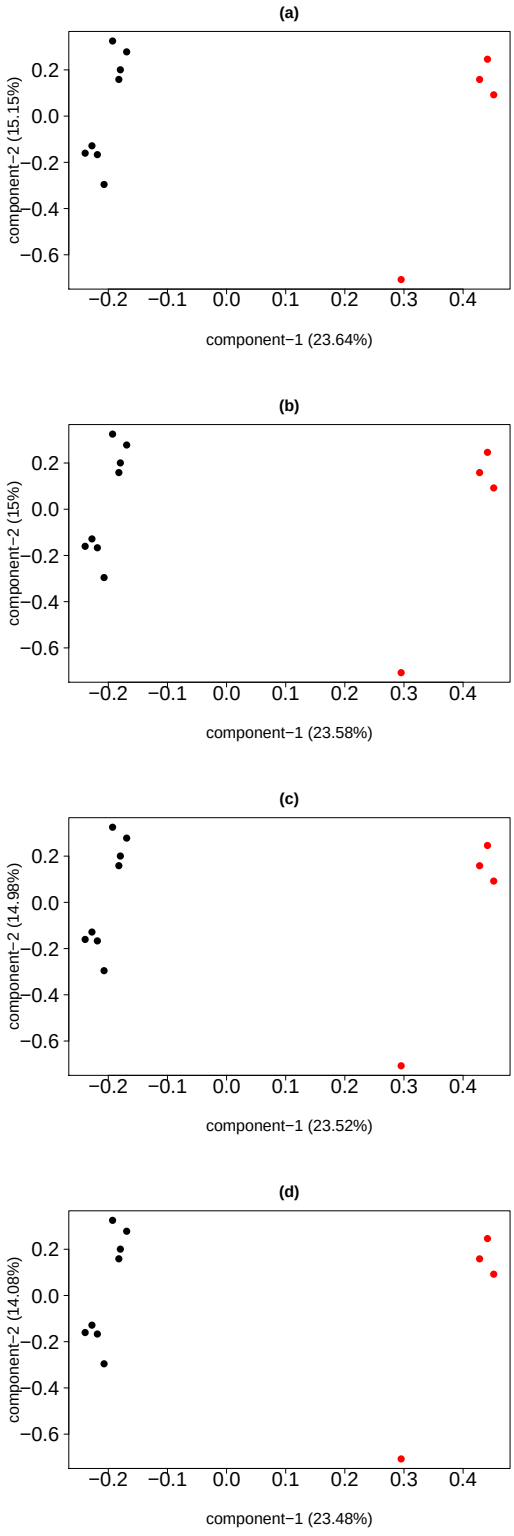


FIG 2. *Sparse PLS-DA scores plot. The sparse PLS-DA algorithm is forced to select top; (a) 500, (b) 300, (c) 200, and (d) 100 differentially expressed genes for the first component that represents variation between the two groups and all 6307 genes for second component that represent with-in group variation. The red dots represent subtype BCR-ABL and the black dots represent subtype E2A-rearranged (EP).*

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Rohart, F., Gautier, B., Singh, A., & Le Cao, K.-A. (2017). mixomics: An r package for 'omics feature selection and multiple data integration. *PLoS computational biology*, *13*(11), e1005752.

Shen, H., & Huang, J. Z. (2008). Sparse principal component analysis via regularized low rank matrix approximation. *Journal of multivariate analysis*, *99*(6), 1015–1034.

