Comment on Estimation Methods of Absorbed Dose due to Tritium

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ABSTRACT

It was attempted to estimate the absorbed doses in cell after incorporation of tritium with the constant absorption model. The point source dose distribution, the dose distributions due to tritium locally incorporated, the fraction of self-absorption in spherical source and the fraction absorbed in sphere of energy emitted from the source around the sphere could be analytically calculated with the constant absorption model, where the beta-spectrum was divided into eighteen one-keV intervals and the beta-particles were supposed to have the middle energy in each energy group.

Furthermore the results were compared with the other ones and the questionable points contained in these estimation methods investigated. Then it was pointed out that the stopping power and the transmission probability of low-energy electrons would be required to be established theoretically and experimentally.

INTRODUCTION

The incorporation of low-energy $\beta$-emitters into a cell leads to a locally large energy absorption of the cellular and sub-cellular structures because the ranges of the $\beta$-particles are short and the emitters selectively incorporated into various sites with the difference of the chemical form. $^3$H-labeled thymidine and water are mainly incorporated into cell nucleus and cytoplasm, respectively, and the dose distributions within cell must be taken into account as the maximum energy of $\beta$-par-

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particles is 18 keV and the average 5.69 keV.

The dose distribution in or around a localized $\beta$-particle source is calculated by a suitable summation of a point source dose distribution. As to the function which expresses the $\beta$-particle point source dose distribution, the expression which the exponential absorption model\textsuperscript{1,2} was modified by Loevinger\textsuperscript{3,4} has been generally used. However, the constants $\nu$ and $c$ are contained in the expression, and $\nu$ can be only calculated for the energy of more than 36 keV and $c$ has been only obtained for 0.17 to 3 MeV. On the other hand the theoretical calculations of electron energy dissipation have been made by Spencer\textsuperscript{5}. However the calculations will not be always able to apply for the estimation of absorbed doses in cell due to $^3$H because of the complication of the derived formula and the insufficient theory of the stopping power for low-energy electrons.

Therefore the estimations of the absorbed doses due to $^3$H incorporated into cell have been performed on the basis of different assumptions by some workers\textsuperscript{6-9} and also their articles reviewed\textsuperscript{10,11}.

Now uniform and homogeneous infinite medium which contains a sphere will be considered. It is assumed that the densities of the medium and sphere are equally 1 gm/cm$^3$. This assumption may be considered to be a good approximation to cell into which $^3$H was incorporated because the maximum range of the $\beta$-particles is about 5.5 $\mu$m in tissue.

Then the energy absorption by $\beta$-particles of energy $E$ which its range is $R$ is supposed to be constant, $E/R$, over the range. In order to calculate the absorbed doses with the constant absorption model $^3$H $\beta$-spectrum is classified at 1 keV interval and the $\beta$-particles which belong to each energy classes are supposed to have the middle energy of the class. The energies of the $\beta$-particles are completely absorbed along its paths as the radiative loss can be neglected for $^3$H $\beta$-particles. The estimation of absorbed dose in cell after incorporation of $^3$H is done on the basis of the constant absorption model with the additional assumptions as follows: the uniform distribution of $^3$H, and the isotropical emissions and the passages along straight lines of its $\beta$-particles.

The contribution of the recoil energy of $^3$He atom to the total dose is small enough to be negligible as its maximum kinetic energy is about 3.3 eV, and therefore it is not contained in present dose calculations.

Therefore, the absorbed doses can be analytically estimated with the constant absorption model, but it is explicit that the model will be rough approximation for the energy dissipation of electrons. It will be discussed later.

The results will be compared with the other ones. It should be emphasized that the main purpose in present work is to indicate some questionable points contained in these approximate calculations.

Now it will be assumed that the concept of rad is applicable when the dimensions are in microns. Although it may be appropriate to raise a question as to the minimum dimensions within which the rad concept is valid, the problem would not
be discussed in present work.

ENERGY SPECTRUM OF TRITIUM BETA-PARTICLES

$^3$H $\beta$-spectrum is well known and in good agreement with a Fermi plot when the maximum energy is taken as being 18 keV$^{12}$. Fig. 1 shows the relative frequencies of emissions of $^3$H $\beta$-particles of different energies when the spectrum is classified at 1 keV interval.

The average energy is obtained as being 5.69 keV by calorimetry$^{13}$.

