

FILE S3**Equations for calculating the expected gametic and allelic frequencies
under Model 1**

The expected frequencies F_c of the different 1n-2c configurations can be derived from the recombination frequencies between the three S_I genes and between the S_I complex locus and the centromere (see schematic representation 1 below), and are given in Table S3-1.

The expected frequencies of each gamete that belong to a specific 1n-2c configuration are equal to $F_c/4$, where F_c is the expected frequency of the specific 1n-2c configuration.

Let D be the sum of the frequencies of the viable gametes. Under model 1, D simplifies to:

$$D = (1 - 2r_3) \left(\frac{r_2 + 1}{2} \right) + 2r_3 \left[\frac{1}{2} + \frac{r_1}{2} - r_1 r_2 \right]$$

The expected frequencies of *viable* gametes are obtained by dividing their corresponding expected frequency by D .

Thus, the final frequency of gametes bearing the S_1^s allele, $f(S_1^s)$, is obtained by summing the individual frequencies of viable gamete that bear this allele. Under Model 1, this simplifies to:

$$f(S_1^s) = \left[\frac{r_2}{2} + r_3(r_1 - r_2 - 2r_1 r_2) \right] / D$$

Similar equations can be derived for Models 2 to 4, with numerical examples based on observed data given in File “S1 - Genetic models.xlsx”.

Under all models, the estimated for recombination fractions r_1 and r_2 are naturally given by dividing the observed recombination fractions, $r_1 \max$ and $r_2 \max$, by $2D$.

$$\hat{r}_1 = \frac{r_1 \max}{2D}; \hat{r}_2 = \frac{r_2 \max}{2D}.$$

In the case of the presence of the additional factor $S_I C$ (see schematic representation 2 below), the effect of $S_I C$ is applied after the initial selection due to the $S_I A$ - S_I - $S_I B$ locus action.

As $S_I C$ only affects the S_I^s gametes when a recombination event occurs between $S_I B$ and $S_I C$, the expected frequencies of these gametes are obtained by multiplying them by $1 - r_3$, where r_3 is the recombination fraction between $S_I B$ and $S_I C$, and dividing them by the relative sum of all gamete frequencies, $1 - r_3 f(S_1^s)$. The frequencies of other viable gametes are obtained by dividing them by the relative sum of all gamete frequencies, $1 - r_3 f(S_1^s)$.

TABLE S3-1

Expected frequencies F_c of the different 1n-2c configuration expressed in function of the recombination frequencies between the three S_I genes, and associated gametic frequencies under no selection and Model 1 selection

1n-2c configuration	<i>Expected configuration frequency without selection (F_c)</i>	<i>Expected gamete frequency without selection (F_g)</i>	<i>Survival under Model 1</i>	<i>Expected gamete frequency under selection (Model 1)</i>
$S_I A$ S_I $S_I B$ <div style="display: flex; align-items: center;"> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s s s</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s s s</div> <div style="font-size: 3em; margin: 0 10px;">}</div> <div> $F_c^1 = (1 - 2r_5)[1 - 2(r_1 + r_2 - 2r_1r_2)]$ </div> </div>		$F_c^1/4$ $F_c^1/4$	Aborted Aborted	
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g g g</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g g g</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^1/4$ $F_c^1/4$	Viable Viable	F_g/D F_g/D
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s s s</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g g g</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^2/4$ $F_c^2/4$	Aborted Viable	F_g/D
$F_c^2 = 2r_5[1 - 2(r_1 + r_2 - 2r_1r_2)]$				
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s s s</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g g g</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^2/4$ $F_c^2/4$	Aborted Viable	F_g/D
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s s s</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g g s</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^3/4$ $F_c^3/4$	Viable Viable	F_g/D F_g/D
X				
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s s g</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g g g</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^3/4$ $F_c^3/4$	Aborted Viable	0 F_g/D
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s s s</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g s s</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^4/4$ $F_c^4/4$	Aborted Aborted	0 0
X				
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s g g</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g g g</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^4/4$ $F_c^4/4$	Viable Viable	F_g/D F_g/D
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s s s</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">s g s</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^5/4$ $F_c^5/4$	Viable Viable	F_g/D F_g/D
X X				
<div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g s g</div> <div style="border: 1px solid black; padding: 2px; margin-right: 5px;">g g g</div> <div style="font-size: 3em; margin: 0 10px;">}</div>		$F_c^5/4$ $F_c^5/4$	Aborted Viable	0 F_g/D
$F_c^5 = 4r_1r_2(1 - 2r_5)$				