TABLE 17

*The list of 72 additional genes identified by the MCT at 0.01 level of significance. The genes selected by sparse PCA among top 500, 300, 200 and 100 are ranked, respectively, from one to four stars.*

| | BCR-ABL | E2A-rearranged (EP) | p-values | |
|---|---|---|---|---|
| Probe Set ID | mean (sd) | mean (sd) | MCT | Welch |
| NONO\|200057_s_at | 8.58 (0.21) | 8.12 (0.2) | 0.0085 | 0.0118 |
| TMED2\|200087_s_at | 7.18 (0.24) | 6.64 (0.29) | 0.0083 | 0.0102 |
| CALM\|200655_s_at | 7.83 (0.38) | 6.95 (0.17) | 0.0089 | 0.014 |
| LAPTM4A\|200673_at | 7.7 (0.41) | 6.83 (0.39) | 0.0086 | 0.0123 |
| PGK1\|200737_at | 5.76 (0.34) | 5.01 (0.31) | 0.008 | 0.0116 |
| ARL6IP5\|200761_s_at**** | 5.67 (0.63) | 4.36 (0.6) | 0.0098 | 0.0142 |
| ZNF207\|200828_s_at | 7.93 (0.44) | 7 (0.34) | 0.0093 | 0.0141 |
| IST1\|200851_s_at*** | 7.17 (0.41) | 6.28 (0.46) | 0.0092 | 0.012 |
| PSAP\|200866_s_at*** | 6.58 (0.74) | 4.82 (0.42) | 0.0064 | 0.0114 |
| ACTR3\|200996_at* | 6.34 (0.42) | 5.36 (0.3) | 0.0063 | 0.011 |
| PSMF1\|201052_s_at | 4.78 (0.42) | 3.89 (0.47) | 0.0096 | 0.0128 |
| ATP6V1B2\|201089_at**** | 5.5 (0.59) | 4.15 (0.29) | 0.009 | 0.0142 |
| HNRNPH2\|201132_at | 3.8 (0.44) | 2.83 (0.51) | 0.0095 | 0.0111 |
| BHLHE40\|201170_s_at**** | 7.25 (0.99) | 4.97 (0.57) | 0.0072 | 0.0129 |
| SEC11A\|201290_at**** | 6.04 (0.37) | 5.18 (0.27) | 0.0061 | 0.0103 |
| SLC9A3R1\|201349_at**** | 5.33 (0.72) | 3.75 (0.55) | 0.0084 | 0.0125 |
| CUL3\|201371_s_at** | 7.3 (0.43) | 6.32 (0.57) | 0.0087 | 0.0101 |
| ITGA5\|201389_at**** | 6.04 (0.77) | 4.28 (0.58) | 0.0062 | 0.0107 |
| TRAM1\|201398_s_at* | 6.59 (0.32) | 5.9 (0.36) | 0.0096 | 0.0123 |
| PLEKHB2\|201411_s_at | 5.02 (0.68) | 3.18 (1.33) | 0.01 | 0.0101 |
| ETF1\|201573_s_at | 5.92 (0.44) | 4.89 (0.22) | 0.007 | 0.0127 |
| IRAK1\|201587_s_at | 6.84 (0.59) | 5.58 (0.49) | 0.0084 | 0.013 |
| USP14\|201672_s_at**** | 6.14 (0.41) | 5.2 (0.21) | 0.0082 | 0.0141 |
| EFCAB14\|201778_s_at | 4.68 (0.33) | 3.93 (0.44) | 0.0097 | 0.0109 |
| SEC63\|201914_s_at | 4.8 (0.42) | 3.86 (0.4) | 0.0076 | 0.0109 |
| SLC25A36\|201917_s_at**** | 5.72 (0.51) | 4.56 (0.26) | 0.0078 | 0.0139 |
| KIF5B\|201991_s_at | 6.23 (0.26) | 5.65 (0.22) | 0.0068 | 0.0105 |
| SPG7\|202104_s_at | 3.81 (0.37) | 3 (0.41) | 0.0089 | 0.0114 |
| RAP1A\|202362_at**** | 5.24 (0.63) | 3.71 (0.39) | 0.005 | 0.0101 |
| BASP1\|202391_at | 4.45 (0.64) | 5.86 (0.78) | 0.0094 | 0.0114 |
| SEC24B\|202798_at*** | 5.4 (0.59) | 4.15 (0.49) | 0.0096 | 0.014 |
| CYTH1\|202879_s_at**** | 4.86 (0.6) | 3.57 (0.52) | 0.0094 | 0.0133 |
| RHOBTB3\|202975_s_at* | 3.51 (0.39) | 2.66 (0.32) | 0.008 | 0.0125 |
