SOME FUNDAMENTALS OF MEDICAL ELECTRONICS
II: DIFFERENTIATING AND INTEGRATING CIRCUITS,
SIMPLE AC CIRCUITS
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In the previous article in this series, the charge and discharge of a condenser via a resistance was considered. It was assumed that ample time was available for the charge or discharge to take place. The product (capacity in microfarads × resistance in megohms) has the dimensions of a time in seconds, and is known as the "time-constant" of the circuit. After charging for a period equal to one time-constant, the voltage appearing across the condenser will equal 63 per cent of the final value. Consider the application of a rectangular voltage pulse to the combination of a condenser C in series with a resistance R as in figure 1b. Such a pulse in which the voltage rises suddenly from one value to a higher value, and then returns suddenly after a definite time, is typical of the waveform provided repetitively by many stimulator circuits. If we apply a train of pulses each having a width of T seconds, then the output waveform developed across R depends on the value of the product CR. The waveforms are shown in figure 2. If CR is long compared with T, then the shape of the pulses is transmitted more or less faithfully. As the time-constant of the circuit is reduced, distortion is progressively introduced, until finally the input rectangular pulse is changed at the output to two sharp pulses, one positive and the other negative going. The progressive introduction of exponential curvature arises from the fact that, with a short time-constant, the condenser has time to become almost charged on the leading edge of the pulse, and almost discharged on the trailing edge. Towards the end of charge or discharge, the current flowing through the condenser falls to a low value and the voltage across R falls markedly. When the time-constant is small compared with the pulse width, then the magnitude of the output voltage is approximately proportional to the rate of change of the input voltage. In mathematical terms this rate of change is called the differential of the input voltage, and the circuit is known as a differentiating circuit. Differentiating circuits are frequently employed to produce short sharp pulses for use in triggering other circuits, and time marks from a blood pressure wave. When a truly rectangular pulse is needed, as in a stimulator, care must be taken to avoid accidentally differentiating the pulse.

An integrating circuit again consists of a condenser and resistance in series, the output now being taken from across the condenser as in figure 1a. When the time-constant of the circuit is small compared with the pulse width, the
exponential change and discharge curves of the condenser distort the input pulse as shown in figure 3. When the time constant is long, the initial portions of the curves are virtually linear, and the output waveform is triangular in shape. Under these circumstances the maximum value of the output voltage is proportional to the area under the input pulse waveform. The maximum output voltage of such an integrating circuit is proportional to the integral of the input voltage with respect to time. A case of practical importance in respiratory work occurs in the integrating pneumotachograph (Hill, 1959). The function of the pneumotachograph is to produce a voltage which is proportional to the instantaneous value of the patient’s respiratory flow rate in litres per minute. This voltage is then integrated electrically over a period of one respiratory cycle to give the volume of gas in litres passed in that period, that is the tidal volume. A further stage of integration having a longer time-constant may be employed to give the minute volume. The simpler types of heart-rate meter employ an integrating circuit. Each R-wave of the electrocardiogram is caused to actuate a pulse-forming circuit which then produces a standard rectangular voltage pulse. These pulses are fed into an integrating circuit; the output of the circuit is then proportional to the heart rate. A useful instrument due to Kitchen (1959) gives a linear scale from 0 to 250 beats per minute. Integrating circuits are also used to give a mean pressure reading from a blood pressure contour waveform.

**Rectangular pulses and sine waves.**

The amplification and handling of rectangular pulses forms an important part of electronic engineering. This is particularly true in data processing systems and in equipment for the measurement of radioactivity. Such pulses are also important from the viewpoint of testing equipment (Crowhurst, 1958) since a rectangular pulse can be regarded as being equivalent to a steady level plus a number of harmonically related sine waves. An ideally sharp rectangular pulse would be equivalent to a number of sine waves extending up to an infinite frequency. In practice a circuit will reasonably well transmit a pulse of width $t$ seconds if it can uniformly pass frequencies of up to $1/t$ cycles per second. If the circuit cannot handle frequencies up to this value, then the output pulse will be further rounded, lengthened and reduced in amplitude. If higher frequencies are transmitted then the output pulse will be sharper, and more nearly rectangular. The effectiveness of an amplifier in passing square pulses is often used as a means of evaluating its frequency response.

