## Appendix A: Computation of the fitness score, $S$, for a given chromosome (set of SNPs)

We desire a fitness score, $S$, that is small if the chromosome's $d$ SNPs are unrelated to disease risk and large if they have a strong joint relationship to risk. For $S$, we use a quadratic form, akin to a paired Hotelling's $T^{2}$ statistic in structure. The vector components for the chromosome take one of five values $(-2,-1,0,1,2)$ based on case-minus-control differences in minor allele counts in each family. (We use "control" generically to mean complement for case-parents triads or unaffected sibling for disease-discordant sibling pairs.). Calculation of mean vectors and covariances is weighted to reflect each family's informativeness about a possible joint effect.

## Weights and the Weighted Mean Vector

We incorporate family weights into $S$ to improve detection of multi-SNP disease associations, allowing for the possibility that each SNP has a small marginal association with disease and that the combination of alleles associated with elevated risk may be uncommon. We reasoned that a family is more informative about risk when the case and control differ in allele count at multiple SNPs in the chromosome; therefore, we upweight families with multiple nonzero differences to improve identification of differential joint transmissions of that chromosome's alleles (either major or minor) to the case. Loci where both case and control are heterozygous can also be somewhat informative, so they also contribute some to the weight.

Consider a chromosome including $d$ SNPs. Let:
$i$ index the family, $i \in\{1,2,3 \cdots, N\}$,
$j$ index the SNPs in that chromosome, $j \in\{1,2,3 \cdots, d\}$,
$\boldsymbol{D}_{\boldsymbol{i}}$ be the $d$-vector of minor allele counts for the case in family $i$,
$\boldsymbol{C}_{\boldsymbol{i}}$ be the $d$-vector of minor allele counts for the control in family $i$,
$\boldsymbol{x}_{\boldsymbol{i}}$ be the $d$-vector of differences between case and control allele counts: $\boldsymbol{x}_{\boldsymbol{i}}=\boldsymbol{D}_{\boldsymbol{i}}-\boldsymbol{C}_{\boldsymbol{i}}$,

Using the following indicators,

$$
\delta_{i j}= \begin{cases}1 & \text { if } x_{i j} \neq 0 \\ 0 & \text { otherwise }\end{cases}
$$

$$
b_{i j}= \begin{cases}1 & \text { if } D_{i j}=1 \text { and } C_{i j}=1 \\ 0 & \text { otherwise }\end{cases}
$$

we define the weight for the $i^{\text {th }}$ family as:

$$
w_{i}= \begin{cases}0 & \text { if } \delta_{i j}=0 \forall j \\ f\left(m_{d} \sum_{j} \delta_{i j}+m_{1} \sum_{j} b_{i j}\right) & \text { otherwise }\end{cases}
$$

where $f(x)$ is a function that raises a user-defined integer to the power of $x$ and that defaults to $f(x)=2^{x} ;$ and $m_{d}$ and $m_{1}$ are user-defined tuning parameters with defaults $m_{d}=2$ and $m_{1}=1$. With these weights, we construct a weighted mean case-control genotype difference vector across families as:

$$
\overline{\boldsymbol{x}}_{\boldsymbol{w}}^{*}=\frac{\sum_{i} w_{i} \boldsymbol{x}_{\boldsymbol{i}}}{w} \text { where } w=\sum_{i} w_{i}
$$

Intuitively, if a SNP-set functions epistatically, its alleles should be jointly carried by cases more frequently than by their matched control. Thus, the vector $\overline{\boldsymbol{x}}_{\boldsymbol{w}}^{*}$ should tend to be longer for epistatic SNP-sets and, geometrically, should also tend to point into a particular orthant defined by the component-specific signs, depending on the joint effect on risk.

## Data Driven Adaptive Recoding of SNPs showing a Recessive Mode of Inheritance

The computation of $\overline{\boldsymbol{x}}_{\boldsymbol{w}}^{*}$ treats one copy versus two copies of the variant allele as equivalent to zero copies versus one copy and treats zero versus two copies as a bigger difference. This computation ignores a possibly recessive mode of inheritance, where two copies of an allele might be required to confer increased disease risk. Intuitively, if an overwhelming excess of cases who jointly carry risk alleles at every locus carry two copies for a particular SNP, one can reasonably guess that that SNP should instead be coded as if recessive. We proceed in two steps: (1) nominating risk alleles and (2) assessing evidence for whether they act recessively.

Note that $\overline{\boldsymbol{x}}_{\boldsymbol{w}}^{*}$, via its component signs, indicates which allele was over-transmitted to cases for each SNP in the chromosome. With our initial minor allele coding, a positive-valued component of $\overline{\boldsymbol{x}}_{\boldsymbol{w}}^{*}$ for a given SNP implicates the minor allele as conferring risk, whereas a negative value implicates the
major ('wild type') allele. We refer to the implicated alleles as "provisional risk alleles," shortened here to "risk alleles" for convenience. We define an indicator for cases who carry one or more copies of the risk allele at every locus of that chromosome as:

$$
r_{D i}= \begin{cases}1 & \text { if the case in the } i^{t h} \\ 0 & \text { otherwise }\end{cases}
$$

Similarly for the controls, we define:

$$
r_{C i}= \begin{cases}1 & \text { if the control in the } i^{t h} \text { family carries at least one risk allele for all } d \text { SNPs } \\ 0 & \text { otherwise }\end{cases}
$$

And finally, we define an indicator variable for whether family $i$ is informative:

$$
I_{W i}= \begin{cases}1 & \text { if the } i^{t h} \text { family weight }>0 \\ 0 & \text { otherwise }\end{cases}
$$

We first restrict attention to cases with both $r_{D i}=1$ and $\mathrm{I}_{W i}=1$. Let $n_{D}$ denote the number of these cases: $n_{D}=\sum_{i} r_{D i} I_{W i}$. For the $j^{t h}$ SNP in the chromosome, we compute the proportion of these cases that carry two copies of that SNP's risk allele, $\hat{p}_{D j}$. Then, we restrict attention to controls with both $r_{C i}=1$ and $\mathrm{I}_{W i}=1$, similarly letting $n_{C}=\sum_{i} r_{C i} I_{W i}$, and compute an analogous proportion, $\hat{p}_{C j}$.

Next, we examine whether $\hat{p}_{D j}$ is large enough to warrant recoding the $j^{t h}$ SNP. We begin by computing a test statistic, $\xi_{1 j}=\frac{\hat{p}_{D j}-\tilde{p}}{\left.\sqrt{ } \tilde{p}(1-\tilde{p}) / n_{D}\right\}}$, where $\tilde{p}$ is an analyst-specified reference proportion, which we set to 0.75 as default. If $\xi_{1 j}$ is large, recessive coding may be appropriate. If, however, the $j^{\text {th }} \mathrm{SNP}$ is common and does not follow a recessive risk model, both $\hat{p}_{D j}$ and $\xi_{1 j}$ may still be large. To assess this possibility, we compute $n_{D} \hat{p}_{D j}, n_{D}\left(1-\hat{p}_{D j}\right), n_{C} \hat{p}_{C j}$, and $n_{C}\left(1-\hat{p}_{C j}\right)$. If all of these expected counts are at least 5 , we further compare $\hat{p}_{D j}$ to $\hat{p}_{C j}$ by computing a second statistic, $\xi_{2 j}=$ $\frac{\hat{p}_{D j}-\hat{p}_{C j}}{\left.\sqrt{\left\{\bar{p}_{j}\left(1-\bar{p}_{j}\right)\left(\frac{1}{n_{D}}+\frac{1}{n_{C}}\right\}\right.}\right\}}$, where $\bar{p}_{j}=\frac{n_{D} \hat{p}_{D j}+n_{C} \hat{p}_{C j}}{n_{D}+n_{C}}$. If min $\left(\xi_{1 j}, \xi_{2 j}\right)$ exceeds a reference threshold (defaulting to 1.64), we recode the $j^{\text {th }}$ SNP as recessive. Alternately, if the condition on expected counts is not satisfied, we recode if $\xi_{1 j}$ exceeds the reference threshold. In effect, we recode the SNP if we are at least $95 \%$ confident (using a one-sided confidence interval) that $\hat{p}_{D j}$ exceeds a high
reference threshold, and, where possible, also $95 \%$ confident that the case proportion $\hat{p}_{D j}$ exceeds the control proportion $\hat{p}_{C j}$.

If a SNP is declared to follow a recessive mode of inheritance within a particular chromosome, we proceed by recomputing the family weights and $\overline{\boldsymbol{x}}_{\boldsymbol{w}}^{*}$. Specifically, in computing the difference sign vectors, $\boldsymbol{x}_{\boldsymbol{i}}$, for a recessive locus, we require that either the case or matched control carry two copies of the risk allele for a difference at $\mathrm{SNP} j$ to be counted. Unlike the initial count coding approach, these differences may only take one of three values $\in\{-1,0,1\}$. Likewise, the indicator $b_{i j}$, corresponding to whether the case and complement are both heterozygous at $\operatorname{SNP} j$, is set to zero when computing the family weights for that chromosome.

Re-coding is undertaken once per chromosome per generation and does not propagate to other chromosomes; that is, a SNP can be coded as recessive in some chromosomes and not in others, depending on the other SNPs in the chromosome. Similarly, if a SNP is re-coded in a chromosome for one generation, that SNP will be re-evaluated if the chromosome propagates to the next generation. While this strategy may slow down the algorithm some, we are concerned that a blanket re-coding across chromosomes and generations would lead to poor performance.

## Computation of a Shrinkage Factor for the Weighted Mean Vector

After coding each SNP in the chromosome as recessive or not, we calculate a shrinkage factor, $q$, to apply in re-calculating the weighted mean vector. The shrinkage factor serves as a check that our weighting approach successfully identifies provisional risk genotypes that are jointly carried by cases more often than controls. We base it on the idea that, provided that the provisional risk genotypes truly reflect underlying risk, a high proportion among those who carry a provisional risk genotype at all $d$ loci should be cases compared to controls. In principle, $q$ can range from 0 to 1 . It is designed to shrink $\overline{\boldsymbol{x}}_{\boldsymbol{w}}$ towards the zero-vector when cases make up a small proportion of the total cases plus controls who carry all $d$ risk genotype(s). Specifically, we identify cases and controls who carry the provisional risk genotype (i.e., 2 copies if recessive, 1 or 2 copies if not) at all $d$ loci in the
chromosome in families where only the case or only the control carries the risk genotype(s) at every locus.