| RREB1\|203704_s_at*** | 5.44 (0.26) | 4.88 (0.27) | 0.0093 | 0.0119 |
| PDE4B\|203708_at**** | 6.49 (1.3) | 3.7 (0.88) | 0.0088 | 0.0145 |
| CSF2RB\|205159_at | 3.71 (1.2) | 6.43 (0.6) | 0.0086 | 0.0145 |
| AAK1\|205434_s_at | 5.27 (0.23) | 4.78 (0.26) | 0.0094 | 0.0116 |
| CTDSP2\|208735_s_at | 5.36 (0.57) | 4.1 (0.59) | 0.0081 | 0.011 |
| SAP18\|208742_s_at | 8.38 (0.3) | 7.73 (0.25) | 0.008 | 0.0122 |
| REEP5\|208872_s_at | 5.51 (0.43) | 4.56 (0.31) | 0.0087 | 0.0136 |
| KPNB1\|208974_x_at | 6 (0.32) | 5.3 (0.29) | 0.0084 | 0.0124 |
| STX3\|209238_at**** | 4.99 (0.79) | 3.21 (0.74) | 0.0065 | 0.0104 |
| SAT1\|210592_s_at*** | 8.45 (0.81) | 6.73 (0.87) | 0.0099 | 0.0128 |
| UBR4\|211950_at** | 5.79 (0.47) | 4.79 (0.49) | 0.01 | 0.013 |
| KBTBD2\|212447_at | 5.58 (0.48) | 4.52 (0.24) | 0.0096 | 0.0158 |
| RMND5A\|212482_at | 5.41 (0.35) | 4.68 (0.24) | 0.0099 | 0.0153 |
| DENND5A\|212561_at**** | 6.54 (0.47) | 5.47 (0.26) | 0.0086 | 0.014 |
| AUTS2\|212599_at | 5.18 (0.49) | 6.25 (0.36) | 0.0082 | 0.0128 |
| DNMBP\|212838_at**** | 4.88 (0.6) | 3.54 (0.38) | 0.0081 | 0.0137 |
| GNPTAB\|212959_s_at**** | 5.11 (0.64) | 3.71 (0.48) | 0.0083 | 0.0132 |
| CASP8\|213373_s_at* | 5.4 (0.92) | 3.44 (0.61) | 0.0096 | 0.0149 |
| POLR2E\|213887_s_at | 5.22 (0.59) | 3.91 (0.49) | 0.0073 | 0.0116 |
| LST1\|214181_x_at | 5.4 (1.55) | 2.12 (1.18) | 0.0095 | 0.0143 |
| SUB1\|214512_s_at*** | 7.6 (0.45) | 6.49 (0.27) | 0.0058 | 0.0105 |
| TBC1D9B\|215994_x_at* | 4.99 (0.18) | 4.6 (0.18) | 0.0086 | 0.0114 |
| WDR83OS\|217780_at | 6.35 (0.26) | 5.75 (0.33) | 0.0085 | 0.0101 |
| KCMF1\|217938_s_at*** | 7.17 (0.36) | 6.4 (0.28) | 0.0084 | 0.0133 |
| NOSIP\|217950_at | 4.71 (0.23) | 4.19 (0.2) | 0.0069 | 0.0108 |
| BCL2L13\|217955_at | 3.63 (0.57) | 2.36 (0.59) | 0.0074 | 0.0104 |
| TSPAN13\|217979_at | 6.44 (0.67) | 5.02 (0.45) | 0.0097 | 0.0158 |
| ZFAND3\|218020_s_at*** | 5.17 (0.32) | 4.45 (0.3) | 0.0067 | 0.01 |
| ZDHHC6\|218249_at | 3.81 (0.07) | 3.22 (0.48) | 0.0096 | 0.01 |
| NDE1\|218414_s_at | 5.26 (0.38) | 4.38 (0.33) | 0.0064 | 0.0102 |
| PSMG2\|218467_at | 7.49 (0.27) | 6.89 (0.31) | 0.0087 | 0.0106 |
| COQ10B\|219397_at | 5.59 (0.39) | 4.74 (0.44) | 0.0089 | 0.0114 |
| BNIP3L\|221478_at | 5.37 (0.45) | 4.38 (0.49) | 0.0084 | 0.0109 |
| YTHDF3\|221749_at | 4.97 (0.41) | 4.08 (0.38) | 0.008 | 0.0116 |
| FGFR1\|222164_at** | 4.7 (0.28) | 4.1 (0.26) | 0.0094 | 0.0135 |
| ACTR10\|222230_s_at* | 4.64 (0.5) | 3.54 (0.4) | 0.008 | 0.0123 |
| PDCD6\|222380_s_at | 3.35 (0.6) | 4.66 (0.51) | 0.0089 | 0.0127 |
| SAFB2\|32099_at*** | 5.42 (0.35) | 4.65 (0.31) | 0.007 | 0.011 |
| KDM6B\|41387_r_at* | 5.42 (0.33) | 4.69 (0.39) | 0.0093 | 0.0113 |

