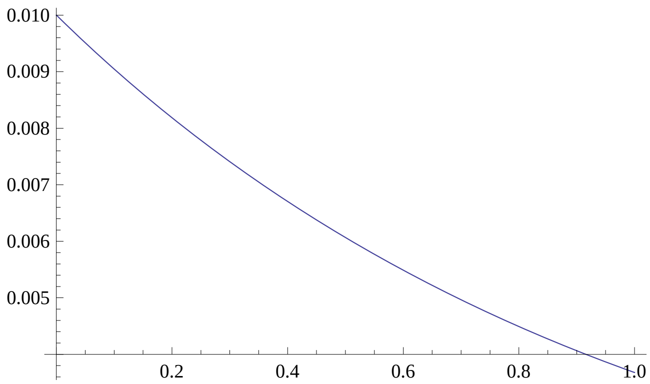
In[1]:=
```
s[r_] := -r / 100 + .01
FixAB[r_, N_] := (1 - Exp[2 s[r]]) / (1 - Exp[2 N * s[r]])
FixBA[r_, N_] := (1 - Exp[-2 s[r]]) / (1 - Exp[-2 N * s[r]])
K[r_, N_, U_] :=
 U * N (FixAB[r, N] * FixBA[r, N] + FixBA[r, N] * FixAB[r, N]) / (FixAB[r, N] + FixBA[r, N])

s[r_] := Exp[- (r) + Log[1 / 100]];
Plot[s[r], {r, 0, 1}]
```

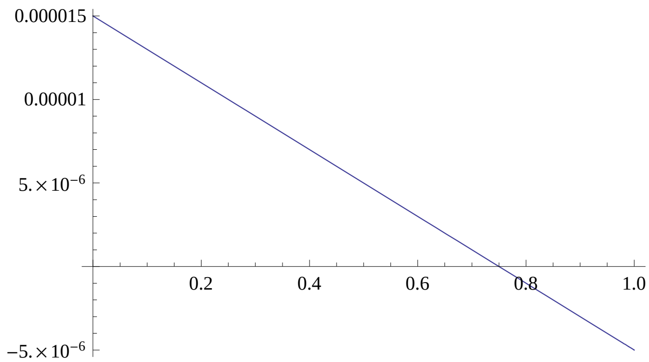



```
Limit[FixAB[r, 100], r → 1]
```

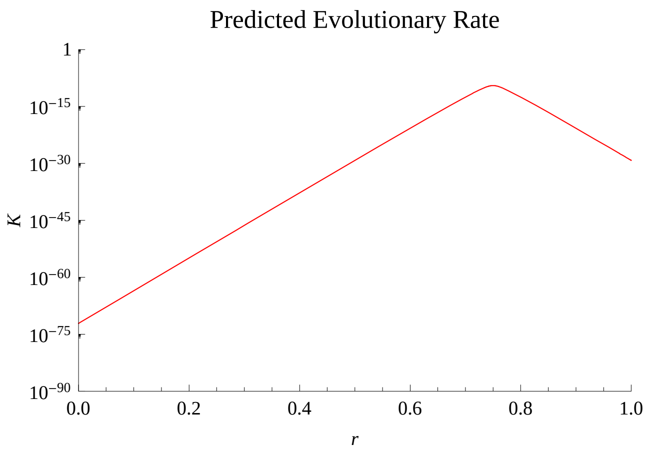
```
0.01
```

In[17]:=
```
s[r_] := -r / 50 000 + .000015
Plot[s[r], {r, 0, 1}]
Linear = LogPlot[K[r, 5 * 10^6, 3.3 * 10^-10], {r, 0, 1}, PlotStyle → Red,
  PlotRange → {{0, 1}, {10^-90, 1}}, Frame → {{True, False}, {True, False}},
  FrameLabel → {r, K}, Axes → {False, False}, PlotLabel → "Predicted Evolutionary Rate"]
g = OpenWrite["C:\Users\dcr476\Desktop\linear.txt", FormatType → OutputForm]
Do[Write[g, r, " ", NumberForm[K[r, 5 * 10^6, 3.3 * 10^-10],
   NumberFormat → (SequenceForm[#1, "e", #3] &)], " "], {r, 0, 1, .001}]
Close[
 g]
```