RANGE-ENERGY RELATION FOR ELECTRONS

A range-energy relation for low-energy electrons such as $^3$H $\beta$-particles is not satisfactorily known though curves of Brown$^{14}$ which the available data for the ranges of homogeneous cathode rays were compiled and Katz et al.$^{15}$ which was obtained from the reliable published values for absorption in Al are found. Except the two curves, none of the empirical formulas$^{16,17}$ which have been used gives a satisfactory fit to the experimental data in the low-energy domain. The two curves, however, show considerably different values and in addition contain some questiona-ble points, because Brown has estimated the maximum energy of $^3$H $\beta$-particles to be 9.5 keV by a comparison of its maximum range in He gas with the compiled data and Katz et al. have only discussed down to 10 keV.
Fig. 2 compares the curves of Brown and Katz et al. with the theoretical ranges in muscle. The values of Katz et al. were obtained by extrapolating down to 0 keV according to their equation. These ranges are shown in \( \mu m \) by dividing the ranges given in gm/cm\(^2\) with 1 gm/cm\(^3\). The discrepancy among these range-energy relations seems to show explicitly that the rate of energy loss for low-energy electrons is less well known.

The relation of Katz and Penfold is used in present work. Since the absorption and scattering of electrons do not change greatly with small changes in atomic number, the ranges in Al may apply for purposes of dosimetry to tissue, though there is not sufficient evidence that it is also established for low-energy electrons.

**DOSE DISTRIBUTION DUE TO TRITIUM POINT SOURCE**

The dose distribution as a function of distance from \(^3\)H point source was calculated by

\[
D(r) = \frac{k}{4\pi r^2} \sum_{i=10}^{18} \frac{E_i}{R_i} \cdot P_i
\]

where \(D(r)\): the absorbed dose at a point of distance \(r\) from \(^3\)H point source.
\(k\): the constant.
\(E_i\): the energy for \(i\)-th fraction \(P_i\), normalized to 1.0, of \(^3\)H \(\beta\)-spectrum divided into eighteen one-keV intervals. \(= (i-1/2) \text{ keV}\).
\(R_i\): the range of \(\beta\)-particles of energy \(E_i\).
\(i_0\): the minimum value of \(i\) determined by \(R_i \geq r\).

In Fig. 3 the results, which are represented in rad/disintegration, are shown by the solid line and compared with the values obtained by Robertson et al.\(^6\) and Künkel\(^7\). The absorbed dose due to \(^3\)H point source becomes zero at the distance of 5.2 \(\mu m\) from the source corresponding to the range of 17.5 keV electrons. These values will be discussed later.

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**Fig. 3.** Point source dose distributions per disintegration of tritium.
DOSE DISTRIBUTION DUE TO TRITIUM UNIFORMLY DISTRIBUTED
IN OR AROUND A SPHERE

The absorbed dose, $D_p$, at a point due to $^3$H uniformly distributed is calculated
by integrating Eqn.(1) over the region which contains $^3$H. That is,

$$D_p = \frac{k}{4\pi} \sum_{i=i_0}^{18} \frac{E_i}{R_i} \cdot P_i \cdot \int_V \frac{dv}{r^2} = k \sum_{i=i_0}^{18} G_i E_i P_i$$

$$G_i = \frac{1}{4\pi} \frac{1}{R_i} \int_V \frac{dv}{r^2}$$

where $V$ represents the region which contains $^3$H and $dv$ a small volume element
in $V$. $G_i$ is generally called as geometrical factor.

The equations which give the geometrical factors for $^3$H uniformly distributed
in or around a sphere are summerized as follows, where $d$ is the sphere radius, $x$
the distance from the center of the sphere, $a=d/R_i$ and $b=x/R_i$. The calculation
method of $G_i$ is shown in Appendix I.

A. Source distributed in the sphere

(a) $G_i$ for $0 < x < d$

1) $R_i \leq d - x$

$$G_i = 1.0$$

2) $d - x \leq R_i \leq d + x$

$$G_i = \frac{1}{4} \left\{ a + \frac{a^2 - 3b^2 + 4b - 1}{2b} - \frac{a^2 - b^2}{b} \ln \left( \frac{a-b}{a+b} \right) \right\}$$

3) $R_i \geq d + x$

$$G_i = \frac{1}{4} \left( 2a - \frac{a^2 - b^2}{b} \ln \left( \frac{a-b}{a+b} \right) \right)$$

(b) $G_i$ for $x > d$

1) $R_i \leq x - d$

$$G_i = 0$$

2) $x - d \leq R_i \leq \sqrt{x^2 - d^2}$

$$G_i = \frac{1}{2} \left\{ 1 - \frac{b-a}{2} + \frac{b^2 - a^2 + 1}{4b} + \frac{b^2 - a^2}{2b} \ln \left( \frac{b-a}{b+a} \right) \right\}$$