**Sinusoidal waveforms.**

A zero frequency or DC voltage is one whose magnitude is constant with respect to time. For example, we should expect the voltage of an accumulator to stay constant until it starts to run down. In addition to direct currents and voltages, alternating currents and voltages occur frequently in electronic engineering. An alternating voltage is one which acts alternately in different directions and whose instantaneous values undergo a definite cycle of changes in regular intervals of time. Most usually, alternating voltages (and currents) follow a sinusoidal waveform. A plot of a typical voltage against time is shown in figure 4. It is seen that at regular intervals the value of the voltage becomes zero. In between it attains alternately a positive and then a negative maximum value. One cycle of this periodic waveform occurs between alternate zeros. A complete cycle is traced out in the periodic time $t$ seconds. The number of cycles completed per second is called the frequency, and this is numerically equal to $1/t$. The normal mains supply in Great Britain is of an alternating nature, having a frequency of 50 cycles per second, and thus a periodic time of 20 milliseconds. Four hundred cycle supplies are often used in aircraft.

**Root mean square and peak values.**

Most applications of electricity are concerned with getting it to perform work, that is to supply
power, the formal definition of power being "rate of doing work". When we pay our electricity bill in terms of the amount of kilowatt hours of electricity used, we are paying for the supply of a given amount of work. Thus if we were to change from a direct to an alternating mains voltage, we should like to know the value of the alternating voltage which would give the same heating power from an electric fire as would the original direct voltage. It turns out that a sine wave voltage having a peak amplitude of $\sqrt{2}$ (1.414), times that of the direct voltage will have the same heating power. The power dissipated in a resistance depends on the square of the voltage and will always be positive, whether the voltage is positive or negative. The square of the instantaneous voltage will be proportional to the power dissipation at that instant. The mean value of the square of the voltage throughout a cycle will lead to a power value which is constant throughout the cycle, that is equivalent to a direct voltage. To find the voltage associated with this power the square root is then taken of the mean voltage squared value. This is called the root mean square or r.m.s. value. Let us consider a 230-volt DC supply. The 230-volt AC supply which gives the same heating power has a value of 230 volts root mean square (230V r.m.s.). Since the peak value of this sine wave is 1.414 times the r.m.s. value, a 230-volt r.m.s. mains supply will have a peak value of 325 volts. This peak value should be borne in mind when considering the provision of electrical insulation and the safe working voltage of condensers. The heater voltage of most thermionic valves is 6.3 volts AC which is 6.3 volts r.m.s. If this voltage is observed on a cathode ray tube, the positive and negative peak values of 8.9 volts can easily be measured.

The concept of phase difference.
Alternating current mains supplies are produced by alternators in the power stations. Basically a coil of wire is rotated about a diameter in a strong magnetic field as shown in figure 5. One complete revolution of the coil produces one complete cycle of a sine wave. The peak voltages are produced
in the positions CC, where the coil is cutting the maximum number of magnetic lines of force because it is moving at right angles to them. At positions AA the voltage falls to zero since here the coil is moving parallel to the lines of force. Faraday's Law states that the induced e.m.f. in a coil is proportional to the rate of change of the lines of force linking the coil. Lenz's Law states that the direction of the induced current is always such that it opposes the motion of change producing it. Hence it is convenient to discuss the development of a sine wave cycle in terms of the angle of rotation of the generating coil, each cycle now being divided up into 360 degrees. It will be remembered that if a coil turns through a quarter of a revolution this is the same as saying it will have turned through an angle of 90°, and for half a revolution the angle will be 180°. One of the waveforms shown in figure 4 is a sine wave, and it is characterized by an equation of the form