For cases, slightly modifying previously defined notation to now accommodate recessive SNPs, let

$$
r_{D i}= \begin{cases}1 & \text { if the case in the } i^{t h} \text { family carries a risk genotype for all } d \text { SNPs } \\ 0 & \text { otherwise }\end{cases}
$$

Analogously, for controls, let

$$
r_{C i}= \begin{cases}1 & \text { if the control in the } i^{t h} \text { family carries a risk genotype for all } d \text { SNPs } \\ 0 & \text { otherwise }\end{cases}
$$

Further define an indicator variable, $I_{r i}$,:

$$
I_{r i}= \begin{cases}1 & \text { only the case or only the control in the } i^{t h} \text { family carries a risk genotype for all } d \text { SNPs } \\ 0 & \text { otherwise }\end{cases}
$$

We compute the shrinkage factor, $q$, as:

$$
q=\frac{\sum_{i} I_{r i} r_{D i}}{\sum_{i} I_{r i} r_{D i}+\sum_{i} I_{r i} r_{C i}}
$$

If $q \leq 0.5$, indicating at least as many controls as cases carry the risk genotypes, we automatically reset $q$ to a very small positive value, $10^{-10}$. We do not allow $q=0$ because the fitness score, as defined below, must be positive. We finally use $q$ to shrink the weighted mean vector:

$$
\overline{\boldsymbol{x}}_{\boldsymbol{w}}=q \overline{\boldsymbol{x}}_{\boldsymbol{w}}^{*}
$$

## Computation of the Fitness Score

Using $\bar{x}_{w}$, we define a covariance matrix: $\widehat{\boldsymbol{\Sigma}}=\frac{\sum_{i} w_{i}\left(x_{i}-\bar{x}_{w}\right)\left(x_{i}-\bar{x}_{w}\right)^{T}}{w}$ where we assign elements $\widehat{\boldsymbol{\Sigma}}_{\boldsymbol{p}, \boldsymbol{q}}=0$ if SNPs $p$ and $q$ are not in linkage. We assume SNPs located on different biological chromosomes are not in linkage, but the GADGETS software allows analysts to flexibly assign pairs of SNPs to have zero covariance. Using $\overline{\boldsymbol{x}}_{\boldsymbol{w}}$ and $\widehat{\boldsymbol{\Sigma}}$, we construct the fitness score: $S=w \bar{x}_{w}^{T} \hat{\Sigma}^{-1} \bar{x}_{w}$. If $\widehat{\boldsymbol{\Sigma}}$ is not invertible, we instead use the pseudoinverse. Because fitness scores are often very large, the software reports fitness scores divided by 1000 to improve readability.

## Appendix B. Imposing crossover and mutation.

A tuning parameter determines the proportion of chromosomes passed from the current generation to the next that are subjected to crossover; the default is $80 \%$. Crossover requires two distinct chromosomes (a chromosome cannot be crossed with a duplicate of itself), so the number selected for crossover must be even. In this setting "crossover" refers to the swapping of some subset of SNPs between two chromosomes. Let $p_{s}$ be the number of chromosomes in the population and recall that one chromosome with the highest fitness is passed unchanged. If $80 \%$ of $p_{s}-1$ is odd, we choose the next higher integer. For that subset, we randomly partition the chromosomes into pairs. We execute crossover with the following steps.

1. Suppose chromosome 1 and chromosome 2 are assigned to crossover, and chromosome 1 has a higher fitness score than chromosome 2. We identify any SNPs contained in both and remove them from consideration to avoid the possibility of a new chromosome containing duplicate SNPs. The remaining SNPs are each either in chromosome 1 or in chromosome 2. Suppose each chromosome has $k$ such SNPs. If the chromosomes originally had no SNPs in common, then $k=$ $d$, the chromosome size. In this case, to avoid potentially exchanging two full chromosomes, we reset $k=d-1$. We then randomly sample an integer, $n_{\text {cross }} \in\{1, \ldots, k\}$ as the number of SNPs that will be crossed over.
2. Given $n_{\text {cross }}$ and the SNPs eligible for crossover in those two chromosomes, we evaluate the contributions of each to the fitness of their current chromosomes using the statistic $t_{j}=\frac{\left|\bar{x}_{w j}\right|}{\widehat{\sigma}_{j}}$, where $\bar{x}_{w j}$ is the element of the weighted mean vector corresponding to SNP $j$ and $\hat{\sigma}_{j}$ is the square root of the $j^{\text {th }}$ diagonal element of $\widehat{\boldsymbol{\Sigma}}$ ( $\overline{\boldsymbol{x}}_{\boldsymbol{w}}$ and $\widehat{\boldsymbol{\Sigma}}$ are as defined in the main text). We regard $t_{j}$ as indicative of a SNP's contribution to a chromosome's fitness score, with higher values signaling higher contributions.
3. With $t_{j}$ for each of the eligible SNPs in each chromosome, we exchange SNPs between chromosomes. Recalling that chromosome 1 has a higher fitness score than chromosome 2, we
begin by exchanging the eligible SNP with the lowest $t_{j}$ on chromosome 1 and the eligible SNP with the highest $t_{j}$ on chromosome 2 , which should tend to improve the fitness of the higher scorer (although improvement is not guaranteed). We continue through all $n_{\text {cross }}$ eligible SNPs in sequence, replacing the SNP with the next lower $t_{j}$ in the higher fitness chromosome by the SNP with the next higher $t_{j}$ in the lower fitness chromosome.

The remaining $20 \%$ of chromosomes not crossed over are subjected to mutation. By default, SNPs inserted as mutations are sampled independently from the pool of input SNPs, with probability proportional to the $\sqrt{ } \chi^{2}$ statistics from likelihood ratio tests of their marginal effects, as estimated by conditioning on the set of transmitted and untransmitted genotypes (20). We choose SNPs in this way to reflect an assumption that the marginal association provides some information about the likelihood a SNP could be a member of a true risk set. Alternatively, analysts could manually specify the SNP sampling probabilities as a way to incorporate subject matter expert opinion or prior biological information. Mutation proceeds as follows.

1. Suppose there are $n_{\text {total }}$ total SNPs in the input data. At each new generation, we take a sample of size $n_{\text {total }}$ with replacement from the input SNPs, where sampling probabilities are proportional to the square roots of the marginal association $\chi^{2}$ statistics. SNPs in this pool are eligible to be inserted into mutated chromosomes for the current generation. SNPs with a higher sampling probability may occur multiple times in the pool. This step avoids repeated sampling without replacement with non-uniform probabilities from the input SNPs, thereby substantially speeding up computation while retaining the benefit of favoring SNPS with larger marginal effects.
2. For a given chromosome, we randomly sample an integer, $n_{m} \in\{1, \ldots, d\}$, and choose to mutate $n_{m}$ of the chromosome's SNPs.
3. We identify the SNP with the lowest $t_{j}$ in the chromosome and replace it with a sampled SNP from the eligible pool generated in step 1 (prohibiting duplicate SNPs on the same chromosome). We mutate each SNP with the next smaller $t_{j}$ sequentially until completing $n_{m}$ mutations.

## Appendix C: Application of the genetic algorithm:

## Initialization:

1. We begin by pre-processing the input autosomal genetic data. For case-parents data, we combine all the parents' genetic data to identify the minor allele for each SNP. For affected/unaffected sibling pairs, we combine the siblings' genetic data to identify the minor alleles. We then re-code all SNPs such that we count copies of the minor allele. For case-parents data, we then compute each SNP's minor allele count in the complement, defined as mother count + father count - case count.
2. For each SNP in the input data, we estimate the minor allele frequency (MAF) based on the combined parental genotypes or the combined sibling pair genotypes and remove SNPs with MAFs below a pre-specified threshold. This threshold is a tuning parameter set to 0 (no filtering) by default.
3. We estimate SNP-specific marginal associations with disease status under a log-additive risk model, conditioning on the set of transmitted and untransmitted genotypes (20).

## Island Model:

Any genetic algorithm requires a mechanism to generate a diverse set of chromosomes that enable the algorithm to effectively explore the solution space and avoid premature convergence. We implement a technique known as an island model (21). A genetic algorithm using an island model, instead of evolving a single population with a large number of chromosomes, simultaneously evolves many subpopulations, or islands, each with a smaller number of chromosomes. These islands evolve independently for many generations, with migration of top scoring chromosomes permitted among small clusters of islands at intervals of a predetermined number of generations. Because the islands evolve largely independently with minimal migration, the island model enables a diverse set of potential solutions and can take advantage of distributed computing resources to reduce run-times.

Once all islands have converged, we aggregate the final evolved populations to identify a final set of top chromosomes. We implement this island-model approach as follows:

1. We randomly generate 200 chromosomes for each island. For chromosomes containing fewer than five SNPs, we use 1000 islands; to accommodate the larger search spaces for chromosomes with five SNPs, we use 1500 islands; and we use 2000 islands for chromosomes with six SNPs.
2. For each island, that initial set of chromosomes is created by sampling SNPs at random (with equal probability) without replacement from a large collection of input SNPs. Users must specify the number of chromosomes and the number of SNPs per chromosome. By default, a given SNP can appear in at most one chromosome in each initial island subpopulation, but the same SNP can appear in multiple islands. If the input data has too few SNPs to accommodate this restriction (e.g., we have 10,000 input SNPs and want 4,000 chromosomes with three SNPs each), then SNPs are instead randomly sampled with replacement from the input data, allowing the same SNP to appear in more than one chromosome.
3. Islands are randomly partitioned into distinct clusters, each containing four islands. Islands in different clusters evolve completely independently, allowing use of distributed computing to simultaneously evolve many island clusters and significantly speed computation.
4. Within each island cluster, each island subpopulation independently evolves over 50 generations as described below in the section Iteration Over Generations. After the $50^{\text {th }}$ generation, we determine whether stopping criteria have been satisfied.
5. If stopping criteria are not satisfied, migration occurs among islands in the cluster. For each island, the chromosomes with the 20 highest and the 20 lowest fitness scores in the current subpopulation are identified and the lowest scoring chromosomes are removed. The top 20 chromosomes are copied and the copies migrate to the neighboring island (island one migrates to island two,..., island four migrates to island one).
6. This 50 -generation cycle continues until stopping criteria or a specified maximum number of generations, defaulting to 500 , is reached. The number of islands, population size per island, number of islands per cluster, and number of chromosomes that migrate between islands are all tunable parameters.

