APPENDIX A

**Algorithm A.1.** The high-level *Proof-of-Hierarchy (PoH)* algorithm of HierarchicalChain.

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**Input:** The local data $D$, the polling time period $\Delta$, the waiting time period $\Theta$, the maximum per-level iteration $\Omega$, the total number of participating sites $N$, and the number of levels $H$.

**Output:** The consensus hierarchical batch machine learning models, including the single models $M_V$ ($V = 1, 2, ..., H$), the horizontal ensemble model $M_{\text{HORIZONTAL}}$, and the vertical ensemble model $M_{\text{VERTICAL}}$.

**Step 1.** Submit a hierarchy transaction, including the unique name/identifier, the hierarchy of this site, and the number of records $R_S$ in the local data $D$, to the blockchain.

**Step 2.** Check the blockchain every time $\Delta$ until the initial transactions from all $N$ sites are received, and compute the hierarchical network structure $K$.

**Step 3.** Wait for time $\Theta$ to let every site construct the hierarchical network structure $K$.

**Step 4.** Run PoH-Modeling (**Algorithm 2**) with the parameters $(D, \Delta, \Theta, \Omega, N, H, K)$ and collect the models $M_{PQ}$ from all levels and sites ($P = 1, 2, ..., H$, $Q = 1, 2, ..., N$).

**Step 5.** Wait for time $\Theta$ to let every site collect the models $M_{PQ}$.

**Step 6.** Run PoH-Ensemble (**Algorithm 3**) with the parameters $(N, H, K, M_{PQ})$ and collect the models $M_{\text{FLAT}}$, $M_{\text{HORIZONTAL}}$, and $M_{\text{VERTICAL}}$. 
Algorithm A.2. The high-level PoH-Modeling algorithm of HierarchicalChain, based on GloreChain-LevelWise, the level-wise adaption of GloreChain.\[1\]

Input: The local data \( D \), the polling time period \( \Delta \), the waiting time period \( \Theta \), the maximum per-level iteration \( \Omega \), the total number of participating sites \( N \), the number of levels \( H \), and the hierarchical network structure \( K \).

Output: The models \( M_{PQ} \) from all levels and sites \( (P = 1, 2, \ldots, H, Q = 1, 2, \ldots, N) \).

Step 1. Loop from level \( V = 1 \) to the number of levels \( H \).

Step 1.1. Compute the sub-network of this site \( S \) in level \( V \) as \( B_V \), the number of sites in \( B_V \) as \( N_V \) and the learning order in \( B_V \) as \( O_V \), from the hierarchical network structure \( K \).

Step 1.2. Wait for time \( \Theta \) to let every site determine \( B_V, N_V \) and \( O_V \).

Step 1.3. Run GloreChain-LevelWise on sub-network \( B_V \) with the parameters \( (D, \Delta, \Theta, \Omega, V, N_V, O_V) \) to compute the consensus model of this site \( S \) in level \( V \) as \( M_{SV} \), and submit \( M_{SV} \) to the blockchain.

Step 1.4. Wait for time \( \Theta \) to let every site submit model to the blockchain.

Step 1.5. Check the blockchain every time \( \Delta \), until the consensus models from all sites in all sub-networks are collected as \( M_{VQ} \) \( (Q = 1, 2, \ldots, N) \).

Step 1.6. Wait for time \( \Theta \) and re-submit \( M_{SV} \) to the blockchain, to let every site collect the models.

Step 2. Collect the models \( M_{PQ} \) from all levels and sites \( (P = 1, 2, \ldots, H, Q = 1, 2, \ldots, N) \).
Algorithm A.3. The high-level PoH-Ensemble algorithm of HierarchicalChain.

**Input:** The total number of participating sites $N$, the number of levels $H$, the hierarchical network structure $K$, and the models $M_{PQ}$ from all levels and sites ($P = 1, 2, ..., H$, $Q = 1, 2, ..., N$).

**Output:** The consensus hierarchical batch machine learning models, including the single models $M_V$ ($V = 1, 2, ..., H$), the horizontal ensemble model $M_{HORIZONTAL}$, and the vertical ensemble model $M_{VERTICAL}$.

**Step 1.** Retrieve the number of records $R_Q$ of all sites ($Q = 1, 2, ..., N$) from the hierarchical network structure $K$.

**Step 2.** Loop from level $V = 1$ to the number of levels $H$.

**Step 2.1.** Compute $M_V = M_{VS}$ of this site $S$.

**Step 3.** Compute $M_{HORIZONTAL} =$ averaged $M_{1Q}$ ($Q = 1, 2, ..., N$) weighted by $R_Q$ ($Q = 1, 2, ..., N$).

**Step 4.** Compute $M_{VERTICAL} =$ averaged $M_{PS}$ ($P = 1, 2, ..., H$) weighted by the number of records of each level of the sub-networks of $S$ computed from $R_P$. 

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REFERENCES OF APPENDIX A