ALVEOLAR GAS RELATIONSHIPS DURING THE USE OF SEMI-CLOSED REBREATHING ANAESTHETIC SYSTEMS

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SUMMARY
A black box and a geometrical approach have been used to deduce the relationships between alveolar oxygen and carbon dioxide concentrations when rebreathing occurs during the use of semi-closed anaesthetic rebreathing systems.

Semi-closed rebreathing systems are popular for the administration of inhalation anaesthetic agents. The various arrangements of reservoir bag, corrugated tubing, fresh gas inflow, expiratory valve and mask have been classified by Mapleson (1954), and the functional behaviour of these systems in terms of the fresh gas flow necessary to prevent rebreathing has been examined during both spontaneous and controlled ventilation (Mapleson, 1958; Waters and Mapleson, 1961; Nunn and Newman, 1964). Rebreathing of alveolar gas will tend to increase the alveolar carbon dioxide concentration, decrease the alveolar oxygen concentration and decrease the inspired anaesthetic concentration. In a recent paper (Conway, 1976), a simple formula relating alveolar oxygen and carbon dioxide concentrations during rebreathing was determined for the Magill (Mapleson A) circuit. The present analysis is intended to provide a general alveolar air equation when any semi-closed rebreathing system is used.

Figure 1 illustrates the use of a "black box" approach to these systems. In this approach the function of a complex system is analysed in terms of its effect on a known input.

The anaesthetic system as a black box
All semi-closed rebreathing systems receive an input of fresh gas. From all of these systems gas escapes from valves or vents to atmosphere. Because both composition and flow of fresh gas are controllable and because gas vented from the system can easily be collected and analysed, a "black box" approach to these systems can be applied. In this approach the function of a complex system is analysed in terms of its effect on a known input.

In figure 1 the central box represents an anaesthetic system connected to a patient. Input consists of a fresh gas supply and output is gas vented from the system. Whilst behaviour of the central patient-anaesthetic system interaction may be complicated, it can be assessed largely by consideration of input and output alone.

Input of fresh gas will be at a known or measurable flow \( \dot{V}_F \). Fresh gas contains oxygen at a known concentration, \( F_{F O_2} \). It will also contain inert anaesthetic gases and nitrogen. For convenience, the inert fraction may be considered as part of the nitrogen fraction \( F_{F N_2} \).

Outflow of vented gas will occur at a flow \( \dot{V}_V \). In the absence of gas exchange within the central box of figure 1, \( \dot{V}_V \) will equal \( \dot{V}_F \). Since gas exchange occurs normally these flows will usually differ. The relationship between them can be expressed as:

\[
\dot{V}_V = \dot{V}_F - \dot{V}_{IA} + \dot{V}_{AE}
\]

where \( \dot{V}_{IA} \) and \( \dot{V}_{AE} \) refer to inspired and expired alveolar ventilation.
Vented gas contains oxygen, carbon dioxide and inert gases in concentrations $F_{Vo2}$, $F_{vco2}$, $F_{vN2}$. Vented gas must consist partly of fresh gas and partly of alveolar gas. These are the only two gases of different composition contributing to the mixture or mixtures within the system from which vented gas is derived. If the fraction of alveolar gas in vented gas is denoted as $f$, then the fraction of fresh gas in vented gas is $(1 - f)$. Vented gas concentrations can now be expressed in terms of fresh and alveolar gas concentrations:

$$F_{Vo2} = f \cdot F_{Ao2} + (1 - f) \cdot F_{Fo2} \quad (1)$$
$$F_{vco2} = f \cdot F_{Ac02} \quad (2)$$
$$F_{vN2} = f \cdot F_{An2} + (1 - f) \cdot F_{FN2} \quad (3)$$

Gas exchange in the central patient-anaesthetic system complex is reflected in differences in the amounts of various gases between input and output. Oxygen uptake and carbon dioxide output may be expressed as:

$$\dot{V}_{O2} = \dot{V}_{F\text{F}O2} - \dot{V}_{F\text{v}O2} \quad (4)$$
$$\dot{V}_{CO2} = \dot{V}_{F\text{v}CO2} \quad (5)$$

The information derived so far is summarized in figure 1. In the absence of inert gas exchange $\dot{V}_{F\text{F}N2}$ will equal $\dot{V}_{F\text{v}N2}$. From the data derived above, an expression for the respiratory exchange ratio can be obtained and hence an equation for alveolar oxygen concentration which is independent of gas flows or the composition of vented gas:

$$F_{Ao2} = F_{Fo2} - F_{Ac02} \left( F_{Fo2} + \frac{1 - F_{Fo2}}{R} \right) \quad (6)$$

A formal proof of this equation is given in Appendix 1.