## Iteration Over Generations:

Let $p_{s}$ be the subpopulation size (number of chromosomes per island per generation).

1. The fitness score is computed for each chromosome in the subpopulation.
2. The top scoring chromosome is identified. If there is a tie, one chromosome is arbitrarily selected. One copy of the top scoring chromosome is automatically propagated unchanged to the next generation to guarantee that the top fitness score will not decrease from generation to generation.
3. The unique chromosomes in the subpopulation, including the top scorer, are identified (i.e., any duplicates are purged). A sample of $p_{s}-1$ of these is chosen with replacement and probability proportional to their fitness scores. These sampled chromosomes will serve as the 'parents' of the next generation. Specifically, we subject the 'parents' to crossover and mutation as described in Appendix B and propagate the resulting 'child' chromosomes to the subsequent generation.
4. We check stopping criteria at 50 -generation intervals. That is, we determine whether the top scoring chromosome has changed over the last 50 generations, independently for each island in a given cluster. If the top-scorer has not changed for any island, we stop iteration for all islands in the cluster. Note the top scoring chromosome does not need to be the same across the islands in the cluster. Otherwise, the algorithm continues for 500 generations.

At termination, we save the top scoring chromosome from the final generation population of each island and combine them into a final overall list. The number of distinct chromosomes in the overall list is typically substantially smaller than the total number saved because many chromosomes are identified in multiple islands.

## Appendix D: Quantifying Evidence for Epistasis

We examine whether a chromosome's fitness score is driven by marginal rather than synergistic effects of its component SNPs as follows. We begin by computing the fitness score for the chromosome, $S$, as specified in the main text. Then, to generate a no-epistasis null distribution of fitness scores, we execute the following permutation procedure. To begin, suppose none of the $d$ SNPs are in linkage disequilibrium (which can be analyst defined). Let:

I be a $N$-vector of the family indices in the observed data
$\boldsymbol{M}_{\boldsymbol{j}}$ be a $N$-vector of case minor allele counts for the $j^{\text {th }}$ SNP
$\boldsymbol{U}_{\boldsymbol{j}}$ be a $N$-vector of control minor allele counts for the $j^{t h}$ SNP
We create a permuted index vector for the $j^{t h}$ SNP, $\boldsymbol{I}_{\boldsymbol{p} \boldsymbol{j}}$, by randomly sampling $N$ integers without replacement from $\boldsymbol{I}$. Next, we use $\boldsymbol{I}_{\boldsymbol{p} \boldsymbol{j}}$ to re-order the genotypes in $\boldsymbol{M}_{\boldsymbol{j}}$ and $\boldsymbol{U}_{\boldsymbol{j}}$. Let:
$\boldsymbol{M}_{\boldsymbol{p} \boldsymbol{j}}$ be $\boldsymbol{M}_{\boldsymbol{j}}$ with genotypes in the order specified by $\boldsymbol{I}_{\boldsymbol{p} \boldsymbol{j}}$
$\boldsymbol{U}_{\boldsymbol{p} \boldsymbol{j}}$ be $\boldsymbol{U}_{\boldsymbol{j}}$ with genotypes in the order specified by $\boldsymbol{I}_{\boldsymbol{p} \boldsymbol{j}}$
Note this operation in effect permutes family labels, rather than disease status.

We conduct this procedure separately for each of the $d$ SNPs in the chromosome. Finally, we concatenate $\boldsymbol{M}_{\boldsymbol{p} \mathbf{1}}, \ldots, \boldsymbol{M}_{\boldsymbol{p} \boldsymbol{d}}$ into a $N$ by $d$ matrix of pseudo-family case genotypes, $\boldsymbol{M}_{\boldsymbol{p}}$, and, we similarly concatenate $\boldsymbol{U}_{\boldsymbol{p} 1}, \ldots, \boldsymbol{U}_{\boldsymbol{p} \boldsymbol{d}}$ into a $N$ by $d$ matrix of pseudo-family control genotypes, $\boldsymbol{U}_{\boldsymbol{p}}$. These genotype matrices preserve the marginal effects for each individual SNP, but any epistatic effects should be destroyed. Based on the genotypes in $\boldsymbol{M}_{\boldsymbol{p}}$ and $\boldsymbol{U}_{\boldsymbol{p}}$, we compute a fitness score, $S_{p}$. We repeat this entire procedure 10,000 times to generate a null distribution of fitness scores that we compare to the observed fitness score, $S$. If any of the $d$ SNPs are in linkage, we proceed similarly, except we use the same $\boldsymbol{I}_{\boldsymbol{p} \boldsymbol{j}}$ for each set of linked SNPs.

## Appendix E: Software Settings for Comparisons to Competitors

We used TrioFS as implemented in the Trio R package, version 3.28.0, available through Bioconductor (https://www.bioconductor.org/packages/release/bioc/html/trio.html). We used the MDR-PDT software, version 2.0.1.21, available at https://ritchielab.org/software/mdr-download. The EPISFA-LD method is not available as a standalone software package, but R scripts implementing the method are available at https://github.com/doublexism/episfa/blob/master/Simulation/functions.R. We used the version of this file available on May 14, 2021.

Each method requires a number of user-specified tuning parameters. We used default software arguments with the following exceptions. First, although not generally required for running GADGETS or TrioFS, for simplicity in carrying out comparisons we presumed omniscience by correctly specifying the risk-set size for GADGETS, MDR-PDT, and TrioFS. That is, we specifically searched for 3-SNP interactions for scenario 2 and 4-SNP interactions for scenario 5. We note that TrioFS requires specifying the maximum SNP-set size, but can dynamically return smaller sets. EPISFA-LD does not a require a user specified risk-set size and can return sets of any size.

For GADGETS in this application, because we used a limited number of candidate SNPs, we also used fewer islands. For scenario 2, we used 100 islands for 500 candidate SNPs and 20 islands for smaller numbers of candidate SNPs. For scenario 5, we used 300 islands for 500 candidate SNPs and 60 islands otherwise. TrioFS requires specification of the number of algorithm iterations and has documentation suggesting a number in the hundreds of thousands. For scenario 2, we used 500,000 iterations for 500 candidate SNPs and 100,000 iterations otherwise. For scenario 5, we used 1,500,000 iterations for 500 candidate SNPs and 300,000 iterations otherwise. We did not compute p-values for GADGETS or MDR-PDT and therefore did not execute permutations for either method. We used the 'episfa' function in the
authors' github R script to run EPISFA-LD with the tuning parameters specified in function 'episfa_sim'. Specifically, we set argument 'nfolds' to 10 , 'recursion' to 5 , 'criteria' to 'ebic', and used the matrix of complement pseudo-sibling genotypes for argument 'contrast'.

Each method was run on a single processor to assess relative performance with comparable computing resources. We note, however, that GADGETS can simultaneously use multiple processors via distributed computing to dramatically improve run times compared to single processor use. TrioFS lacks this capability. MDR-PDT can make use of multiple threads but only to re-run the method on permuted datasets, not for a single run on the original study data. Because TrioFS and GADGETS both use stochastic search approaches, their runtimes largely reflect choices regarding tuning parameters controlling convergence. Because MDR-PDT searches exhaustively, its run-times reflect the size of the search space. We did not carry out analyses for MDR-PDT applied to 500 candidate SNPs because the runs on smaller SNP sets conservatively suggest run-times that would exceed one-month.

Table S1. Simulation scenarios

|  | Scenario 1 | Scenario 2 |
| :---: | :---: | :---: |
| SNPs in SNP-set 1 <br> rs (SNP number) | rs6537495, rs7098516, rs4910793, rs10501820 $(960,1656,2625,4169)$ | $\begin{aligned} & \text { rs7090929, rs } 12421071, \text { rs } 17031482 \\ & (656,4688,6886) \end{aligned}$ |
| SNPs in SNP-set 2 <br> rs (SNP number) | $\begin{aligned} & \text { rs1731422, rs4237892, rs7985535, } \\ & \text { rs1487251 }(5877,6743,7979,8646) \end{aligned}$ | $\begin{aligned} & \text { rs2065089, rs7911843, rs1994548, rs10863137, } \\ & \text { rs953130 }(111,2009,3132,6600,8001) \end{aligned}$ |
| SNP frequency in the original data | SNP-set 1: 0.060 0.058 0.065 0.065 <br> SNP-set 2: 0.061 0.064 0.059 | $\begin{array}{cccc} 0.052 & 0.036 & 0.038 & 0.110 \\ 0.112 & 0.114 & 0.116 & 0.113 \end{array}$ |
| SNP genetic model <br> (dominant/recessive $\mathrm{D} / \mathrm{R}$ ) | $\begin{aligned} & \text { SNP-set 1: D-D-D-D } \\ & \text { SNP-set 2: D-D-D-D } \end{aligned}$ | $\begin{aligned} & \text { D-D-D-D } \\ & \text { D-D-D-D } \end{aligned}$ |
| Risk with neither SNP-set | 1.66/1000 | 1.66/1000 |
| Risk with SNP-set 1 | 401/1000 | 119.2/1000 |
| Risk with SNP-set 2 | 401/1000 | 197.8/1000 |
| Risk with both SNP-sets | 401/1000 | 952.6/1000 |
| Cases with SNP-set $1^{\text {a }}$ | 33, 28, 35, 38, 46 | 30, 32, 36, 32, 39 |
| Controls with SNP-set $1^{\text {a }}$ | 0, 0, 0, 0, 0 | 1, 0, 0, 0, 1 |
| Cases with SNP-set $2^{\text {a }}$ | 36, 37, 31, 45, 33 | 38, 44, 43, 52, 40 |
| Controls with SNP-set $2^{\text {a }}$ | $0,0,0,0,0$ | $0,0,0,0,0$ |

${ }^{\mathrm{a}}$ Separate entries separated by commas correspond to different replicates of the same scenario.