<div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;"> s s s s s g </div>	}	$F_c^6 = 4r_2r_5(1 - 2r_1)$	$F_c^6/4$	Aborted	0
			$F_c^6/4$	Aborted	0
X					
<div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;"> g g s g g g </div>			$F_c^6/4$	Viable	F_g/D
			$F_c^6/4$	Viable	F_g/D
<div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;"> s s s s g g </div>	}	$F_c^7 = 4r_1r_5(1 - 2r_2)$	$F_c^7/4$	Viable	F_g/D
			$F_c^7/4$	Viable	F_g/D
X					
<div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;"> g s s g g g </div>			$F_c^7/4$	Aborted	0
			$F_c^7/4$	Viable	F_g/D
<div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;"> s s s g s g </div>	}	$F_c^8 = 8r_1r_2r_5$	$F_c^8/4$	Aborted	0
			$F_c^8/4$	Aborted	0
X X					
<div style="display: inline-block; border: 1px solid black; padding: 2px; margin: 2px;"> s g s g g g </div>			$F_c^8/4$	Viable	F_g/D
			$F_c^8/4$	Viable	F_g/D

The sum of the frequencies of the viable gametes thus simplifies to:

$$D' = D / (1 - f(S_1^s)) r_3,$$

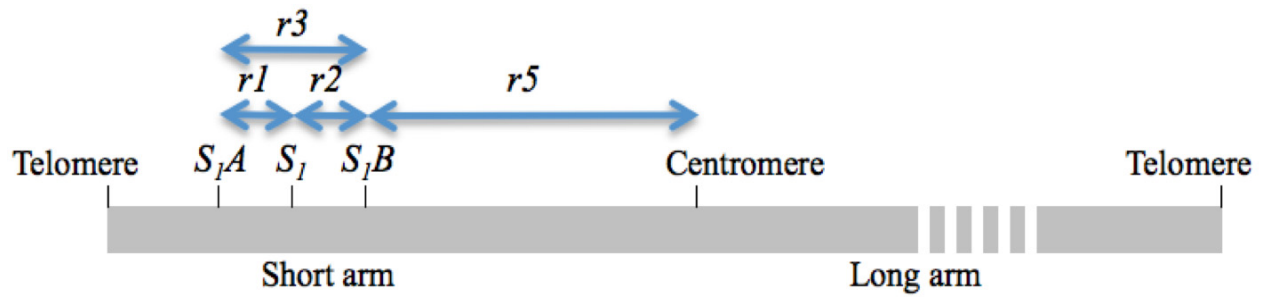
while the final frequency of gametes bearing the S_1^s allele converts to:

$$f'(S_1^s) = (1 - r_3) f(S_1^s) / (1 - f(S_1^s) r_3),$$

and the estimates for r_1 and r_2 convert to:

$$\hat{r}_1 = \frac{r_1 \max}{2D'}; \hat{r}_2 = \frac{r_2 \max}{2D'}.$$

Schematic representation 1: positions of the three S_I genes on rice chromosome 6, expressed as recombination fractions between each other and between the S_I locus and the centromere.



Schematic representation 2: positions of the three S_I genes and the $S_I C$ gene on rice chromosome 6, expressed as recombination fractions between each other and between the S_I locus and the centromere.