Out[18]=



Out[19]=



Out[20]= `OutputStream[C:\Users\dcr476\Desktop\linear.txt, 19]`

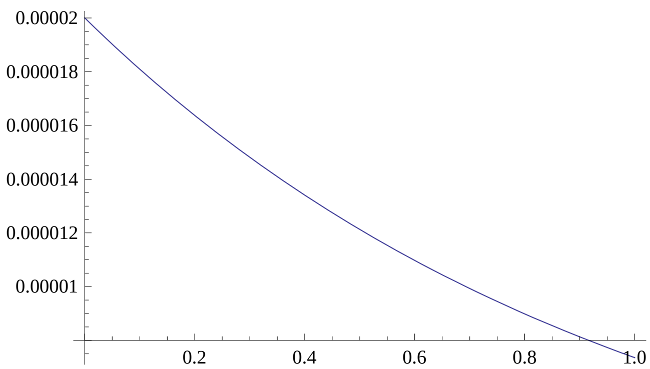
Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

∞::indet : Indeterminate expression 0. ComplexInfinity encountered. ≫

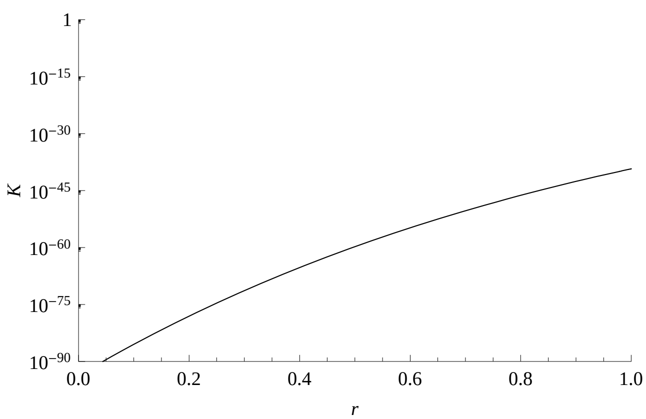
Out[22]= `C:\Users\dcr476\Desktop\linear.txt`

In[23]:=
```
s[r_] := Exp[-(r) + Log[1 / 50 000]];
Plot[s[r], {r, 0, 1}]
Exponential = LogPlot[K[r, 5 * 10^6, 3.3 * 10^-10],
  {r, 0, 1}, PlotStyle → Black, PlotRange → {{0, 1}, {10^-90, 1}},
  Frame → {{True, False}, {True, False}}, FrameLabel → {r, K}, Axes → {False, False}]
g = OpenWrite["C:\Users\dcr476\Desktop\exponential.txt", FormatType → OutputForm]
Do[Write[g, r, " ", NumberForm[K[r, 5 * 10^6, 3.3 * 10^-10],
   NumberFormat → (SequenceForm[#1, "e", #3] &)], " "], {r, 0, 1, .001}]
Close[
 g]
```

Out[24]=



Out[25]=

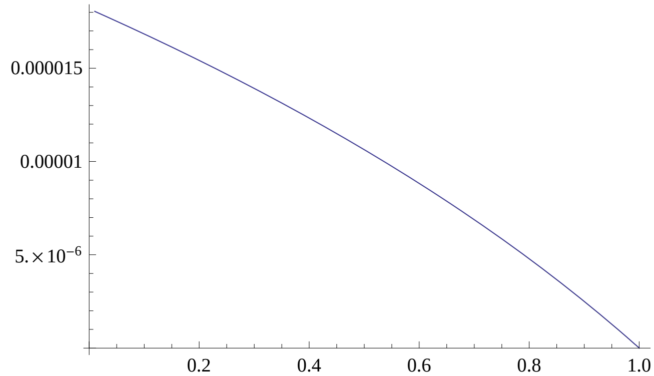


Out[26]= `OutputStream[C:\Users\dcr476\Desktop\exponential.txt, 20]`

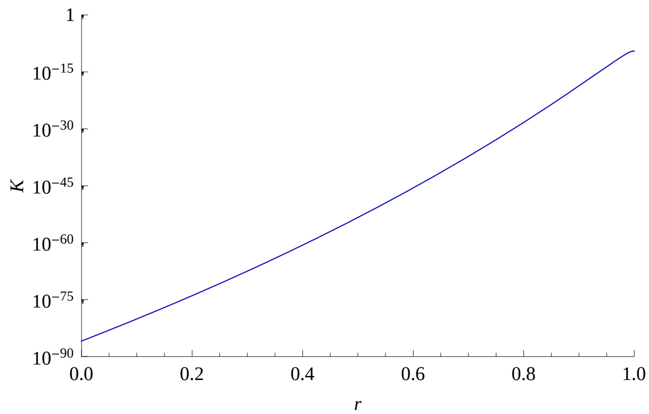
Out[28]= `C:\Users\dcr476\Desktop\exponential.txt`

