FILE S3: GENERAL COALESCENCE PROBABILITIES IN THE NON-CONDITIONAL APPROXIMATION

The probability of coalescence for two individuals originally in two different classes k and k', as defined in Eq. (48) can be rewritten as

$$P_c^{k,k'\to k'-\ell} = \frac{1}{1+2Nh_{k-\ell}s(k-\ell)}\left[I_1+I_2\right], \tag{S.27}$$

where we have defined

$$I_1 = \int_0^\infty Q_{k'}^{k-\ell}(t_1)e^{-s(k-\ell)t_1}\int_0^{t_1} Q_k^{k-\ell}(t_2)e^{s(k-\ell)t_2}dt_2dt_1 \tag{S.28}$$

$$I_2 = \int_0^\infty Q_{k}^{k-\ell}(t_2)e^{-s(k-\ell)t_2}\int_0^{t_2} Q_{k'}^{k-\ell}(t_1)e^{s(k-\ell)t_1}dt_1dt_2. \tag{S.29}$$

Note that both I_1 and I_2 involve integrals of the form

$$I_a = \int_0^t Q_a^b(t')e^{sbt'}dt'. \tag{S.30}$$

Plugging in the results for the non-conditional distributions of mutant timings, Eq. (S.26), and making use of the binomial expansion formula for $(1+x)^n$ noted in File S2, we find this integral becomes

$$\begin{aligned}
I_a &= s(a-b)\binom{a}{b}\int_0^t e^{s(b-a)t'}\left(e^{st'}-1\right)^{a-b-1}dt' && \text{(S.31)}\\
&= s(a-b)\binom{a}{b}\sum_{i=0}^{a-b-1}(-1)^{a-b-1+i}\binom{a-b-1}{i}\int_0^t e^{s(b-a+i)t'}dt' && \text{(S.32)}\\
&= (a-b)\binom{a}{b}(-1)^{a-b}\sum_{i=0}^{a-b-1}\frac{(-1)^i}{a-b}\binom{a-b}{i}\left(e^{s(b-a+i)t}-1\right) && \text{(S.33)}\\
&= \binom{a}{b}(-1)^{a-b}\sum_{i=0}^{a-b}(-1)^i\binom{a-b}{i}\left(e^{s(b-a+i)t}-1\right) && \text{(S.34)}\\
&= \binom{a}{b}(-1)^{a-b}e^{s(b-a)t}\sum_{i=0}^{a-b}\left(-e^{st}\right)^i\binom{a-b}{i} && \text{(S.35)}\\
&= \binom{a}{b}e^{s(b-a)t}\left(e^{st}-1\right)^{a-b}. && \text{(S.36)}
\end{aligned}$$

We now substitute this result for I_a into our expressions for I_1 and I_2. We note that both have terms of the form

$$I_b = \int_0^\infty Q_a^b(t)\binom{c}{b}e^{-sct}\left(e^{st}-1\right)^{c-b}dt. \tag{S.37}$$

Using similar manipulations to those above, we find

$$
\begin{aligned}
I_b &= (a-b)\binom{a}{b}\binom{c}{b}\int_0^\infty e^{-s(a+c)t}\left(e^{st}-1\right)^{a+c-2b-1}dt && \text{(S.38)}\\
&= s(a-b)\binom{a}{b}\binom{c}{b}(-1)^{a+c-1}\sum_{i=0}^{a+c-2b-1}\binom{a+c-2b-1}{i}(-1)^i\int_0^\infty e^{-s(a+c-i)t}dt && \text{(S.39)}\\
&= (a-b)\binom{a}{b}\binom{c}{b}(-1)^{a+c-1}\sum_{i=0}^{a+c-2b-1}(-1)^i\binom{a+c-2b-1}{i}\frac{1}{a+c-i}. && \text{(S.40)}
\end{aligned}
$$

Using the partial fraction decomposition

$$\frac{1}{\binom{n+x}{n}} = \sum_{i=1}^{n}(-1)^{i-1}\binom{n}{i}\frac{i}{x+i}, \qquad \text{(S.41)}$$

we find

$$I_b = \frac{\frac{a-b}{a+c-2b}\binom{a}{b}\binom{c}{b}(-1)^{a+c}}{\binom{-2b-1}{a+c-2b}} = \frac{\frac{a-b}{a+c-2b}\binom{a}{b}\binom{c}{b}(-1)^{2b}}{\binom{a+c}{a+c-2b}}. \qquad \text{(S.42)}$$

We can now use this result for I_b to determine I_1 and I_2, and hence compute $P_c^{k,k'\to k'-\ell}$. We find

$$P_c^{k,k'\to k'-\ell} = \frac{1}{1+2Nh_{k-\ell}s(k-\ell)}\frac{\binom{k'}{k-\ell}\binom{k}{k-\ell}}{\binom{k+k'}{2\ell+k'-k}}. \qquad \text{(S.43)}$$

As we noted in the main text, this is just

$$P_c^{k,k'\to k-\ell} = \frac{1}{1+2Nh_{k-\ell}s(k-\ell)}A_\ell^{k,k'}, \qquad \text{(S.44)}$$

with $A_\ell^{k,k'}$ as defined in Eq. (16). Note that when $k=k'$, this result simplifies to $P_c^{k,k\to k-\ell}$ as defined in the main text, as expected.