\[ V = V_{\text{max}} \cdot \sin \theta \]

where \( \theta \) is a function of time. Let us take the angle \( \theta \) to be 0° at the start of a cycle. Then trigonometry tables show that the sine of 0° has a value of zero, hence here the voltage is also zero. After a quarter of a cycle, \( \theta \) equals 90° and sine 90° equals unity. So now the voltage has reached its maximum positive value of \( V_{\text{max}} \). At half a cycle, sine 180° equals zero and the voltage again falls to zero. At three-quarters of a revolution, sine 270° equals minus one, and the voltage reaches its negative maximum value of \( -V_{\text{max}} \). A wave having a similar shape can be characterized by the equation

\[ V = V_{\text{max}} \cdot \cos \theta \]

Figure 4 shows a plot of the sine and cosine waveforms. Since cosine 0° equals unity, the cosine wave starts its cycle with the voltage having a positive maximum value which falls to zero at 90°. Inspection of figure 4 reveals that the two waves are 90° out of phase. The concept of a phase difference between two electrical signals is very important in considering the function of feedback in oscillators and amplifiers. If a phase difference of 180° exists between the two waves, then at any given instant the voltage of each wave will be of opposite sign. If they were added together then cancellation would tend to occur. Complete cancellation will occur when the magnitudes of the voltages are equal. If the two voltages are completely in phase (in step), they simply add up to yield a wave having an amplitude equal to the sum of the amplitudes of the individual waves. If a sine wave is passed into a differentiating circuit it is differentiated to yield a cosine wave. Thus RC networks introduce a phase shift. For the phase shift to be less than 5°, the time-constant should be greater than twice the period of the lowest frequency to be passed.

**Fundamental and harmonics.**

Next to a steady level, a sinusoidal waveform is the simplest encountered. Sine wave generators are in common use for providing test signals for evaluating the frequency response of amplifiers, filters and recorders. However, many signals, such as music or a blood pressure contour, when converted to a corresponding electrical signal are seen to consist of the combination of several sine waves having related frequencies. If we sound a tuning fork, we produce a pure sine wave note. The same note made by bowing a violin string will sound differently, due to the simultaneous production of harmonic frequencies. The tuning fork produces the fundamental frequency alone. A frequency of double the fundamental is known as the second harmonic, a frequency of three times the fundamental being the third harmonic, and so on. As an example, a blood pressure waveform of 120 beats per minute would have a fundamental frequency of 2 cycles per second. To reproduce the contour faithfully, the recording system should
have a frequency response capable of handling the tenth harmonic, that is 20 c.p.s. In a musical note, the harmonics provide the quality or timbre. Thus it is essential that a high fidelity amplifier be able to reproduce all the harmonics. The addition of a proportion of harmonics will distort a sine wave as shown in figure 6. This "harmonic distortion" is obviously a thing that must not be introduced by an amplifier itself.

The effect of capacitance in an AC circuit.

If either a DC or an AC voltage is applied to a circuit consisting only of a pure resistance, then Ohm's Law applies (in the AC case, using r.m.s. values) and the current and voltage are in phase. When a condenser is connected across a direct voltage supply, a charging current flows at first, but once the condenser is charged this current ceases. There is now a potential difference equal to the supply voltage appearing across the condenser. Thus the condenser effectively isolates or "blocks" the high potential line of the supply from the low tension line. However, if the condenser is connected across an AC supply, it is found that a current continues to flow through the condenser. To find the magnitude of the r.m.s. current passed by the condenser, we divide the r.m.s. voltage by the reactance of the condenser. The reactance is given by the formula

\[ X_c = \frac{1}{2\pi fC} \]