|  | Scenario 3 | Scenario 4 | Scenario 5 |
| :---: | :---: | :---: | :---: |
| SNPs in SNP-set 1 <br> rs (SNP number) | $\begin{aligned} & \text { rs } 10508738, \text { rs 17565737, } \\ & \text { rs1473938, rs7124944 } \\ & (715,1743,2562,4105) \end{aligned}$ | rs6537495 rs10748546 rs7117223 rs105018 $20(960,1729,2704,4169)$ | ```rs6537495 rs10748546 rs71172 23 rs10501820 (960,1729,2624, 4169)``` |
| SNPs in SNP-set 2 <br> rs (SNP number) | $\begin{aligned} & \text { rs16915128, rs 1005890, } \\ & \text { rs10492405 (5429, 6717, } \\ & \text { 7937)(three singleton } \\ & \text { SNPs) } \end{aligned}$ | $\begin{aligned} & \text { rs1731422,rs4761726,rs7985535, rs359334 } \\ & (5877,6709,7979,8658) \end{aligned}$ | $\begin{aligned} & \text { rs1731422,rs4761726,rs798553 } \\ & 5, \text { rs359334 } \\ & (5877,6709,7979,8658) \end{aligned}$ |
| SNP frequency in the original data | $\begin{aligned} & 0.0990 .0990 .0990 .100 \\ & 0.0980 .0980 .099 \end{aligned}$ | $\begin{array}{llll} 0.060 & 0.278 & 0.265 & 0.065 \\ 0.061 & 0.276 & 0.059 & 0.279 \end{array}$ | $\begin{array}{llll} 0.060 & 0.278 & 0.612 & 0.065 \\ 0.061 & 0.276 & 0.059 & 0.279 \end{array}$ |
| SNP genetic model (dominant/recessive $\mathrm{D} / \mathrm{R}$ ) | $\begin{aligned} & \text { D-D-D-D } \\ & \text { D, D, D } \end{aligned}$ | $\begin{aligned} & \text { D-R-R-D } \\ & \text { D-R-D-R } \end{aligned}$ | $\begin{aligned} & \text { D-R-R-D } \\ & \text { D-R-D-R } \end{aligned}$ |
| Risk with neither SNP-set | 1.66/1000 | 0.8/1000 | 0.8/1000 |
| Risk with SNP-set 1 | 401/1000 | 354/1000 | 52/1000 |
| Risk with SNP-set 2 | 4.96/1000 (one singleton SNP) | 401/1000 | 401/1000 |
| Risk with both SNP-sets | 668/1000 (SNP-set 1 \& all three singleton SNPs) | 998/1000 | 980/1000 |
| Cases with SNP-set 1 | 48, 56, 42, 56, 50 | 35, 38, 36, 50, 32 | 36, 45, 49, 41 |
| Controls with SNP-set 1 | 1, $0,1,0,2$ | 0, $0,0,1,0$ | 1, 0, 2, 0 |
| Cases with SNP-set 2 | NA | 31, 37, 25, 34, 33 | 49, 31, 36, 38 |
| Controls with SNP-set 2 | NA | $0,0,0,0,0$ | 0, $0,0,0$ |

Table S2. Marginal log relative risk (standard error) for SNPs in simulated risk-related SNPsets. Estimates are from conditional logistic regression using a log additive model.

## Replicate

 SNP-set 1 SNPs
## SNP-set 2 SNPs

## Simulation 1

| 1 | $0.23(0.1), 0.12(0.1), 0.41(0.1), 0.36(0.1)$ |
| :--- | :--- |
| 2 | $0.36(0.1), 0.09(0.1), 0.27(0.1), 0.24(0.1)$ |
| 3 | $0.14(0.1), 0.42(0.1), 0.38(0.1), 0.15(0.1)$ |
| 4 | $0.35(0.1), 0.35(0.1), 0.43(0.1), 0.17(0.1)$ |
| 5 | $0.20(0.1), 0.44(0.1), 0.29(0.1), 0.33(0.1)$ |

## Simulation 2

| 1 | $0.33(0.1), 0.36(0.2), 0.24(0.2)$ |
| :--- | :--- |
| 2 | $0.18(0.1), 0.24(0.2), 0.45(0.2)$ |
| 3 | $0.36(0.1), 0.36(0.2), 0.17(0.1)$ |
| 4 | $0.30(0.1), 0.12(0.2), 0.61(0.2)$ |
| 5 | $0.43(0.1), 0.10(0.2), 0.63(0.2)$ |

## Simulation 3

$1 \quad 0.07(0.1), 0.25(0.1), 0.44(0.1), 0.12(0.1)$
$2 \quad-0.04(0.1), 0.39(0.1), 0.31(0.1), 0.38(0.1)$
3
0.23 ( 0.1 ), 0.08 ( 0.1 ), $0.15(0.1), 0.16(0.1)$

4
0.19 (0.1), 0.02 (0.1), 0.26 (0.1), 0.18 (0.1)
$5 \quad 0.04(0.1), 0.11(0.1), 0.34(0.1), 0.20(0.1)$

## Simulation 4

$1 \quad 0.16(0.1), 0.09(0.1), 0.14(0.1), 0.23(0.1)$
$20.33(0.1), 0.09(0.1), 0.08(0.1), 0.47(0.1)$
3
0.30 ( 0.1 ), 0.00 ( 0.1 ), $0.09(0.1), 0.35(0.1)$

4
0.31 ( 0.1 ), 0.06 ( 0.1 ), 0.07 ( 0.1$), 0.41$ ( 0.1 )
$5 \quad 0.23(0.1), 0.11(0.1), 0.11(0.1), 0.24(0.1)$

$$
\begin{gathered}
0.13(0.1), 0.05(0.1), 0.36(0.1), 0.02(0.1) \\
0.24(0.1), 0.27(0.1), 0.23(0.1), 0.11(0.1) \\
-0.06(0.1), 0.00(0.1), 0.16(0.1),-0.01(0.1) \\
0.24(0.1), 0.04(0.1), 0.43(0.1), 0.17(0.1) \\
0.45(0.1), 0.16(0.1), 0.36(0.1), 0.01(0.1)
\end{gathered}
$$

## Simulation 5

$10.15(0.1), 0.08(0.1), 0.08(0.1), 0.20(0.1)$
$20.17(0.1), 0.11(0.1), 0.03(0.1), 0.34(0.1)$
3
0.46 ( 0.1 ), - $0.06(0.1), 0.07(0.1), 0.22(0.1)$
$4 \quad 0.29(0.1), 0.18(0.1), 0.03(0.1), 0.28(0.1)$

Table S3. Simulation 1 recovery of risk-related SNP-sets directly from GADGETS chromosomes: entry is the rank ( $1=$ highest $)$ by fitness score of the first chromosome containing the number of risk set SNPs specified by the column. NF indicates that the highest ranked chromosome to meet the conditions did not appear among the number of unique chromosomes reported by GADGETS.

| Number of SNPs from SNP-set 1 |  |  |  | Number of SNPs from SNP-set 2 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\geq 1$ | $\geq 2$ | $\geq 3$ | 4 | $\geq 1$ | $\geq 2$ | $\geq 3$ |

## Replicate 1

| $\mathrm{d}=2$ | NF | NF | - | - | 1 | 1 | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 3 | 3 | 3 | - | 1 | 1 | 1 | - |
| $\mathrm{d}=4$ | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=5$ | 17 | 17 | 17 | 17 | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=6$ | 251 | 251 | 251 | 251 | 1 | 1 | 1 | 1 |

## Replicate 2

| $\mathrm{d}=2$ | NF | NF | - | - | $N F$ | $N F$ | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 2 | 2 | 2 | - | 1 | 1 | 1 | - |
| $\mathrm{d}=4$ | 17 | NF | NF | NF | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=5$ | 11 | 11 | 11 | 11 | 1 | 1 | 1 | 1 |
| $d=6$ | 37 | 37 | 37 | 37 | 1 | 1 | 1 | 1 |

Replicate 3

| $d=2$ | 2 | 2 | - | - | $N F$ | $N F$ | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d=3$ | 1 | 1 | 1 | - | $N F$ | $N F$ | $N F$ | - |
| $d=4$ | 1 | 1 | 1 | 1 | $N F$ | $N F$ | $N F$ | $N F$ |
| $d=5$ | 1 | 1 | 1 | 1 | $N F$ | $N F$ | $N F$ | $N F$ |
| $d=6$ | 1 | 1 | 1 | 1 | 176 | $N F$ | $N F$ | $N F$ |

Replicate 4

| $\mathrm{d}=2$ | 1 | 1 | - | - | 2 | 2 | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 2 | 2 | 2 | - | 1 | 1 | 1 | - |
| $\mathrm{d}=4$ | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| $\mathrm{~d}=5$ | 1 | 1 | 1 | 1 | 3 | 3 | 3 | 3 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 1 | 30 | 30 | 30 | 30 |

Replicate 5

| $\mathrm{d}=2$ | 1 | 1 | - | - | 5 | 5 | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | - | 6 | 6 | 6 | - |
| $\mathrm{d}=4$ | 1 | 1 | 1 | 1 | 10 | 10 | 10 | 10 |
| $\mathrm{~d}=5$ | 1 | 1 | 1 | 1 | 34 | 34 | 34 | 34 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 1 | 213 | 213 | 213 | 213 |

Table S4. Simulation 2 recovery of risk-related SNP-sets directly from GADGETS chromosomes: entry is the rank ( $1=$ highest $)$ by fitness score of the first chromosome containing the number of risk set SNPs specified by the column. NF indicates that the highest ranked chromosome to meet the conditions did not appear among the number of unique chromosomes reported by GADGETS.

| Number of SNPs from SNP-set 1 |  |  | Number of SNPs from SNP-set 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\geq 1$ | $\geq 2$ | 3 | $\geq 1$ | $\geq 2$ | $\geq 3$ | $\geq 4$ | 5 |

## Replicate 1

| $\mathrm{d}=2$ | 1 | 1 | - | 3 | 3 | - | - | - |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{d}=3$ | 1 | 1 | 1 | 2 | 2 | 2 | - | - |
| $\mathrm{d}=4$ | 1 | 1 | 1 | 4 | 4 | 4 | 4 | - |
| $\mathrm{d}=5$ | 1 | 1 | 1 | 3 | 3 | 3 | 3 | 3 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 5 | 5 | 5 | 5 | 5 |

## Replicate 2

| $\mathrm{d}=2$ | 1 | 1 | - | 3 | NF | - | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | 9 | NF | NF | - | - |
| $\mathrm{d}=4$ | 1 | 1 | 1 | 7 | 28 | NF | NF | - |
| $\mathrm{d}=5$ | 1 | 1 | 1 | 50 | 50 | 50 | 50 | 50 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 74 | 74 | 74 | 74 | 74 |