In[42]:=
```
s[r_] := Log[2 - r] (1 / (55 000 * Log[2]));
Plot[s[r], {r, .01, 1}]
Logarithmic = LogPlot[K[r, 5 * 10^6, 3.3 * 10^-10],
  {r, 0, 1}, PlotStyle → Blue, PlotRange → {{0, 1}, {10^-90, 1}},
  Frame → {{True, False}, {True, False}}, FrameLabel → {r, K}, Axes → {False, False}]
g = OpenWrite["C:\Users\dcr476\Desktop\logarithmic.txt", FormatType → OutputForm]
Do[Write[g, r, " ", NumberForm[K[r, 5 * 10^6, 3.3 * 10^-10],
   NumberFormat → (SequenceForm[#1, "e", #3] &)], " "], {r, 0, 1, .001}]
Close[
 g]
```

Out[43]=



Out[44]=



Out[45]= `OutputStream[C:\Users\dcr476\Desktop\logarithmic.txt, 23]`

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

∞::indet : Indeterminate expression 0. ComplexInfinity encountered. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

∞::indet : Indeterminate expression 0. ComplexInfinity encountered. ≫

Power::infy : Infinite expression $\frac{1}{0.}$ encountered. ≫

General::stop : Further output of Power::infy will be suppressed during this calculation. ≫

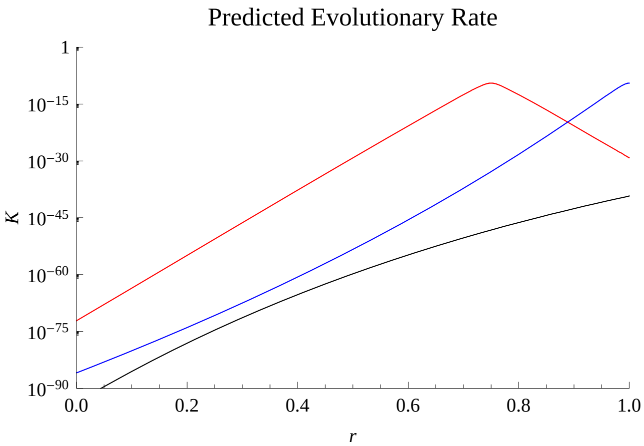
∞::indet : Indeterminate expression 0. ComplexInfinity encountered. ≫

General::stop : Further output of ∞::indet will be suppressed during this calculation. ≫

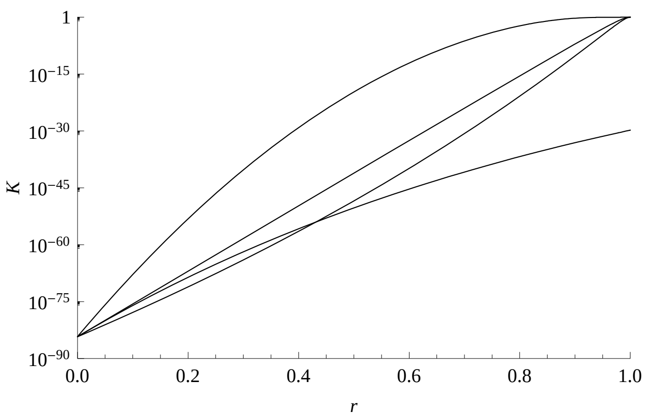
Out[47]= `C:\Users\dcr476\Desktop\logarithmic.txt`

In[48]:=
```
Show[{Linear, Exponential, Logarithmic}]
```

Out[48]=




```
Show[{Linear, Quadratic, Exponential, Logarithmic}]
```




```
Factor[2 ((1 - Exp[2 s]) / (1 - Exp[2 N * s]) * (1 - Exp[-2 s]) / (1 - Exp[-2 N * s])) /
    ((1 - Exp[2 s]) / (1 - Exp[2 N * s]) + (1 - Exp[-2 s]) / (1 - Exp[-2 N * s]))]
```

$$\frac{2\,e^{2 N s}\,(-1+e^{s})\,(1+e^{s})}{(-1+e^{N s})\,(1+e^{N s})\,(e^{2 s}+e^{2 N s})}$$