where \( \pi = 3.14 \), \( f \) is the frequency of the AC supply in cycles per second, and \( C \) is the capacity of the condenser in farads. The capacitive reactance \( X_c \) is in ohms. Thus a 1-microfarad condenser connected across a 230-volt r.m.s. 50-cycle supply would have a capacitance reactance of 3,185 ohms, and would pass a current of 72 mA r.m.s. For a 3,185-ohm resistance passing a 72 mA current, the power dissipated would be 16.5 watts. The great advantage of using a condenser to restrict current in an AC circuit lies in the fact that the power loss is much less than for a resistance having an ohmic value the same as that of the condenser reactance. In practice a condenser may be regarded as a pure capacitance in series with a loss resistance which is normally small. A good quality paper condenser might have a loss resistance of the order of 5 to 10 ohms, so that the power dissipated in this resistance is small. A large capacitance can be used to limit the armature current in \( \frac{1}{2} \) horse-power servomotor when the motor runs slowly (Hill, Hook and Bell, 1961). If a double-beam oscilloscope is used to compare the waveforms of the voltage across the condenser and the current passing through it, then it will be seen that they are both sinusoidal in shape, the current leading the voltage by 90°. Physically, one can think of the fact that that current must flow into the condenser before the voltage across it can start to rise. In this case the current is said to lead the voltage.

The effect of inductance in an AC circuit.

The reactance of a condenser arises from the condenser reacting so as to oppose changes in circuit conditions. On positive half cycles the voltage appearing across the condenser tends to oppose the flow of current into the condenser. On negative half cycles the fall in voltage across the condenser tends to oppose the current flow. A similar opposition arises to any change in the flow of current through a coil of wire. When the current through the coil is changed, a voltage, "the back-e.m.f." is developed across the coil in such a sense as to oppose the current changes. Thus if the coil current is stopped, the action of the back e.m.f. is to try to maintain the current. The magnitude of the back-e.m.f. is proportional to the rate of change of the current. It is quite possible for this e.m.f. to reach a value sufficient to ionize the air and produce a spark across the switch contacts of the circuit. Because of this opposition effect a coil also possesses a reactance. The inductive reactance of a coil is given by the formula

\[ X_L = \frac{2\pi fL}{L} \]

\( X_L \) is in ohms when \( f \) is in cycles per second, and the inductance of \( L \) of the coil is in henrys. The unit of inductance is defined in terms of the induced voltage produced across a coil when the current changes. The relation is

\[ V = -L \frac{di}{dt} \]

The minus sign shows that the induced voltage is acting to oppose the change in current. Thus if the current is changing uniformly in an inductance of 1 henry at a rate of 1 ampere per second, then the induced voltage will have a value of
1 volt. In an inductance, the current lags behind the applied voltage by 90°. The lag arises from the necessity of the applied voltage to overcome the back-e.m.f.

As will be explained in a later article dealing with the conversion of alternating to direct current, large iron-cored inductances are used as smoothing chokes to assist in the removal of residual AC from the DC output of the converter. A typical choke might have an inductance of 20 henrys when carrying a DC current of 100 mA. The ohmic resistance of the choke is 500 ohms. In general, the effect of the DC current is to reduce the inductance of the choke. For a full-wave power supply, the ripple current will have a frequency in Britain of 100 cycles, so that for this frequency the reactance of the choke will be 12,560 ohms. The 500-ohm DC resistance of the choke will dissipate 5 watts at 100 mA. The 12,500-ohm reactance effectively chokes back the AC, whilst only the 500-ohm resistance is presented to the DC. Inductances are frequently used in radio frequency circuits. A typical value would be 200 microhenrys at 1 megacycle.