Replicate 3

| $\mathrm{d}=2$ | 1 | 1 | - | $N F$ | $N F$ | - | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | NF | NF | NF | - | - |
| $\mathrm{d}=4$ | 1 | 1 | 1 | NF | NF | NF | NF | - |
| $\mathrm{d}=5$ | 1 | 1 | 1 | 61 | 61 | 61 | 61 | 61 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 126 | 126 | 126 | 126 | 126 |

Replicate 4

| $\mathrm{d}=2$ | 1 | 1 | - | $N F$ | $N F$ | - | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | 2 | 2 | 2 | - | - |
| $\mathrm{d}=4$ | 4 | 4 | 4 | 1 | 1 | 1 | 1 | - |
| $\mathrm{d}=5$ | 13 | 13 | 13 | 1 | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=6$ | 20 | 20 | 20 | 1 | 1 | 1 | 1 | 1 |

Replicate 5

| $d=2$ | 2 | 2 | - | $N F$ | $N F$ | - | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d=3$ | 1 | 1 | 1 | $N F$ | $N F$ | $N F$ | - | - |
| $d=4$ | 1 | 1 | 1 | $N F$ | $N F$ | $N F$ | $N F$ | - |
| $d=5$ | 1 | 1 | 1 | $N F$ | $N F$ | $N F$ | $N F$ | $N F$ |
| $d=6$ | 1 | 1 | 1 | $N F$ | $N F$ | $N F$ | $N F$ | $N F$ |

Table S5. Simulation 3 recovery of risk-related SNP-sets directly from GADGETS chromosomes: entry is the rank ( $1=$ highest $)$ by fitness score of the first chromosome containing the number of risk set SNPs specified by the column. NF indicates that the highest ranked chromosome to meet the conditions did not appear among the number of unique chromosomes reported by GADGETS.

| Number of SNPs from SNP-set 1 |  |  |  |  |  |  |  |  |  | Singleton 1 | Singleton 2 | Singleton 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\geq 1$ | $\geq 2$ | $\geq 3$ | 4 |  |  |  |  |  |  |  |  |  |

## Replicate 1

| $\mathrm{d}=2$ | 1 | NF | - | - | NF | NF | 4 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~d}=3$ | 1 | 1 | 1 | - | NF | NF | 9 |
| $\mathrm{~d}=4$ | 1 | 1 | 1 | 7 | NF | 64 | 13 |
| $\mathrm{~d}=5$ | 1 | 1 | 1 | 2 | NF | 223 | 56 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 1 | 302 | 212 | 121 |

## Replicate 2

| $\mathrm{d}=2$ | 1 | 1 | - | - | 2 | NF | NF |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | - | 6 | NF | 22 |
| $\mathrm{~d}=4$ | 1 | 1 | 1 | 1 | 13 | 96 | 31 |
| $\mathrm{~d}=5$ | 1 | 1 | 1 | 1 | 54 | 200 | 68 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 1 | 144 | 191 | 151 |

## Replicate 3

| $\mathrm{d}=2$ | NF | NF | - | - | $N F$ | $N F$ | $N F$ |
| :--- | :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~d}=3$ | NF | NF | NF | - | 27 | NF | NF |
| $\mathrm{d}=4$ | NF | NF | NF | NF | 114 | NF | NF |
| $\mathrm{d}=5$ | NF | NF | NF | NF | 5 | 424 | NF |
| $\mathrm{d}=6$ | 780 | NF | NF | NF | 23 | 257 | NF |

## Replicate 4

| $\mathrm{d}=2$ | NF | NF | - | - | $N F$ | $N F$ | 6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $d=3$ | NF | NF | NF | - | $N F$ | $N F$ | 2 |
| $\mathrm{~d}=4$ | 154 | NF | NF | NF | NF | NF | 1 |
| $\mathrm{~d}=5$ | 363 | 363 | NF | NF | NF | NF | 1 |
| $\mathrm{~d}=6$ | 153 | 153 | 399 | NF | NF | NF | 1 |

## Replicate 5

| $\mathrm{d}=2$ | NF | NF | - | - | NF | NF | NF |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | - | NF | NF | NF |
| $\mathrm{d}=4$ | 1 | 1 | 1 | 1 | NF | 71 | NF |
| $\mathrm{d}=5$ | 1 | 1 | 1 | 1 | NF | 144 | NF |
| $\mathrm{d}=6$ | 1 | 1 | 1 | 1 | NF | 23 | NF |

Table S6. Simulation 4 recovery of risk-related SNP-sets directly from GADGETS chromosomes: entry is the rank ( $1=$ highest $)$ by fitness score of the first chromosome containing the number of risk set SNPs specified by the column. NF indicates that the highest ranked chromosome to meet the conditions did not appear among the number of unique chromosomes reported by GADGETS.

|  | Number of SNPs from SNP-set 1 |  |  | Number of SNPs from SNP-set 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\geq 1$ | $\geq 2$ | $\geq 3$ | 4 | $\geq 1$ | $\geq 2$ | $\geq 3$ | 4 |
| Replicate 1 |  |  |  |  |  |  |  |  |
| $\mathrm{d}=2$ | NF | NF | - | - | NF | NF | - | - |
| $\mathrm{d}=3$ | NF | NF | NF | - | 1 | NF | NF | - |
| $\mathrm{d}=4$ | NF | NF | NF | NF | 1 | NF | NF | NF |
| $\mathrm{d}=5$ | NF | NF | NF | NF | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=6$ | NF | NF | NF | NF | 1 | 1 | 1 | 1 |

Replicate 2

| $\mathrm{d}=2$ | 1 | 1 | - | - | 8 | NF | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 2 | 2 | 4 | - | 1 | 1 | 1 | - |
| $\mathrm{d}=4$ | 2 | 2 | 2 | 2 | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=5$ | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 1 | 20 | 20 | 20 | 20 |

Replicate 3

| $\mathrm{d}=2$ | 1 | 1 | - | - | $N F$ | $N F$ | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | - | $N F$ | $N F$ | $N F$ | - |
| $\mathrm{d}=4$ | 1 | 1 | 1 | NF | NF | NF | NF | NF |
| $\mathrm{d}=5$ | 1 | 1 | 1 | NF | NF | NF | NF | NF |
| $\mathrm{d}=6$ | 1 | 1 | 1 | NF | NF | NF | NF | NF |

Replicate 4

| $\mathrm{d}=2$ | 2 | 2 | - | - | 1 | 1 | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | - | 3 | 3 | 3 | - |
| $\mathrm{d}=4$ | 1 | 1 | 1 | 1 | 2 | 2 | 2 | 2 |
| $\mathrm{~d}=5$ | 1 | 1 | 1 | 1 | 14 | 14 | 14 | 14 |
| $\mathrm{~d}=6$ | 1 | 1 | 1 | 1 | 62 | 62 | 62 | 62 |

Replicate 5

| $d=2$ | $N F$ | $N F$ | - | - | 1 | 1 | - | - |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d=3$ | 65 | $N F$ | $N F$ | - | 1 | 1 | 1 | - |
| $d=4$ | $N F$ | $N F$ | $N F$ | $N F$ | 1 | 1 | 1 | 1 |
| $d=5$ | $N F$ | $N F$ | $N F$ | $N F$ | 1 | 1 | 1 | 1 |
| $d=6$ | 200 | $N F$ | $N F$ | $N F$ | 1 | 1 | 1 | 1 |

Table S7. Simulation 5 recovery of risk-related SNP-sets directly from GADGETS chromosomes: entry is the rank ( $1=$ highest) by fitness score of the first chromosome containing the number of risk set SNPs specified by the column. NF indicates that the highest ranked chromosome to meet the conditions did not appear among the number of unique chromosomes reported by GADGETS.

|  | Number of SNPs from SNP-set 1 |  |  | Number of SNPs from SNP-set 2 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\geq 1$ | $\geq 2$ | $\geq 3$ | 4 | $\geq 1$ | $\geq 2$ | $\geq 3$ | 4 |
| Replicate 1 |  |  |  |  |  |  |  |  |
| $\mathrm{d}=2$ | NF | NF | - | - | 7 | 7 | - | - |
| $\mathrm{d}=3$ | NF | NF | NF | - | 1 | 1 | 1 | - |
| $\mathrm{d}=4$ | NF | NF | NF | NF | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=5$ | 6 | 6 | 6 | NF | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=6$ | 36 | 36 | 36 | NF | 1 | 1 | 1 | 1 |

Replicate 2

| $\mathrm{d}=2$ | 4 | 4 | - | - | $N F$ | $N F$ | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d=3$ | 1 | 1 | 1 | - | $N F$ | $N F$ | $N F$ | - |
| $d=4$ | 1 | 1 | 1 | $N F$ | 4 | 4 | 4 | $N F$ |
| $d=5$ | 1 | 1 | 1 | $N F$ | 7 | 7 | 7 | 7 |
| $d=6$ | 1 | 1 | 1 | 23 | 5 | 5 | 5 | 5 |

Replicate 3

| $\mathrm{d}=2$ | 4 | 4 | - | - | $N F$ | $N F$ | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{d}=3$ | 1 | 1 | 1 | - | 2 | 2 | 2 | - |
| $\mathrm{d}=4$ | 4 | 4 | 4 | NF | 1 | 1 | 1 | 1 |
| $\mathrm{~d}=5$ | 13 | 13 | 13 | NF | 1 | 1 | 1 | 1 |
| $d=6$ | 40 | 40 | 40 | 96 | 1 | 1 | 1 | 1 |

Replicate 4

| $d=2$ | 1 | 1 | - | - | $N F$ | $N F$ | - | - |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d=3$ | 1 | 1 | 1 | - | $N F$ | $N F$ | $N F$ | - |
| $d=4$ | 1 | 1 | 1 | 5 | $N F$ | $N F$ | $N F$ | $N F$ |
| $d=5$ | 1 | 1 | 1 | 13 | 119 | $N F$ | $N F$ | $N F$ |
| $d=6$ | 4 | 4 | 4 | 4 | 1 | 1 | 1 | 1 |

Table S8. P-values based on permutation, both for a global test that combines chromosome sizes (d) 2-6 and tests for respective specific chromosome sizes. These are based on the observed data set and 100 permuted data sets. The first column specifies the maximal number of top chromosomes that could be used to construct the test. The next five columns report the number of top chromosomes that were used in practice. The maximal number exceeds the number used when GADGETS returns fewer chromosomes than the maximal number specified for the observed data or any permute.