GEOMETRICAL ANALYSIS

A more useful solution to the alveolar air equation under conditions of rebreathing is to adapt the geometrical approach to respiratory gas equations suggested by Leigh and Tyrrell (1968). As inspired, expired and vented gases may all be considered as mixtures of varying proportions of fresh and alveolar gases, their co-ordinate points, when plotted on an oxygen–carbon dioxide diagram (Rahn and Fenn, 1955), in terms of either fractional concentration or tension will all lie on the straight line joining the co-ordinate points of fresh and alveolar gases (fig. 2). By drawing horizontal and vertical lines through these points, a series of similar triangles can be constructed whose heights and bases can be expressed in terms of fractional gas concentrations. Thus:

$$\frac{F_{Ac02}}{F_{Fo2} - F_{Ao2}} = \frac{F_{vco2}}{F_{Fo2} - F_{vO2}}$$

Solving this for $F_{Ao2}$ gives:

$$F_{ Ao2 } = F_{ Fo2 } - F_{ Ao2 } \left( \frac{F_{ Fo2 } - F_{ vO2 } }{F_{vO2}} \right) \quad (7)$$

Similarly, it can be shown that:

$$F_{ Ao2 } = F_{ Fo2 } - F_{ Ao2 } \left( \frac{F_{ Fo2 } - F_{ vO2 } }{F_{vCO2}} \right) \quad (8)$$
$$F_{ Ao2 } = F_{ Fo2 } - F_{ Ao2 } \left( \frac{F_{ Fo2 } - F_{ vO2 } }{F_{vCO2}} \right) \quad (9)$$

The gas exchange line on this diagram has a slope which is related to but not equal to the respiratory exchange ratio. Leigh, Strickland and Conway (1972) have given expressions relating the mathematical slope of the gas $R$ line on an oxygen–carbon dioxide diagram ($RS$) to the respiratory exchange ratio ($R$) under conditions of no rebreathing. It can be shown (Appendix 2) that, under conditions of rebreathing, $R$ and $RS$ are related by:

$$\frac{1}{RS} = F_{Fo2} + \frac{1 - F_{Fo2}}{R}$$
RS in figure 2 is given by:

\[ RS = \frac{F_{ACO_2}}{FF_{O_2} - FA_{O_2}} \]

thus

\[ FA_{O_2} = FF_{O_2} - FA_{CO_2} \left( \frac{1}{RS} \right) \]

or

\[ FA_{O_2} = FF_{O_2} - FA_{CO_2} \left( \frac{1 - FF_{O_2}}{R} \right) \]

**DISCUSSION**

It will be seen from these analyses that the alveolar air equation previously derived for the Magill circuit (Conway, 1976) can be applied to all semi-closed rebreathing systems. The descriptive equations state that, under all circumstances of rebreathing within such systems, alveolar oxygen and carbon dioxide concentrations will be related to each other by the fresh gas oxygen concentration and the slope of the gas exchange line. Equation (6) closely resembles the classical alveolar air equation (Riley et al., 1946) whilst equation (8) is a variant of the form of equation described by Filley (Filley, MacIntosh and Wright, 1954) and popularized by Nunn (1963). The various equations described in this paper vary in the ease with which they can be applied in clinical experimental conditions. For all of these equations, fresh gas composition must be known. Equation (6) requires measurement of the respiratory exchange ratio at an equilibrium state. Although carbon dioxide output and oxygen uptake can be measured easily if all gas vented from the circuit is collected, complete inert gas equilibrium is unlikely to be present during the course of any anaesthetic administration. Equations (7)–(9) do not invoke in their proof a state of inert gas equilibrium. These equations should apply even in the presence of anaesthetic or other inert gas exchange if gas concentrations are measured within a single breath. Whilst equation (8) may appeal by virtue of being familiar, it and equation (9) depend upon the measurement of mean expired and inspired gas concentrations respectively. Expired gas is always inhomogeneous and during rebreathing the composition of inspired gas will usually vary during the inspiratory cycle. Measurement of mean inspired and expired gas concentrations requires simultaneous measurement of gas concentration and flow during each respiratory cycle and is an impracticable procedure. Equation (7) requires vented gas concentrations to be known. Vented gas can easily be collected and analysed without interfering with system function. It is not necessary in applying this formula to estimate mixed vented gas composition. As has been shown under different circumstances by Leigh, Strickland and Conway (1972) analysis of any sample of vented gas (or indeed of expired gas) will provide co-ordinate points to determine the gas exchange line.

