File S6 A conditioned diffusion process

Our results in the main text indicate that the growth rate under the conditioned population dynamics depends mostly on the absolute value and not so much on the sign of the growth rate under the unconditioned population dynamics. For mathematically interested readers, we now explore a simple model where this fact can be proven easily. We consider a diffusion process on the interval $[0, 1]$ with constant infinitesimal mean $\mu(x) = \mu$ and constant infinitesimal variance $\sigma^2(x) = \sigma^2$. We will show that the associated diffusion process conditioned on hitting 1 before 0 is independent of the sign of μ and that its infinitesimal mean increases with $|\mu|$.

Our task is to compute the infinitesimal mean and variance of the conditioned diffusion process. Following the formulas given by Karlin and Taylor (1981, p. 263), the infinitesimal mean of the conditioned diffusion process is

$$\mu^*(x) = \mu(x) + \frac{s(x)}{S(x)} \cdot \sigma^2(x), \tag{S10}$$

where $S(x)$ is the scale function and $s(x)$ is its derivative. Using the definitions of these functions (e.g. Karlin and Taylor 1981, p. 262) and plugging in the parameters of our diffusion, we obtain

$$s(x) = \exp\left(-\int_0^x \frac{2\mu(\eta)}{\sigma^2(\eta)} d\eta\right) = \exp\left(-\frac{2\mu x}{\sigma^2}\right) \tag{S11}$$

and

$$S(x) = \int_0^x s(\eta) d\eta = \frac{\sigma^2}{2\mu} \cdot \left[1 - \exp\left(-\frac{2\mu x}{\sigma^2}\right)\right]. \tag{S12}$$

Substituting Equation (S11) and (S12) into Equation (S10), we obtain

$$\mu^*(x) = \mu \cdot \frac{\exp(2\mu x/\sigma^2) + 1}{\exp(2\mu x/\sigma^2) - 1} =: f(\mu), \tag{S13}$$

a function that is symmetric about 0, i.e. $f(-\mu) = f(\mu)$, and increases with the absolute value of μ (Figure S9). The variance $\sigma^{2*}(x)$ equals the original variance $\sigma^2(x)$.

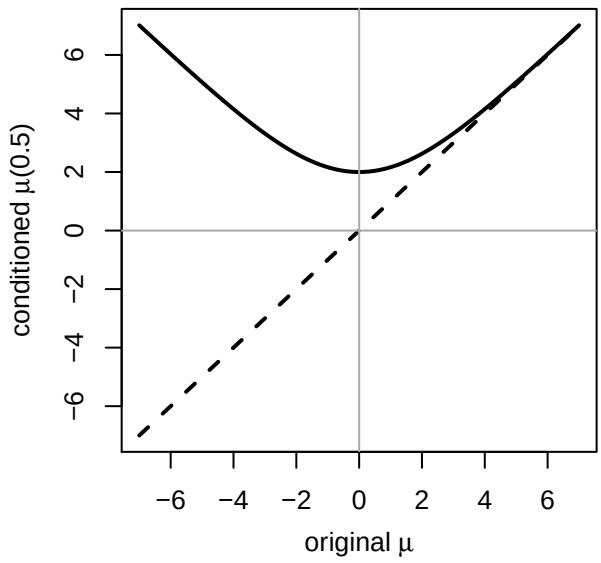

Figure S9 The infinitesimal mean of the conditioned diffusion process at 0.5 (solid line), i.e. in the middle of the interval, as a function of the infinitesimal mean of the original process. On the dashed line, the infinitesimal means of original and conditioned process would be equal.

File S7 Source code and analysis scripts

164 File S7 is available for download as a zip-folder at

`http://www.genetics.org/lookup/suppl/doi:10.1534/genetics.114.167551/-/DC1`.