| Top <br> Chromosomes | $k$ Used per Chromosome Size (d) |  |  |  |  | Global <br> Test P | Tests Based on Specific Chromosome Sizes |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 3 | 4 | 5 | 6 |  | 2 | 3 | 4 | 5 | 6 |
| Simulation 1, Replicate 2 |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 3 | 10 | 10 | 10 | 10 | 0.02 | 0.41 | 0.13 | 0.02 | 0.01 | 0.01 |
| 30 | 3 | 10 | 30 | 30 | 30 | 0.02 | 0.41 | 0.13 | 0.15 | 0.01 | 0.01 |
| 50 | 3 | 10 | 50 | 50 | 50 | 0.02 | 0.41 | 0.13 | 0.19 | 0.01 | 0.01 |
| Simulation 2, Replicate 3 |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 1 | 6 | 10 | 10 | 10 | 0.01 | 0.31 | 0.02 | 0.01 | 0.01 | 0.01 |
| 30 | 1 | 6 | 30 | 30 | 30 | 0.01 | 0.31 | 0.02 | 0.01 | 0.01 | 0.01 |
| 50 | 1 | 6 | 47 | 50 | 50 | 0.01 | 0.31 | 0.02 | 0.01 | 0.01 | 0.01 |
| Simulation 3, Replicate 1 |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 4 | 10 | 10 | 10 | 10 | 0.01 | 0.76 | 0.17 | 0.01 | 0.01 | 0.02 |
| 30 | 4 | 18 | 30 | 30 | 30 | 0.01 | 0.76 | 0.20 | 0.03 | 0.01 | 0.02 |
| 50 | 4 | 18 | 50 | 50 | 50 | 0.02 | 0.76 | 0.20 | 0.05 | 0.01 | 0.02 |
| Simulation 4, Replicate 2 |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 2 | 8 | 10 | 10 | 10 | 0.01 | 0.05 | 0.01 | 0.01 | 0.01 | 0.01 |
| 30 | 2 | 8 | 30 | 30 | 30 | 0.01 | 0.05 | 0.01 | 0.01 | 0.01 | 0.01 |
| 50 | 2 | 8 | 50 | 50 | 50 | 0.01 | 0.05 | 0.01 | 0.01 | 0.01 | 0.01 |
| Simulation 5, Replicate 2 |  |  |  |  |  |  |  |  |  |  |  |
| 10 | 2 | 10 | 10 | 10 | 10 | 0.01 | 0.08 | 0.01 | 0.01 | 0.01 | 0.01 |
| 30 | 2 | 12 | 30 | 30 | 30 | 0.01 | 0.08 | 0.01 | 0.01 | 0.01 | 0.01 |
| 50 | 2 | 12 | 50 | 50 | 50 | 0.01 | 0.08 | 0.01 | 0.01 | 0.01 | 0.01 |

Table S9. Epistasis h-values for the top ranked chromosome for each chromosome size (d) from a data set representing each scenario considered.

| Simulation(Replicate) | $d=2$ | $d=3$ | $d=4$ | $d=5$ | $d=6$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $1(2)$ | $0.0179 * *$ | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| $2(3)$ | 0.0002 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| $3(1)$ | 0.0033 | 0.0014 | 0.0003 | 0.0001 | 0.0001 |
| $4(2)$ | 0.0001 | 0.0001 | 0.0001 | 0.0001 | 0.0001 |
| $5(2)$ | $--*$ | 0.0001 | 0.0001 | 0.0001 | 0.0001 |

*The epistasis h-value could not be computed because all SNPs were located on the same biological chromosome and considered to be in linkage.
**No SNPs from a risk-related SNP-set were contained in the top ranked chromosome.

Table S10. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 2 among 347 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 889 families from Asian populations. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a single distinct chromosome of size 2. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association $(\mathrm{p}=0.01)$. Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a risk-related genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a no-epistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  | RSID |  | Relative Risk |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP1 | SNP2 | SNP1 | SNP2 | Joint |  |  |
| ABCA4(1) | IRF6(1) | rs560426 | rs2013162 | 1.4 | 1.2 | 1.5 | -- |  |

*Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.
**-- indicates SNP sets where the epistasis h-value could not be computed because all SNPs were located on the same biological chromosome.

Table S11. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 3 among 347 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 889 families from Asian populations. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a total of 3 distinct chromosomes of size 3. Chromosomes are sorted by fitness score in descending order. The global test of the omnibus null of no association across chromosome sizes $2-6$ for these data indicated the presence of an association ( $p=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a risk-related genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a no-epistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  |  | RSID |  |  | Relative Risk |  |  |  | $\mathrm{h}^{* *}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP3 | SNP1 | SNP2 | SNP3 | SNP1 | SNP2 | SNP3 | Joint |  |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | rs560426 | rs4951357 | rs2013162 | 1.4 | 1.1 | 1.2 | 1.7 | -- |
| ABCA4(1) | IRF6(1) | GRID2(4) | rs560426 | rs2013162 | rs12506428 | 1.4 | 1.2 | 1.2 | 1.8 | 0.2032 |
| ABCA4(1) | IRF6(1) | $\begin{gathered} \text { LOC10272496 } \\ 8(20) \end{gathered}$ | rs560426 | rs2013162 | rs13041247 | 1.4 | 1.2 | 1.2 | 1.8 | 0.1701 |

[^0]Table S12. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 4 among 347 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 889 families from an Asian population. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a total of 7 distinct chromosomes of size 4. Chromosomes are sorted by fitness score in descending order. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association ( $p=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a risk-related genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a no-epistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  |  |  | RSID |  |  |  | Relative Risk |  |  |  |  | $\mathrm{h}^{* *}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP3 | SNP4 | SNP1 | SNP2 | SNP3 | SNP4 | SNP1 | SNP2 | SNP3 | SNP4 | Joint |  |
| ABCA4(1) | IRF6(1) | GRID2(4) | 20:40652989 | rs560426 | rs2013162 | rs12506428 | rs6102085 | 1.4 | 1.2 | 1.2 | 1.2 | 2.1 | 0.0579 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | 17:9016313 | rs560426 | rs4951357 | rs2013162 | rs9788972 | 1.4 | 1.1 | 1.2 | 1.4 | 2.7 | 0.0318 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | GRID2(4) | rs560426 | rs4951357 | rs2013162 | rs 12506428 | 1.4 | 1.1 | 1.2 | 1.2 | 1.9 | 0.1697 |
| ABCA4(1) | IRF6(1) | GRID2(4) | 20:40652989 | rs952499 | rs2013162 | rs12506428 | rs6102085 | 1.2 | 1.2 | 1.2 | 1.2 | 1.9 | 0.0060 |
| ABCA4(1) | IRF6(1) | GRID2(4) | NTN1(17) | rs952499 | rs2013162 | rs12506428 | rs9915089 | 1.2 | 1.2 | 1.2 | 1.6 | 2.7 | 0.0054 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | 20:40652989 | rs560426 | rs4951357 | rs2013162 | rs6102085 | 1.4 | 1.1 | 1.2 | 1.2 | 1.9 | 0.4819 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | 1:209814702 | rs560426 | rs4951357 | rs2013162 | rs10863790 | 1.4 | 1.1 | 1.2 | 1.1 | 1.8 | -- |

*Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.
**-- indicates SNP sets where the epistasis h-value could not be computed because all SNPs were located on the same biological chromosome.

Table S13. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 5 among 347 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 889 families from an Asian population. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a total of 24 distinct chromosomes of size 5. Chromosomes are sorted by fitness score in descending order. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association ( $\mathrm{p}=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a risk-related genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated riskrelated genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a noepistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  |  |  |  | RSID |  |  |  |  | Relative Risk |  |  |  |  |  | h |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | Joint |  |
| ABCA4(1) | IRF6(1) | GRID2(4) | 17:9016313 | 20:40652989 | rs952499 | rs2013162 | rs12506428 | rs9788972 | rs6102085 | 1.2 | 1.2 | 1.2 | 1.4 | 1.2 | 3.2 | 0.0004 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | GRID2(4) | 17:9016313 | rs560426 | rs4951357 | rs2013162 | rs12506428 | rs9788972 | 1.4 | 1.1 | 1.2 | 1.2 | 1.4 | 3.2 | 0.0103 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | 17:9016313 | $\begin{gathered} \text { LOC102724968 } \\ (20) \end{gathered}$ | rs560426 | rs4951357 | rs2013162 | rs9788972 | rs13041247 | 1.4 | 1.1 | 1.2 | 1.4 | 1.2 | 3.5 | 0.0021 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | GRID2(4) | 17:9016313 | rs952499 | rs4951357 | rs2013162 | rs12506428 | rs9788972 | 1.2 | 1.1 | 1.2 | 1.2 | 1.4 | 2.9 | 0.0015 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | 1:209814702 | GRID2(4) | rs560426 | rs4951357 | rs2013162 | rs10863790 | rs12506428 | 1.4 | 1.1 | 1.2 | 1.1 | 1.2 | 2.1 | 0.1399 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | 17:9016313 | $\begin{gathered} \text { LOC102724968 } \\ (20) \end{gathered}$ | rs560426 | rs4951357 | rs2013162 | rs9788972 | rs11696257 | 1.4 | 1.1 | 1.2 | 1.4 | 1.2 | 3.5 | 0.0032 |
| ABCA4(1) | IRF6(1) | C3orf52(3) | 3:194959271 | $\begin{gathered} \text { LOC102724968 } \\ (20) \end{gathered}$ | rs560426 | rs2073485 | rs16859207 | rs711993 | rs6102074 | 1.4 | 1.2 | 1.6 | 1.3 | 1.3 | 5.2 | 0.0008 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | GRID2(4) | 20:40652989 | rs560426 | rs4951357 | rs2013162 | rs12506428 | rs6102085 | 1.4 | 1.1 | 1.2 | 1.2 | 1.2 | 2.5 | 0.1131 |
| ABCA4(1) | SYT14(1) | C3orf52(3) | 3:194959271 | $\underset{(20)}{\operatorname{LOC} 102724968}$ | rs560426 | rs11119388 | rs16859207 | rs711993 | rs6065259 | 1.4 | 1.1 | 1.6 | 1.3 | 1.2 | 5.0 | 0.0001 |
| ABCA4(1) | IRF6(1) | GRID2(4) | 17:9017282 | 20:40652989 | rs952499 | rs2013162 | rs12506428 | rs4791330 | rs6102085 | 1.2 | 1.2 | 1.2 | 1.3 | 1.2 | 2.6 | 0.0012 |