Because the equations described apply to any circumstance when inspired gas is a mixture of fresh and alveolar gases, their applicability is not limited to semi-closed rebreathing anaesthetic systems. These equations can be applied to rebreathing situations during the use of semi-open systems and to semi-closed absorption systems when a carbon dioxide absorber is not used. They are not applicable directly to closed or semi-closed systems using carbon dioxide absorption. These equations can be applied also to situations where for therapeutic or experimental reasons rebreathing is employed, such as the use of an added deadspace during ventilator therapy or during tube breathing.

Under certain limited circumstances figure 2 can be used to deduce respiratory and apparatus deadspaces. If the inspired gas mixture is of constant composition, physiological deadspace can be derived by a modification of the Bohr equation:

\[ V_{D_{phys}} = \frac{FA_{CO_2} - FF_{CO_2}}{FF_{O_2} - FA_{O_2}} \]

With most semi-closed rebreathing circuits, inspired gas composition will vary during the respiratory cycle. This formula can be applied only under these circumstances if the exact composition of gas which fills the deadspace at end-inspiration is known.

The inspired concentrations of oxygen and carbon dioxide may be considered as reflecting the presence of apparatus deadspace. Again, such apparatus deadspace can be assessed only if the composition of gas filling it at end-expiration can be forecast. Thus, if the apparatus acts as a simple added deadspace it will be filled at end-expiration by alveolar gas, and apparatus deadspace will be given by:

\[ V_{D_{app}} = \frac{FA_{CO_2}}{FF_{O_2} - FA_{O_2}} \]

If the apparatus deadspace acts as a mixing device which at end-expiration contains mixed expired gas...
without the addition of fresh gas then the equation applying is:

$$\frac{V_{D_{app}}}{V_1} = \frac{F_{IO_2}}{F_{IO_2}}$$

Whilst equations (6)–(9) demonstrate the relationship between alveolar oxygen and carbon dioxide concentrations, they do not allow prediction of the values of these gas concentrations when different systems are used. For this to be predicted, fresh gas flow and the composition of vented gas must be known. Equations (11) and (12) of Appendix 1 may be rearranged as follows:

$$FA_{O_2} = F_{FO_2} - \frac{V_{O_2}}{\frac{F_{IO_2}}{F_{FN_2}} - \frac{V_{CO_2}}{F_{FN_2}}}$$

As $FN_2$ and $FN_2$ differ little from each other these equations may be simplified by considering the ratio of the two nitrogen concentrations as unity and their difference as zero. Then:

$$FA_{CO_2} = V_{CO_2} - \frac{F_{IO_2} \cdot V_{CO_2}}{F_{FN_2}}$$

$$FA_{O_2} = F_{FO_2} - \frac{V_{O_2}}{\frac{F_{IO_2}}{F_{FN_2}} - \frac{V_{CO_2}}{F_{FN_2}}}$$

The factor $f$ can be predicted as equal to unity when the Magill circuit is used during spontaneous ventilation and at low fresh gas flows, when vented gas has the same composition as alveolar gas. In all other semi-closed rebreathing systems $f$ will have a value less than unity. No simple methods are available for predicting the value of this factor in these systems.