*among top 500 **among top 300 ***among top 200 ****among top 100

TABLE 18

*The list of 72 additional genes identified by MCT at 0.01 level of significance. The genes selected by sparse PLS-DA among top 500, 300, 200 and 100 are ranked, respectively, from one to four stars.*

| | BCR-ABL | E2A-rearranged (EP) | p-value | |
|---|---|---|---|---|
| Probe Set ID | mean (sd) | mean (sd) | MCT | Welch |
| NONO\|200057_s_at* | 8.58 (0.21) | 8.12 (0.2) | 0.0085 | 0.0118 |
| TMED2\|200087_s_at | 7.18 (0.24) | 6.64 (0.29) | 0.0083 | 0.0102 |
| CALM\|200655_s_at**** | 7.83 (0.38) | 6.95 (0.17) | 0.0089 | 0.014 |
| LAPTM4A\|200673_at | 7.7 (0.41) | 6.83 (0.39) | 0.0086 | 0.0123 |
| PGK1\|200737_at* | 5.76 (0.34) | 5.01 (0.31) | 0.008 | 0.0116 |
| ARL6IP5\|200761_s_at | 5.67 (0.63) | 4.36 (0.6) | 0.0098 | 0.0142 |
| ZNF207\|200828_s_at* | 7.93 (0.44) | 7 (0.34) | 0.0093 | 0.0141 |
| IST1\|200851_s_at | 7.17 (0.41) | 6.28 (0.46) | 0.0092 | 0.012 |
| PSAP\|200866_s_at**** | 6.58 (0.74) | 4.82 (0.42) | 0.0064 | 0.0114 |
| ACTR3\|200996_at*** | 6.34 (0.42) | 5.36 (0.3) | 0.0063 | 0.011 |
| PSMF1\|201052_s_at | 4.78 (0.42) | 3.89 (0.47) | 0.0096 | 0.0128 |
| ATP6V1B2\|201089_at**** | 5.5 (0.59) | 4.15 (0.29) | 0.009 | 0.0142 |
| HNRNPH2\|201132_at | 3.8 (0.44) | 2.83 (0.51) | 0.0095 | 0.0111 |
| BHLHE40\|201170_s_at*** | 7.25 (0.99) | 4.97 (0.57) | 0.0072 | 0.0129 |
| SEC11A\|201290_at*** | 6.04 (0.37) | 5.18 (0.27) | 0.0061 | 0.0103 |
| SLC9A3R1\|201349_at** | 5.33 (0.72) | 3.75 (0.55) | 0.0084 | 0.0125 |
| CUL3\|201371_s_at | 7.3 (0.43) | 6.32 (0.57) | 0.0087 | 0.0101 |
| ITGA5\|201389_at** | 6.04 (0.77) | 4.28 (0.58) | 0.0062 | 0.0107 |
| TRAM1\|201398_s_at | 6.59 (0.32) | 5.9 (0.36) | 0.0096 | 0.0123 |
| PLEKHB2\|201411_s_at | 5.02 (0.68) | 3.18 (1.33) | 0.01 | 0.0101 |