**Tuned circuits.**

The formulae for the reactance of a capacitance and an inductance reveal that the inductive reactance of a coil increases with increasing frequency, whereas the capacitative reactance of a condenser falls with increasing frequency. If we consider a coil in series with a condenser as in figure 7a, the effective opposition to a AC current is given by the impedance of the coil-condenser combination. The impedance $Z$ is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

where $R$ is the total resistance of the circuit. Although inductive and capacitative reactance and resistance are all measured in ohms, it is not possible to add them directly owing to the phase differences existing between the voltages and currents. In a capacitative reactance the current leads the voltage by 90°, and in an inductive reactance the current lags by 90°. Thus at any instant the currents due to a particular voltage will be 180° out of phase and will be in opposite directions. The reactances should therefore be subtracted. For a given combination of capacitance and inductance it is possible to find a frequency for which the capacitative and inductive reactances are equal. At this frequency the impedance of the combination simply equals the resistance of the circuit. The frequency at which this occurs is known as the resonant frequency of the circuit. Normally a circuit is adjusted to resonance by adjusting the capacitance value; the process is known as tuning the circuit to a given frequency. At resonance we speak of a tuned circuit. A series combination of inductance and capacitance is called an acceptor circuit because it presents a low impedance at resonance. Such circuits may be used as tuned shunts for a particular frequency when placed across an AC supply.

A very important case arises when the inductance and capacitance are placed in parallel across the AC supply as in figure 7b. At resonance the currents in $L$ and $C$ will be oppositely directed at any instant. If there is no resistance present ($R = 0$), then no current flows into or out of the circuit, there is simply a large oscillatory current surging inside the tuned circuit. Ideally the impedance presented to the supply is then infinite.

![Fig. 7](image)

(a) Series tuned circuit. (b) Parallel tuned circuit.

In practice there is always some resistance associated with the coil and condenser, so that a small make-up current has to flow into the circuit to make up power losses. Hence the impedance although high is not infinite. A parallel tuned circuit is known as the rejector circuit, because it will reject currents having a frequency equal to the resonance value. The resonant frequency of either a series or a parallel tuned circuit is given by

$$f = \frac{1}{2\pi \sqrt{LC}}$$

As an example, a medium band wireless coil of 190 microhenrys inductance can be tuned to a frequency of 1 megacycle by a variable condenser.
of 133 picofarads capacity. At resonance, the reactance is cancelled and only the resistance is left so that the current passed is large. When this large current flows, a large signal voltage is developed across the parallel tuned circuit. This magnification of the signal strength by a parallel tuned circuit enables a radio receiver to be tuned to a particular station. Tuned circuits are also used in telemetry transmitters and receivers for the transmission of physiological parameters. The effectiveness of a coil in producing this magnification in a tuned circuit is expressed in terms of its magnification factor or Q. Q is given by the ratio $2\pi fL/R$ where $L$ is the inductance in henrys of the coil and $R$ its resistance. The resistance here is the AC resistance of the coil, which is usually rather higher than its DC resistance. At high frequencies, current tends to concentrate on the outside of the coil wire causing the resistance to appear high; this is the "skin effect". The coil mentioned previously would have a Q of 50 and an AC resistance of 24 ohms at 1 megacycle. The DC resistance would be 4.5 ohms.

The next article in the series will discuss the operation of thermionic valves and valve circuits.

REFERENCES

BOOK REVIEW


This monograph has been designed to give the practising anaesthetist a firm background to the clinical application of electroencephalography to anaesthesia. It also aims at providing a source of information to aid in acquiring the skill and judgment necessary to interpret the resultant electroencephalogram intelligently.

With these aims in mind it is perhaps unfortunate that there is no uniformity of presentation of the specimen tracings, some of which would benefit by a considerable degree of magnification. The failure to erase blots on some of the tracings gives some authenticity to the practical nature of this book.

One major omission is that where multichannel tracings are given, as in the first chapter, no explanation of the symbols indicating the location of the electrode placement is given either in the diagram or in the text.

Chapters are devoted to basic considerations and general physiological variables affecting the electroencephalograph, instruments and recorders, electroencephalographic patterns produced by anaesthetic agents, abnormal physiological conditions, extracorporeal circulation, hypothermia and unexpected cardiac arrest.

In the final comments a fair appraisal of the value of electroencephalography is presented.

The topics covered by the authors are of considerable interest to anaesthetists and it is unfortunate that this book is marred by poor presentation of tracings.

I. C. Geddes