APPENDIX 1

Alveolar gas equation

In the absence of net inert gas exchange:

$$\frac{V_{FFN_2}}{V_{FFN_2}} = \frac{V_{FFN_2}}{V_{FFN_2}}$$

Substituting equation (3) and rearranging,

$$\nu = \frac{V_{FFN_2}}{f \cdot FN_2 + (1-f) \cdot FN_2}$$

(10)

Substituting equations (2) and (10) in equation (5) gives:

$$\frac{V_{CO_2}}{f \cdot FN_2 + (1-f) \cdot FN_2} = \frac{F_{FFN_2}(f - F_{CO_2})}{f \cdot FN_2 + (1-f) \cdot FN_2}$$

(11)

Substituting equations (1) and (10) in equation (4) gives:

$$\frac{V_{O_2}}{f \cdot FN_2 + (1-f) \cdot FN_2} = \frac{V_{FFN_2}(f - F_{O_2})}{f \cdot FN_2 + (1-f) \cdot FN_2}$$

which simplifies to:

$$\frac{V_{O_2}}{V_{FFN_2}} = \frac{f \cdot V_F(F_{CO_2} - F_{CO_2} - F_{CO_2} \cdot F_{CO_2})}{f \cdot FN_2 + (1-f) \cdot FN_2}$$

(12)

Substituting (11) and (12) and solving for $FA_{O_2}$ gives:

$$FA_{O_2} = F_{FO_2} - F_{CO_2} \left( F_{FO_2} + 1 - F_{FO_2} \right)$$

APPENDIX 2

Equation of the R line

From figure 2, the gas exchange line has a slope $RS$ given by:

$$RS = \frac{F_{CO_2}}{F_{FO_2} - F_{FO_2}}$$

The respiratory exchange ratio is given by:

$$\frac{V_{O_2}}{V_{CO_2}} = \frac{V_{FFO_2}}{V_{FFO_2}} - \frac{V_{FFCO_2}}{V_{FFCO_2}}$$

(13)

In the absence of net inert gas exchange,

$$\nu(1 - F_{CO_2}) = \nu(1 - F_{O_2} - F_{CO_2})$$

thus

$$\nu = \frac{F_{CO_2}}{1 - F_{O_2}}$$

Substituting this in equation (13) and rearranging gives:

$$F_{FO_2} - F_{FO_2} = \frac{R}{1 - F_{O_2} + RF_{O_2}}$$

or

$$\frac{1}{RS} = F_{FO_2} + 1 - F_{FO_2}$$

REFERENCES


ALVEOLAR GAS RELATIONSHIPS WITH SEMI-CLOSED SYSTEMS


RELATIONS DU GAZ ALVEOLAIRE PENDANT L'USAGE DE SYSTEMES ANESTHESIQUES DE RE-RESPIRATION SEMI-FERMES

RESUME
On a adopté une attitude géométrique et utilisé une boîte noire pour calculer les relations qui existent entre les concentrations d'oxygène alvéolaire et celles de gaz carbonique, lorsque se produit la re-respiration pendant l'usage de systèmes anesthésiques de re-respiration semi-fermés.

ALVEOLARE GASBEZIEHUNGEN WÄHRENDE DER VERWENDUNG HALBGESCHLOSSENER WIEDEREINATMUNGS-NARKOSESYSTEME

ZUSAMMENFASSUNG
Eine "schwarze Kiste" und eine geometrische Methode wurden verwendet, um die Beziehungen zwischen alveolaren Sauerstoff- und Kohlendioxydkonzentrationen zu ermitteln, wenn es während der Verwendung von halbgeschlossenen Wiedereinatmungs-Narkose systemen zur Wiedereinatmung kommt.

RELACIONES ENTRE LOS GASES ALVEOLARES DURANTE EL USO DE SISTEMAS ANESTESICOS DE REINHALACION SEMICERRADOS

SUMARIO
Se han utilizado enfoques de caja negra y de geométrico para deducir las relaciones entre las concentraciones alveolares de oxígeno y anhídrido carbónico cuando se produce reinhalación durante el empleo de sistemas anestésicos semicerrados con reinhalación.