| ETF1\|201573_s_at**** | 5.92 (0.44) | 4.89 (0.22) | 0.007 | 0.0127 |
| IRAK1\|201587_s_at* | 6.84 (0.59) | 5.58 (0.49) | 0.0084 | 0.013 |
| USP14\|201672_s_at**** | 6.14 (0.41) | 5.2 (0.21) | 0.0082 | 0.0141 |
| EFCAB14\|201778_s_at | 4.68 (0.33) | 3.93 (0.44) | 0.0097 | 0.0109 |
| SEC63\|201914_s_at* | 4.8 (0.42) | 3.86 (0.4) | 0.0076 | 0.0109 |
| SLC25A36\|201917_s_at**** | 5.72 (0.51) | 4.56 (0.26) | 0.0078 | 0.0139 |
| KIF5B\|201991_s_at* | 6.23 (0.26) | 5.65 (0.22) | 0.0068 | 0.0105 |
| SPG7\|202104_s_at | 3.81 (0.37) | 3 (0.41) | 0.0089 | 0.0114 |
| RAP1A\|202362_at**** | 5.24 (0.63) | 3.71 (0.39) | 0.005 | 0.0101 |
| BASP1\|202391_at | 4.45 (0.64) | 5.86 (0.78) | 0.0094 | 0.0114 |
| SEC24B\|202798_at* | 5.4 (0.59) | 4.15 (0.49) | 0.0096 | 0.014 |
| CYTH1\|202879_s_at* | 4.86 (0.6) | 3.57 (0.52) | 0.0094 | 0.0133 |
| RHOBTB3\|202975_s_at* | 3.51 (0.39) | 2.66 (0.32) | 0.008 | 0.0125 |
| RREB1\|203704_s_at | 5.44 (0.26) | 4.88 (0.27) | 0.0093 | 0.0119 |
| PDE4B\|203708_at** | 6.49 (1.3) | 3.7 (0.88) | 0.0088 | 0.0145 |
| CSF2RB\|205159_at**** | 3.71 (1.2) | 6.43 (0.6) | 0.0086 | 0.0145 |
| AAK1\|205434_s_at | 5.27 (0.23) | 4.78 (0.26) | 0.0094 | 0.0116 |
| CTDSP2\|208735_s_at | 5.36 (0.57) | 4.1 (0.59) | 0.0081 | 0.011 |
| SAP18\|208742_s_at* | 8.38 (0.3) | 7.73 (0.25) | 0.008 | 0.0122 |
| REEP5\|208872_s_at** | 5.51 (0.43) | 4.56 (0.31) | 0.0087 | 0.0136 |
| KPNB1\|208974_x_at* | 6 (0.32) | 5.3 (0.29) | 0.0084 | 0.0124 |
| STX3\|209238_at* | 4.99 (0.79) | 3.21 (0.74) | 0.0065 | 0.0104 |
| SAT1\|210592_s_at | 8.45 (0.81) | 6.73 (0.87) | 0.0099 | 0.0128 |
| UBR4\|211950_at | 5.79 (0.47) | 4.79 (0.49) | 0.01 | 0.013 |
| KBTBD2\|212447_at*** | 5.58 (0.48) | 4.52 (0.24) | 0.0096 | 0.0158 |
| RMND5A\|212482_at** | 5.41 (0.35) | 4.68 (0.24) | 0.0099 | 0.0153 |