[^1]Table S14. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 6 among 347 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 889 families from Asian populations. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a total of 71 distinct chromosomes of size 6. Chromosomes are sorted by fitness score in descending order. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association ( $p=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a risk-related genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated riskrelated genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a noepistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  |  |  |  |  | RSID |  |  |  |  |  | Relative Risk |  |  |  |  |  |  | h |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP6 | SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP6 | SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP6 | Joint |  |
| ABCA4(1) | ABCA4(1) | ATP2B4(1) | IRF6(1) | 4:125640770 | NTN1(17) | rs560426 | rs2297636 | rs4951357 | rs2013162 | rs13140903 | rs9915089 | 1.4 | 1.1 | 1.1 | 1.2 | 1.3 | 1.6 | 7.7 | 0.0052 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | 1:209814702 | GRID2(4) | 17:9016313 | rs952499 | rs4951357 | rs2013162 | rs10863790 | rs12506428 | rs9788972 | 1.2 | 1.1 | 1.2 | 1.1 | 1.2 | 1.4 | 3.2 | 0.0024 |
| ABCA4(1) | ABCA4(1) | ATP2B4(1) | IRF6(1) | IRF6(1) | NTN1(17) | rs560426 | rs2297636 | rs4951357 | rs2073485 | rs2013162 | rs9915089 | 1.4 | 1.1 | 1.1 | 1.2 | 1.2 | 1.6 | 4.6 | 0.0079 |
| ABCA4(1) | ABCA4(1) | ATP2B4(1) | IRF6(1) | NTN1(17) | $\begin{gathered} \text { LOC10272496 } \\ 8(20) \end{gathered}$ | rs560426 | rs2297636 | rs4951357 | rs2013162 | rs9915089 | rs13041247 | 1.4 | 1.1 | 1.1 | 1.2 | 1.6 | 1.2 | 6.9 | 0.0012 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | 1:209814702 | GRID2(4) | 17:9016313 | rs560426 | rs4951357 | rs2013162 | rs10863790 | rs12506428 | rs9788972 | 1.4 | 1.1 | 1.2 | 1.1 | 1.2 | 1.4 | 3.4 | 0.0100 |
| ABCA4(1) | IRF6(1) | SHTN1(10) | 17:9017282 | 18:31580789 | 20:40652989 | rs560426 | rs2013162 | rs7078160 | rs4791330 | rs1616887 | rs6102085 | 1.4 | 1.2 | 1.2 | 1.3 | 1.2 | 1.2 | 4.8 | 0.0014 |
| ABCA4(1) | 1:209814702 | SYT14(1) | C3orf52(3) | 3:194959271 | $\begin{gathered} \text { LOC10272496 } \\ 8(20) \end{gathered}$ | rs560426 | rs10863790 | rs1119388 | rs16859207 | rs711993 | rs6102074 | 1.4 | 1.1 | 1.1 | 1.6 | 1.3 | 1.3 | 6.4 | 0.0004 |
| ABCA4(1) | ABCA4(1) | IRF6(1) | GRID2(4) | 17:9016313 | 20:40652989 | rs560426 | rs952499 | rs2013162 | rs12506428 | rs9788972 | rs6102085 | 1.4 | 1.2 | 1.2 | 1.2 | 1.4 | 1.2 | 3.8 | 0.0007 |
| ABCA4(1) | ATP2B4(1) | IRF6(1) | IRF6(1) | 1:209814702 | GRID2(4) | rs560426 | rs4951357 | rs2073485 | rs2013162 | rs10863790 | rs12506428 | 1.4 | 1.1 | 1.2 | 1.2 | 1.1 | 1.2 | 2.2 | 0.1036 |
| ABCA4(1) | IRF6(1) | SYT14(1) | C3orf52(3) | 3:194959271 | $\begin{gathered} \text { LOC10272496 } \\ 8(20) \end{gathered}$ | rs560426 | rs2073485 | rs1119388 | rs16859207 | rs711993 | rs6102074 | 1.4 | 1.2 | 1.1 | 1.6 | 1.3 | 1.3 | 6.7 | 0.0002 |

[^2]Table S15. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 2 among 395 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 668 families from European populations. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a single distinct chromosome of size 2. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association ( $\mathrm{p}=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a risk-related genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a no-epistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

|  | Gene(Chromosome)* | RSID |  | Relative Risk |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP1 | SNP2 | SNP1 | SNP2 | Joint | h |
| 8:128933908 | ARHGAP8/PRR5-ARHGAP8(22) | rs987525 | rs5765956 | 1.9 | 1.2 | 2.0 | 0.2651 |

[^3]Table S16. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 3 among 395 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 668 families from European populations. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a total of 3 distinct chromosomes of size 3. Chromosomes are sorted by fitness score in descending order. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association ( $p=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a risk-related genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated riskrelated genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a noepistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  |  | RSID |  |  | Relative Risk |  |  |  | h |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP3 | SNP1 | SNP2 | SNP3 | SNP1 | SNP2 | SNP3 | Joint |  |
| ABCA4(1) | UNC5C(4) | 8:128933908 | rs560426 | rs4254782 | rs987525 | 1.2 | 1.2 | 1.9 | 2.2 | 0.0040 |
| ABCA4(1) | 8:128933908 | ARHGAP8/PRR5-ARHGAP8(22) | rs560426 | rs987525 | rs5765956 | 1.2 | 1.9 | 1.2 | 2.4 | 0.0067 |
| PAX7(1) | 8:128933908 | ARHGAP8/PRR5-ARHGAP8(22) | rs6659735 | rs987525 | rs5765956 | 1.2 | 1.9 | 1.2 | 2.2 | 0.0520 |

[^4]Table S17. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 4 among 395 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 668 families from European populations. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a total of 18 distinct chromosomes of size 4 . Chromosomes are sorted by fitness score in descending order. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association ( $p=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a riskrelated genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a no-epistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  |  |  | RSID |  |  |  | Relative Risk |  |  |  |  | h |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP3 | SNP4 | SNP1 | SNP2 | SNP3 | SNP4 | SNP1 | SNP2 | SNP3 | SNP4 | Joint |  |
| ABCA4(1) | UNC5C(4) | 8:128933908 | ARHGAP8/PRR5-ARHGAP8(22) | rs560426 | rs4254782 | rs987525 | rs5765956 | 1.2 | 1.2 | 1.9 | 1.2 | 2.7 | 0.0004 |
| ABCA4(1) | UNC5C(4) | 8:128933908 | NTN1(17) | rs560426 | rs4254782 | rs987525 | rs8069536 | 1.2 | 1.2 | 1.9 | 1.6 | 4.7 | 0.0009 |
| ABCA4(1) | 3:89485227 | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.9 | 1.6 | 5.6 | 0.0002 |
| 1:18613886 | ABCA4(1) | UNC5C(4) | 8:128933908 | rs4920522 | rs560426 | rs4254782 | rs987525 | 1.2 | 1.2 | 1.2 | 1.9 | 2.7 | 0.0016 |
| 1:18613886 | ABCA4(1) | 8:128933908 | ARHGAP8/PRR5-ARHGAP8(22) | rs4920522 | rs560426 | rs987525 | rs5765956 | 1.2 | 1.2 | 1.9 | 1.2 | 2.9 | 0.0048 |
| 1:18625618 | ABCA4(1) | 8:128933908 | ARHGAP8/PRR5-ARHGAP8(22) | rs 17352100 | rs560426 | rs987525 | rs5765956 | 1.2 | 1.2 | 1.9 | 1.2 | 3.0 | 0.0019 |
| ABCA4(1) | 8:128914415 | 8:128933908 | ARHGAP8/PRR5-ARHGAP8(22) | rs560426 | rs 12542837 | rs987525 | rs5765956 | 1.2 | 1.5 | 1.9 | 1.2 | 2.5 | 0.0027 |
| ABCA4(1) | 8:128933908 | NTN1(17) | ARHGAP8/PRR5-ARHGAP8(22) | rs560426 | rs987525 | rs8069536 | rs5765956 | 1.2 | 1.9 | 1.6 | 1.2 | 5.3 | 0.0011 |
| 1:18625618 | ABCA4(1) | UNC5C(4) | 8:128933908 | rs17352100 | rs560426 | rs4254782 | rs987525 | 1.2 | 1.2 | 1.2 | 1.9 | 2.6 | 0.0006 |
| ABCA4(1) | 8:128903514 | 8:128933908 | ARHGAP8/PRR5-ARHGAP8(22) | rs560426 | rs1519847 | rs987525 | rs5765956 | 1.2 | 1.5 | 1.9 | 1.2 | 2.5 | 0.0044 |

[^5]Table S18. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 5 among 395 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 668 families from European populations. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a total of 59 distinct chromosomes of size 5 . Chromosomes are sorted by fitness score in descending order. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association ( $p=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a riskrelated genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a no-epistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  |  |  |  | RSID |  |  |  |  | Relative Risk |  |  |  |  |  | h |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | Joint |  |
| ABCA4(1) | 3:89485227 | UNC5C(4) | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs4254782 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.2 | 1.9 | 1.6 | 7.6 | 0.0001 |
| 1:18613886 | ABCA4(1) | UNC5C(4) | 8:128933908 | ARHGAP8/PRR5ARHGAP8(22) | rs4920522 | rs560426 | rs4254782 | rs987525 | rs5765956 | 1.2 | 1.2 | 1.2 | 1.9 | 1.2 | 3.6 | 0.0001 |
| PAX7(1) | ABCA4(1) | UNC5C(4) | 8:128933908 | ARHGAP8/PRR5ARHGAP8(22) | rs766325 | rs560426 | rs4254782 | rs987525 | rs5765956 | 1.2 | 1.2 | 1.2 | 1.9 | 1.2 | 3.3 | 0.0001 |
| ABCA4(1) | 3:89485227 | 8:128914415 | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs12542837 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.5 | 1.9 | 1.6 | 6.5 | 0.0001 |
| 1:18625618 | ABCA4(1) | UNC5C(4) | 8:128933908 | ARHGAP8/PRR5ARHGAP8(22) | rs17352100 | rs560426 | rs4254782 | rs987525 | rs5765956 | 1.2 | 1.2 | 1.2 | 1.9 | 1.2 | 3.4 | 0.0003 |
| ABCA4(1) | UNC5C(4) | 8:128933908 | NTN1(17) | ARHGAP8/PRR5ARHGAP8(22) | rs560426 | rs4254782 | rs987525 | rs8069536 | rs5765956 | 1.2 | 1.2 | 1.9 | 1.6 | 1.2 | 6.3 | 0.0002 |
| ABCA4(1) | 3:89485227 | 8:128903514 | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs1519847 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.5 | 1.9 | 1.6 | 6.5 | 0.0002 |
| ABCA4(1) | 3:89485227 | 8:128933908 | 8:128935636 | NTN1(17) | rs560426 | rs7632427 | rs987525 | rs12548036 | rs8069536 | 1.2 | 1.1 | 1.9 | 1.5 | 1.6 | 6.5 | 0.0001 |
| ABCA4(1) | 3:89485227 | 8:128907554 | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs1519841 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.4 | 1.9 | 1.6 | 6.5 | 0.0001 |
| ABCA4(1) | UNC5C(4) | 8:128914415 | 8:128933908 | ARHGAP8/PRR5ARHGAP8(22) | rs560426 | rs4254782 | rs 12542837 | rs987525 | rs5765956 | 1.2 | 1.2 | 1.5 | 1.9 | 1.2 | 2.8 | 0.0004 |

