Within the non-conditional approximation we need to calculate the distribution of mutant timings, as used in Eq. (48). Specifically, we need to calculate

$$Q_k^{k-\ell}(t) = Q_k^{k-1}(t) \star Q_{k-1}^{k-2}(t) \star Q_{k-2}^{k-3}(t) \star \ldots \star Q_{k-\ell+1}^{k-\ell}(t), \tag{S.17}$$

where $\star$ refers to a convolution and

$$Q_{k-\ell+1}^{k-\ell}(t) = s(k-\ell+1)e^{-s(k-\ell+1)t}, \tag{S.18}$$

as given by Eq. (6). In general, the convolution of n exponential distributions with parameters $\lambda_1 \ldots \lambda_n$ is given by

$$\sum_{i=0}^{n-1} \lambda_i e^{-\lambda_i t} \prod_{j=0,\neq i}^{n-1} \frac{\lambda_j}{\lambda_j - \lambda_i}. \tag{S.19}$$

Applying this identity with $\lambda_i = s(k-i)$, we find

$$Q_k^{k-\ell}(t) = \sum_{i=0}^{\ell-1} s e^{-s(k-i)t} \left(\frac{\prod_{j=0}^{\ell-1} k-j}{\prod_{j=0,\neq i}^{\ell-1} i-j} \right) \tag{S.20}$$

We can simplify this expression by noting that

$$\prod_{j=0}^{\ell-1}(k-j) = \frac{k!}{(k-\ell)!}, \tag{S.21}$$

and similarly that

$$\prod_{j=0,\neq i}^{\ell-1} (i-j) = i!(\ell-1-i)!(-1)^{\ell-1-i}. \tag{S.22}$$

This means we have

$$Q_k^{k-\ell}(t) = \sum_{i=0}^{\ell-1} s\ell e^{-s(k-i)t}(-1)^{\ell-i-1} \binom{\ell-1}{i} \binom{k}{k-\ell}. \tag{S.23}$$

We can evaluate this sum by recognizing the binomial expansion formula

$$(1+x)^n = \sum_{i=0}^{n} x^i \binom{n}{i}, \tag{S.24}$$

where we identify $x = -e^{st}$. We find

$$Q_k^{k-\ell}(t) = s\ell \binom{k}{\ell} e^{-skt} \left(e^{st} - 1\right)^{\ell-1}. \tag{S.25}$$

More generally, we have

$$Q_a^b(t) = s(a-b)\binom{a}{b}e^{-sat}\left(e^{st}-1\right)^{a-b-1}. \tag{S.26}$$