$$\mathrm{Simplify}\left[\frac{2\,e^{2 N s}\,(-1+e^{s})\,(1+e^{s})}{(-1+e^{N s})\,(1+e^{N s})\,(e^{2 s}+e^{2 N s})}\right]$$

$$\frac{2\,e^{2 N s}\,(-1+e^{s})\,(1+e^{s})}{(-1+e^{2 N s})\,(e^{2 s}+e^{2 N s})}$$

$$\mathrm{Factor}\left[\mathrm{D}\left[\frac{2\,e^{2*N*s}\,(-1+e^{s})\,(1+e^{s})}{(-1+e^{2*N*s})\,(e^{2 s}+e^{2*N*s})},\ s\right]\right]$$

$$-\frac{4\,e^{2 N s}\left(e^{2 s}-e^{2 s+4 N s}-e^{2 s} N+e^{4 s} N-e^{4 N s} N+e^{2 s+4 N s} N\right)}{(-1+e^{N s})^{2}\,(1+e^{N s})^{2}\,(e^{2 s}+e^{2 N s})^{2}}$$

$$\text{Simplify}\left[D\left[\frac{2\,e^{2*N*s}\,(-1+e^{s})\,(1+e^{s})}{\left(-1+e^{2*N*s}\right)\left(e^{2\,s}+e^{2*N*s}\right)},\,s\right]\right]$$

$$-\frac{4\,e^{2\,N\,s}\left(-e^{2\,s}\,(-1+N)+e^{2\,s+4\,N\,s}\,(-1+N)+e^{4\,s}\,N-e^{4\,N\,s}\,N\right)}{\left(-1+e^{2\,N\,s}\right)^{2}\left(e^{2\,s}+e^{2\,N\,s}\right)^{2}}$$

$$\text{Simplify}\left[D\left[D\left[\frac{2\,e^{2*N*s}\,(-1+e^{s})\,(1+e^{s})}{\left(-1+e^{2*N*s}\right)\left(e^{2\,s}+e^{2*N*s}\right)},\,s\right],\,s\right]\right]$$

$$\text{Factor}\left[D\left[D\left[\frac{2\,e^{2*N*s}\,(-1+e^{s})\,(1+e^{s})}{\left(-1+e^{2*N*s}\right)\left(e^{2\,s}+e^{2*N*s}\right)},\,s\right],\,s\right]\right]$$

$$\frac{1}{\left(-1+e^{2\,N\,s}\right)^{3}\left(e^{2\,s}+e^{2\,N\,s}\right)^{3}}8\,e^{2\,N\,s}\left(-e^{4\,s}\,(-1+N)^{2}+e^{2\,(1+N)\,s}\,(-1+N)^{2}-e^{4\,s+6\,N\,s}\,(-1+N)^{2}+e^{2\,s+8\,N\,s}\,(-1+N)^{2}+e^{6\,s}\,N^{2}-e^{6\,N\,s}\,N^{2}-e^{8\,N\,s}\,N^{2}+e^{2\,(3+N)\,s}\,N^{2}+e^{2\,s+4\,N\,s}\left(-1+6\,N-6\,N^{2}\right)+e^{2\,(2+N)\,s}\left(1+2\,N-2\,N^{2}\right)+e^{2\,s+6\,N\,s}\left(-1-2\,N+2\,N^{2}\right)+e^{4\,(1+N)\,s}\left(1-6\,N+6\,N^{2}\right)\right)$$

$$\left(8\,e^{2\,N\,s}\left(-e^{4\,s}+e^{2\,s+2\,N\,s}+e^{4\,s+2\,N\,s}-e^{2\,s+4\,N\,s}+e^{4\,s+4\,N\,s}-e^{2\,s+6\,N\,s}-e^{4\,s+6\,N\,s}+e^{2\,s+8\,N\,s}+2\,e^{4\,s}\,N-2\,e^{2\,s+2\,N\,s}\,N+2\,e^{4\,s+2\,N\,s}\,N+6\,e^{2\,s+4\,N\,s}\,N-6\,e^{4\,s+4\,N\,s}\,N-2\,e^{2\,s+6\,N\,s}\,N+2\,e^{4\,s+6\,N\,s}\,N-2\,e^{2\,s+8\,N\,s}\,N-e^{4\,s}\,N^{2}+e^{6\,s}\,N^{2}-e^{6\,N\,s}\,N^{2}-e^{8\,N\,s}\,N^{2}+e^{2\,s+2\,N\,s}\,N^{2}-2\,e^{4\,s+2\,N\,s}\,N^{2}+e^{6\,s+2\,N\,s}\,N^{2}-6\,e^{2\,s+4\,N\,s}\,N^{2}+6\,e^{4\,s+4\,N\,s}\,N^{2}+2\,e^{2\,s+6\,N\,s}\,N^{2}-e^{4\,s+6\,N\,s}\,N^{2}+e^{2\,s+8\,N\,s}\,N^{2}\right)\right)\Big/\left(\left(-1+e^{N\,s}\right)^{3}\left(1+e^{N\,s}\right)^{3}\left(e^{2\,s}+e^{2\,N\,s}\right)^{3}\right)$$

