THE INTERRELATIONSHIP BETWEEN FIO₂ AND THE GAS EXCHANGE RATIO IN THE OXYGEN/CARBON DIOXIDE DIAGRAM

J. M. LEIGH, D. A. P. STRICKLAND AND C. M. CONWAY

SUMMARY

The interrelationship between FIO₂, true gas exchange ratio (R) and the mathematical slope of the gas R line (RS) is not at all obvious. This paper shows the derivation of an equation relating these quantities and offers both a graphical presentation and an example of a table enabling R to be measured from knowledge of RS and FIO₂.

The gas exchange ratio (R) represents the ratio of carbon dioxide output to oxygen uptake by the lungs. In the steady state it also represents the ratio of carbon dioxide produced by the tissues to tissue oxygen consumption—the RQ. When this exchange is presented on an oxygen/carbon dioxide diagram (Rahn and Fenn, 1955) it is plotted in terms of fractional concentrations (or tensions) to produce a gas R line which has a slope which is related to but numerically different from R.

While it is difficult to measure R in terms of inspired and expired volume, the mathematical slope of the gas R line gives a convenient estimate of gas exchange which can be used in the accurate formulation of respiratory gas equations (Leigh and Tyrrell, 1968).

However, the interrelationship between the mathematical slope of the gas R line and R which is implicit in the formal derivation of respiratory gas equations (Riley et al., 1946; Riley and Cournand, 1949; Rahn and Fenn, 1955) is unfortunately never explicit. This paper aims to clarify that situation.

DERIVATION

Both the mathematical slope of the gas R line and R are calculated in terms of the ratios between expired carbon dioxide and inhaled oxygen. The mathematical slope of the gas R line (which we will designate as RS) depends upon the differences in fractional concentration between inspired and expired gas, thus:

\[ RS = \frac{FECO₂}{FIO₂ - FE₂} \]  (i)

In contrast, R considers the exchange of the respiratory gases in terms of mass, that is, it takes account of the change in volume between inspirate and expirate:

\[ R = \frac{VE \cdot FECO₂}{V₁ \cdot FIO₂ - VE \cdot FE₂} \]

An alternative form of this equation is:

\[ R = \frac{FECO₂}{FIO₂ \left(1 - \frac{FE₂}{1 - FIO₂}\right)} - FE₂ \]  (ii)

Both equations (i) and (ii) can thus be expressed in terms of the same three basic measurements.

Equation (ii) yields the following:

\[ \frac{FIO₂ - FE₂}{FECO₂} = FIO₂ + \frac{1 - FIO₂}{R} \]

Therefore

\[ \frac{1}{RS} = FIO₂ + \frac{1 - FIO₂}{R} \]  (iii)

This last equation (iii) thus represents the interrelationship between RS and R and can be solved for either:

\[ RS = \frac{R}{1 - [FIO₂ (1 - R)]} \]  (iv)

\[ R = \frac{RS(1 - FIO₂)}{1 - (RS \cdot FIO₂)} \]  (v)

In all these equations E can be substituted by A. In fact any value on the gas R line can be substituted, i.e. FEO₂ and FC₀₂.

Also F can be substituted by P since \( P = F(PBAR - 47) \) and equation (iii) would then be

\[ \frac{PBAR - 47}{RS} = PIO₂ + \frac{(PBAR - 47) - PIO₂}{R} \]

This relatively simple formulae conceal the subtleties of the interrelationship. However, the graphical interpretations in figures 1 and 2 are more revealing. The former plots RS against R with isopleths of FIO₂ from 0.0 to 1.0 and the latter plots R against FIO₂.

DISCUSSION

Graphical presentation.

These relatively simple formulae conceal the subtleties of the interrelationship. However, the graphical interpretations in figures 1 and 2 are more revealing. The former plots RS against R with isopleths of FIO₂ from 0.0 to 1.0 and the latter plots R against FIO₂.
INTERRELATIONSHIP BETWEEN $F_{1O_2}$ AND GAS EXCHANGE RATIO

$F_{1O_2}$ can be prepared so that $R$ can be obtained by simple measurements of $R_S$. For example, Table I gives some values of $R$ derived from $R_S$ during air breathing.

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When subjects breathe a gas mixture of constant oxygen content the analysis of samples taken at any time during expiration will give a measure of $R_S$ which can then be easily converted to $R$. This latter approach to the measurement of $R$ is more convenient than the conventional method which requires collection of mixed expired gas.

Symbols.

$i=$ inspired; $e=$ mixed-expired; $A=$ ideal alveolar; $F=$ gas fraction; $P=$ gas tension;

$(P_{BAR} - 47)=$ barometric pressure (mm Hg) less the saturated vapour pressure of water (mm Hg) at $37°C$.

ACKNOWLEDGEMENT

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REFERENCES


LES RAPPORTS ENTRE LA CONCENTRATION FRACTIONNELLE DE L’ANHYDRIDE CARBONIQUE DANS LE GAZ INSPIRE (FlO₂) ET LA PROPORTION D’ÉCHANGE DE GAZ DANS LE DIAGRAMME OXYGENE/ANHYDRIDE CARBONIQUE

SOMMAIRE

Les rapports entre le FlO₂, la proportion réelle d'échange de gaz (R) et l’inclinaison mathématique de la ligne R du gaz ne sont pas du tout évidents. Ce travail présente la dérivation d'une équation au sujet de ces quantités et offre une présentation graphique, ainsi qu'un exemple d'un tableau qui permet de mesurer R en connaissant RS et FlO₂.

DIE BEZIEHUNG ZWISCHEN FlO₂* UND DEM GASAUSTAUSCHQUOTIENTEN IM O₂/CO₂-DIAGRAMM

ZUSAMMENFASSUNG


Las relaciones entre FlO₂*, proporcion verdadera de intercambio gaseoso (R) e inclinación matemática de la línea R del gas (RS) no son evidentes. Esta comunicación muestra la derivación de una ecuación que relaciona estas cantidades y ofrece una presentación gráfica y el ejemplo de una tabla que permite medir R a partir de RS y FlO₂.

*(=concentration fractional de anhidrido carbónico en el gas inspirado)