| DENND5A\|212561_at*** | 6.54 (0.47) | 5.47 (0.26) | 0.0086 | 0.014 |
| AUTS2\|212599_at** | 5.18 (0.49) | 6.25 (0.36) | 0.0082 | 0.0128 |
| DNMBP\|212838_at*** | 4.88 (0.6) | 3.54 (0.38) | 0.0081 | 0.0137 |
| GNPTAB\|212959_s_at** | 5.11 (0.64) | 3.71 (0.48) | 0.0083 | 0.0132 |
| CASP8\|213373_s_at** | 5.4 (0.92) | 3.44 (0.61) | 0.0096 | 0.0149 |
| POLR2E\|213887_s_at* | 5.22 (0.59) | 3.91 (0.49) | 0.0073 | 0.0116 |
| LST1\|214181_x_at* | 5.4 (1.55) | 2.12 (1.18) | 0.0095 | 0.0143 |
| SUB1\|214512_s_at**** | 7.6 (0.45) | 6.49 (0.27) | 0.0058 | 0.0105 |
| TBC1D9B\|215994_x_at | 4.99 (0.18) | 4.6 (0.18) | 0.0086 | 0.0114 |
| WDR83OS\|217780_at | 6.35 (0.26) | 5.75 (0.33) | 0.0085 | 0.0101 |
| KCMF1\|217938_s_at* | 7.17 (0.36) | 6.4 (0.28) | 0.0084 | 0.0133 |
| NOSIP\|217950_at* | 4.71 (0.23) | 4.19 (0.2) | 0.0069 | 0.0108 |
| BCL2L13\|217955_at | 3.63 (0.57) | 2.36 (0.59) | 0.0074 | 0.0104 |
| TSPAN13\|217979_at** | 6.44 (0.67) | 5.02 (0.45) | 0.0097 | 0.0158 |
| ZFAND3\|218020_s_at* | 5.17 (0.32) | 4.45 (0.3) | 0.0067 | 0.01 |
| ZDHHC6\|218249_at | 3.81 (0.07) | 3.22 (0.48) | 0.0096 | 0.01 |
| NDE1\|218414_s_at* | 5.26 (0.38) | 4.38 (0.33) | 0.0064 | 0.0102 |
| PSMG2\|218467_at | 7.49 (0.27) | 6.89 (0.31) | 0.0087 | 0.0106 |
| COQ10B\|219397_at | 5.59 (0.39) | 4.74 (0.44) | 0.0089 | 0.0114 |
| BNIP3L\|221478_at | 5.37 (0.45) | 4.38 (0.49) | 0.0084 | 0.0109 |
| YTHDF3\|221749_at* | 4.97 (0.41) | 4.08 (0.38) | 0.008 | 0.0116 |
| FGFR1\|222164_at* | 4.7 (0.28) | 4.1 (0.26) | 0.0094 | 0.0135 |
| ACTR10\|222230_s_at** | 4.64 (0.5) | 3.54 (0.4) | 0.008 | 0.0123 |
| PDCD6\|222380_s_at* | 3.35 (0.6) | 4.66 (0.51) | 0.0089 | 0.0127 |
| SAFB2\|32099_at* | 5.42 (0.35) | 4.65 (0.31) | 0.007 | 0.011 |
| KDM6B\|41387_r_at | 5.42 (0.33) | 4.69 (0.39) | 0.0093 | 0.0113 |