$$\text{Solve}\left[\left(8\,e^{2\,N\,s}\left(-e^{4\,s}\,(-1+N)^{2}+e^{2\,(1+N)\,s}\,(-1+N)^{2}-e^{4\,s+6\,N\,s}\,(-1+N)^{2}+e^{2\,s+8\,N\,s}\,(-1+N)^{2}+e^{6\,s}\,N^{2}-e^{6\,N\,s}\,N^{2}-e^{8\,N\,s}\,N^{2}+e^{2\,(3+N)\,s}\,N^{2}+e^{2\,s+4\,N\,s}\left(-1+6\,N-6\,N^{2}\right)+e^{2\,(2+N)\,s}\left(1+2\,N-2\,N^{2}\right)+e^{2\,s+6\,N\,s}\left(-1-2\,N+2\,N^{2}\right)+e^{4\,(1+N)\,s}\left(1-6\,N+6\,N^{2}\right)\right)\right)\Big/\left(\left(-1+e^{2\,N\,s}\right)^{3}\left(e^{2\,s}+e^{2\,N\,s}\right)^{3}\right)=0,\,s\right]$$

Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non−algebraic way. ≫

$$\text{Solve}\left[\frac{1}{\left(-1+e^{2\,N\,s}\right)^{3}\left(e^{2\,s}+e^{2\,N\,s}\right)^{3}}8\,e^{2\,N\,s}\left(-e^{4\,s}\,(-1+N)^{2}+e^{2\,(1+N)\,s}\,(-1+N)^{2}-e^{4\,s+6\,N\,s}\,(-1+N)^{2}+e^{2\,s+8\,N\,s}\,(-1+N)^{2}+e^{6\,s}\,N^{2}-e^{6\,N\,s}\,N^{2}-e^{8\,N\,s}\,N^{2}+e^{2\,(3+N)\,s}\,N^{2}+e^{2\,s+4\,N\,s}\left(-1+6\,N-6\,N^{2}\right)+e^{2\,(2+N)\,s}\left(1+2\,N-2\,N^{2}\right)+e^{2\,s+6\,N\,s}\left(-1-2\,N+2\,N^{2}\right)+e^{4\,(1+N)\,s}\left(1-6\,N+6\,N^{2}\right)\right)=0,\,s\right]$$

$$\text{FindRoot}\left[D\left[D\left[\frac{2\,e^{2*N*s}\,(-1+e^{s})\,(1+e^{s})}{\left(-1+e^{2*N*s}\right)\left(e^{2\,s}+e^{2*N*s}\right)},\,s\right],\,s\right],\,\{s,\,0\}\right]$$

FindRoot::nlnum : The function value

$$\left\{\frac{4.\,2.71828^{0.+0.\,N}}{\left(-1.+2.71828^{0.\,N}\right)\left(1.+\ll 18\gg^{0.\,N}\right)}-\frac{16.\ll 18\gg^{\ll 3\gg+\ll 1\gg}\,N}{(-1.+\ll 1\gg)^{2}\,(1.+\ll 1\gg)}+\frac{\ll 1\gg}{\ll 1\gg}+\ll 6\gg+\ll 1\gg+\frac{\ll 1\gg}{\ll 1\gg}+\ll 2\gg\right\}$$ is not a list of numbers with dimensions {1} at {s} = {0.}. ≫

$$\text{FindRoot}\left[\partial_{s}\partial_{s}\frac{2\,e^{2\,N\,s}\,(-1+e^{s})\,(1+e^{s})}{\left(-1+e^{2\,N\,s}\right)\left(e^{2\,s}+e^{2\,N\,s}\right)},\,\{s,\,0\}\right]$$