[^6]Table S19. Top scoring chromosomes, relative risks, and epistasis test h-values for chromosome size 6 among 395 candidate SNPs from a caseparent triad study of cleft lip (with or without cleft palate) in 668 families from European populations. Candidate SNPs were selected based on the strength of marginal disease associations or being located in the WNT signaling pathway, as described by Li et al. (2015). GADGETS returned a total of 161 distinct chromosomes of size 6 . Chromosomes are sorted by fitness score in descending order. The global test of the omnibus null of no association across chromosome sizes 2-6 for these data indicated the presence of an association ( $\mathrm{p}=0.01$ ). Marginal relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype over the number of complements with a riskrelated genotype among pairs that did not inherit identical alleles (though some such pairs would have had identical risk genotypes based on the assigned dominant or recessive role for the minor allele). Joint relative risks were computed as the number of cases with a GADGETS nominated risk-related genotype at each locus in the chromosome over the number of complements with a risk-related genotype at each locus, where each count was weighted by the GADGETS family weights. When no complements carried the nominated risk genotype, a fraction with the weighted number of cases carrying the risk genotype over zero is reported. Epistasis h-values calculated on GADGETS-selected SNP-sets will tend to be low under a no-epistasis null; one would need to test the identified set of SNPs in independent validation data to properly control type I error.

| Gene(Chromosome)* |  |  |  |  |  | RSID |  |  |  |  |  | Relative Risk |  |  |  |  |  |  | h |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP6 | SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP6 | SNP1 | SNP2 | SNP3 | SNP4 | SNP5 | SNP6 | Joint |  |
| ABCA4(1) | 3:89485227 | UNC5C(4) | 8:128914415 | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs4254782 | rs12542837 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.2 | 1.5 | 1.9 | 1.6 | 9.4 | 0.0001 |
| ABCA4(1) | 3:89485227 | UNC5C(4) | 8:128903514 | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs4254782 | rs 1519847 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.2 | 1.5 | 1.9 | 1.6 | 9.4 | 0.0001 |
| ABCA4(1) | 3:89485227 | UNC5C(4) | 8:128907554 | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs4254782 | rs1519841 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.2 | 1.4 | 1.9 | 1.6 | 9.4 | 0.0001 |
| ABCA4(1) | 3:89485227 | UNC5C(4) | 8:128933908 | 8:128935636 | NTN1(17) | rs560426 | rs7632427 | rs4254782 | rs987525 | rs 12548036 | rs8069536 | 1.2 | 1.1 | 1.2 | 1.9 | 1.5 | 1.6 | 9.4 | 0.0001 |
| ABCA4(1) | WNT9A(1) | 3:89485227 | 8:128914415 | 8:128933908 | NTN1(17) | rs560426 | rs10127943 | rs7632427 | rs12542837 | rs987525 | rs8069536 | 1.2 | 0.9 | 1.1 | 1.5 | 1.9 | 1.6 | 13.5 | 0.0001 |
| ABCA4(1) | 3:13775872 | 3:55429452 | 8:128933908 | NTN1(17) | $\begin{gathered} \text { ARHGAP8/PR } \\ \text { R5- } \\ \text { ARHGAP8(22) } \end{gathered}$ | rs560426 | rs12485574 | rs1822811 | rs987525 | rs8069536 | rs5765956 | 1.2 | 1.1 | 1.1 | 1.9 | 1.6 | 1.2 | 563/0 | 0.0001 |
| ABCA4(1) | WNT9A(1) | 3:89485227 | 8:128903514 | 8:128933908 | NTN1(17) | rs560426 | rs10127943 | rs7632427 | rs 1519847 | rs987525 | rs8069536 | 1.2 | 0.9 | 1.1 | 1.5 | 1.9 | 1.6 | 13.5 | 0.0001 |
| ABCA4(1) | WNT9A(1) | 3:89485227 | 8:128907554 | 8:128933908 | NTN1(17) | rs560426 | rs10127943 | rs7632427 | rs1519841 | rs987525 | rs8069536 | 1.2 | 0.9 | 1.1 | 1.4 | 1.9 | 1.6 | 13.5 | 0.0001 |
| ABCA4(1) | 3:89485227 | 8:128907554 | 8:128914415 | 8:128933908 | NTN1(17) | rs560426 | rs7632427 | rs1519841 | rs12542837 | rs987525 | rs8069536 | 1.2 | 1.1 | 1.4 | 1.5 | 1.9 | 1.6 | 7.2 | 0.0001 |
| ABCA4(1) | 3:89485227 | 8:128914415 | 8:128933908 | 8:128935636 | NTN1(17) | rs560426 | rs7632427 | rs 12542837 | rs987525 | rs 12548036 | rs8069536 | 1.2 | 1.1 | 1.5 | 1.9 | 1.5 | 1.6 | 7.2 | 0.0003 |

[^7]Figure S1. Network plot for simulation scenario 2, replicate 3. Chromosomes were filtered for inclusion using global permutations. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2 , respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate pairs of SNPs located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S2. Network plot for simulation scenario 3, replicate 1. Chromosomes were filtered for inclusion using global permutations. SNP label ' 1 ' indicates membership in epistatic risk set 1. No SNPs from risk set 2 were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate pairs of SNPs located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S3. Network plot for simulation scenario 4, replicate 2. Chromosomes were filtered for inclusion using global permutations. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate pairs of SNPs located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S4. Network plot for simulation scenario 5, replicate 2. Chromosomes were filtered for inclusion using global permutations. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Unlabeled SNPs are not risk-related. Dashed connections indicate pairs of SNPs located on the same biological chromosome with pairwise $R^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S5. Network plot for simulation scenario 1, replicate 1. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S6. Network plot for simulation scenario 1, replicate 3. Global permutations were not available to filter chromosomes. SNP label ' 1 ' indicates membership in epistatic risk set 1 . No SNPs from epistatic risk set 2 were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudosiblings.


Figure S7. Network plot for simulation scenario 1, replicate 4. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S8. Network plot for simulation scenario 1, replicate 5. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S9. Network plot for simulation scenario 2, replicate 1. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S10. Network plot for simulation scenario 2, replicate 2. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S11. Network plot for simulation scenario 2, replicate 4. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S12. Network plot for simulation scenario 2, replicate 5. Global permutations were not available to filter chromosomes. SNP label ' 1 ' indicates membership in epistatic risk set 1 . No SNPs from epistatic risk set 2 were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudosiblings.


Figure S13. Network plot for simulation scenario 3, replicate 2. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in risk sets 1 and 2 , respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S14. Network plot for simulation scenario 3, replicate 3. Global permutations were not available to filter chromosomes. No simulated risk-related SNPs were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S15. Network plot for simulation scenario 3, replicate 4. Global permutations were not available to filter chromosomes. SNP label '2' indicates membership in risk set 2. No SNPs from epistatic risk set 1 were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudosiblings.


Figure S16. Network plot for simulation scenario 3, replicate 5. Global permutations were not available to filter chromosomes. SNP label ' 1 ' indicates membership in epistatic risk set 1 . No SNPs from risk set 2 were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudosiblings.


Figure S17. Network plot for simulation scenario 4, replicate 1. Global permutations were not available to filter chromosomes. SNP label ' 2 ' indicates membership in epistatic risk set 2. No SNPs from epistatic risk set 1 were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudosiblings.


Figure S18. Network plot for simulation scenario 4, replicate 3. Global permutations were not available to filter chromosomes. SNP label ' 1 ' indicates membership in epistatic risk set 1 . No SNPs from epistatic risk set 2 were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudosiblings.


Figure S19. Network plot for simulation scenario 4, replicate 4. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S20. Network plot for simulation scenario 4, replicate 5. Global permutations were not available to filter chromosomes. SNP label ' 2 ' indicates membership in epistatic risk set 2. No SNPs from epistatic risk set 1 were identified. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudosiblings.


Figure S21. Network plot for simulation scenario 5, replicate 1. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Figure S22. Network plot for simulation scenario 5, replicate 3. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.



Pair-Score


Figure S23. Network plot for simulation scenario 5, replicate 4. Global permutations were not available to filter chromosomes. SNP labels ' 1 ' and ' 2 ' indicate membership in epistatic risk sets 1 and 2, respectively. Larger, darker nodes and thicker, darker edges correspond to larger SNP and SNP-pair scores. Dashed connections indicate SNPs are located on the same biological chromosome with pairwise $\mathrm{R}^{2}$ of at least 0.1 in complement pseudo-siblings.


Pair-Score



[^0]:    *Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.
    **-- indicates SNP sets where the epistasis h-value could not be computed because all SNPs are located on the same biological chromosome.

[^1]:    *Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.

[^2]:    *Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.

[^3]:    *Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.

[^4]:    *Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.

[^5]:    *Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.

[^6]:    *Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.

[^7]:    *Genes were annotated based on the dbSNP database from NCBI, build 38, using the rsnps R package (version 0.4.0). For SNPs located in intergenic regions, SNP chromosome:position is reported.