*among top 500 **among top 300 ***among top 200 ****among top 100

TABLE 19

*The list of 13 additional genes identified by the MCT at 0.01 level of significance based on q-values. The genes selected by sparse PCA among top 500, 300, 200 and 100 are ranked, respectively, from one to four stars.*

| | BCR-ABL | E2A-rearranged (EP) | p-values | |
|---|---|---|---|---|
| Probe Set ID | mean (sd) | mean (sd) | MCT | Welch |
| STARD7\|200028_s_at*** | 6.41 (0.15) | 5.47 (0.3) | 0.009 | 0.0151 |
| SH3GL1\|201851_at* | 5.25 (0.16) | 4.34 (0.29) | 0.0097 | 0.0172 |
| ABL1\|202123_s_at**** | 6.85 (0.36) | 4.6 (0.61) | 0.009 | 0.0142 |
| TMEM11\|203437_at | 5.12 (0.09) | 4.07 (0.31) | 0 | 0.014 |
| ADD3\|205882_x_at | 5.39 (0.16) | 4.21 (0.32) | 0.0097 | 0.0131 |
| CD164\|208405_s_at* | 6.96 (0.19) | 5.58 (0.36) | 0.0097 | 0.0131 |
| CCT5\|208696_at | 5.51 (0.04) | 4.6 (0.26) | 0.0097 | 0.0142 |
| TAPBP\|208829_at | 7.37 (0.12) | 6.47 (0.3) | 0 | 0.0147 |
| RNF139\|209510_at | 6.8 (0.14) | 5.92 (0.26) | 0.0097 | 0.0142 |
| NUP98\|210793_s_at* | 5.55 (0.09) | 4.5 (0.32) | 0 | 0.0142 |
| XPO6\|211982_x_at*** | 5.65 (0.09) | 5.03 (0.16) | 0 | 0.0131 |
| KDM3A\|212689_s_at* | 6.41 (0.06) | 5.84 (0.18) | 0.0097 | 0.0142 |
| UBP1\|218082_s_at**** | 4.97 (0.18) | 3.89 (0.23) | 0.009 | 0.0142 |

*among top 500 **among top 300 ***among top 200 ****among top 100

TABLE 20

*The list of 13 additional genes identified by the MCT at 0.01 level of significance based on q-values. The genes selected by sparse PLS-DA among top 500, 300, 200 and 100 are ranked, respectively, from one to four stars.*

| | BCR-ABL | E2A-rearranged (EP) | p-value | |
|---|---|---|---|---|
| Probe Set ID | mean (sd) | mean (sd) | MCT | Welch |
| STARD7\|200028_s_at**** | 6.41 (0.15) | 5.47 (0.3) | 0.009 | 0.0151 |
| SH3GL1\|201851_at**** | 5.25 (0.16) | 4.34 (0.29) | 0.0097 | 0.0172 |
| ABL1\|202123_s_at**** | 6.85 (0.36) | 4.6 (0.61) | 0.009 | 0.0142 |
| TMEM11\|203437_at**** | 5.12 (0.09) | 4.07 (0.31) | 0 | 0.014 |
| ADD3\|205882_x_at**** | 5.39 (0.16) | 4.21 (0.32) | 0.0097 | 0.0131 |
| CD164\|208405_s_at**** | 6.96 (0.19) | 5.58 (0.36) | 0.0097 | 0.0131 |
| CCT5\|208696_at**** | 5.51 (0.04) | 4.6 (0.26) | 0.0097 | 0.0142 |
| TAPBP\|208829_at**** | 7.37 (0.12) | 6.47 (0.3) | 0 | 0.0147 |
| RNF139\|209510_at**** | 6.8 (0.14) | 5.92 (0.26) | 0.0097 | 0.0142 |
| NUP98\|210793_s_at**** | 5.55 (0.09) | 4.5 (0.32) | 0 | 0.0142 |
| XPO6\|211982_x_at**** | 5.65 (0.09) | 5.03 (0.16) | 0 | 0.0131 |
| KDM3A\|212689_s_at**** | 6.41 (0.06) | 5.84 (0.18) | 0.0097 | 0.0142 |
| UBP1\|218082_s_at**** | 4.97 (0.18) | 3.89 (0.23) | 0.009 | 0.0142 |

*among top 500 **among top 300 ***among top 200 ****among top 100