3) $\sqrt{x^2 - d^2} \leq R_i \leq x + d$

$$G_i = \frac{1}{2} \left\{ 1 - \frac{b-a}{2} + \frac{b^2 - a^2 + 1}{4b} + \frac{b^2 - a^2}{2b} \ln \left( \frac{b-a}{b+a} \right) \right\}$$

4) $R_i \geq x + d$

$$G_i = \frac{1}{2} \left( a - \frac{b^2 - a^2}{2b} \ln \left( \frac{b+a}{b-a} \right) \right)$$
(c) \( G_i \) at \( x=0 \)
1) \( R_i \leq d \)
   \[ G_i = 1.0 \]
2) \( R_i \geq d \)
   \[ G_i = a \]

(d) \( G_i \) at \( x=d \)
1) \( R_i \leq 2d \)
   \[ G_i = \frac{1}{2} \left( 1 - \frac{1}{4a} \right) \]
2) \( R_i \geq 2d \)
   \[ G_i = a/2 \]

B. Source distributed around the sphere
(a) \( G_i \) for \( 0 < x < d \)
1) \( R_i \leq d-x \)
   \[ G_i = 0 \]
2) \( d-x \leq R_i \leq d+x \)
   \[ G_i = \frac{1}{4} \left\{ -a - \frac{a^2 - 3b^2 - 4b - 1}{2b} + \frac{a^2 - b^2}{b} \ln (a-b) \right\} \]
3) \( R_i \geq d+x \)
   \[ G_i = 1 - \frac{a}{2} + \frac{a^2 - b^2}{4b} \ln \frac{a-b}{a+b} \]

(b) \( G_i \) for \( x > d \)
1) \( R_i \leq x-d \)
   \[ G_i = 1 \]
2) \( x-d \leq R_i \leq \sqrt{x^2 - d^2} \)
   \[ G_i = \frac{1}{4} \left\{ -a + \frac{3b^2 - a^2 + 4b + 1}{2b} + \frac{a^2 - b^2}{b} \ln (b-a) \right\} \]
3) \( \sqrt{x^2 + d^2} \leq R_i \leq x+d \)
   \[ G_i = \frac{1}{2} \left\{ 1 - \frac{a}{2} + \frac{3b^2 - a^2 + 1}{4b} + \frac{a^2 - b^2}{2b} \ln (b-a) \right\} \]
4) \( R_i \geq x+d \)
   \[ G_i = 1 - \frac{a}{2} + \frac{a^2 - b^2}{4b} \ln \frac{b-a}{b+a} \]

(c) \( G_i \) at \( x=0 \)
1) \( R_i \leq d \)
   \[ G_i = 0 \]
2) \( R_i \geq d \)
   \[ G_i = 1 - a \]
(d) \( G_i \) at \( x = d \)

1) \( R_i \leq 2d \)

\[
G_i = \frac{1}{2} \left( 1 + \frac{1}{4a} \right)
\]

2) \( R_i > 2d \)

\[
G_i = 1 - a/2
\]

The results obtained are shown in Fig. 4 for the spheres of radii of 1 to 5 \( \mu m \) as the spheres are imagined as being cell nucleus. The absorbed dose is given in rad as one disintegration per \( cm^3 \) occurs. The dashed lines show the surfaces of the spheres.

FRACTION OF SELF-ABSORPTION IN SPHERICAL SOURCE

According to the assumptions the fraction of self-absorption in a spherical source, \( F \), is given by

\[
F = \frac{\sum_{i=1}^{18} P_i E_i \frac{\bar{L}_i}{R_i}}{\sum_{i=1}^{18} P_i E_i}
\]

where \( \bar{L}_i \) is a mean distance traversed in the sphere by the \( \beta \)-particles of the energy \( E_i \) and given by

\[
\bar{L}_i = \frac{3d}{4} \quad \text{for} \quad R_i \geq 2d
\]

\[
\bar{L}_i = R_i - \frac{3R_i^2}{8d} + \frac{R_i^4}{64d^3} \quad \text{for} \quad R_i \leq 2d
\]

The results are shown in Fig. 5 by the solid line. The fractions of self-absorption in the spherical sources increase with the radii as expected and are 0.64, 0.79, 0.85 and 0.89 for the radii of 1, 2, 3 and 4 \( \mu m \) respectively.

The comparison with the other results shown in Fig. 5 will be discussed later.

![Fig. 4. Dose distributions per disintegration/cm³ due to tritium uniformly deposited inside or outside the spheres of 1-5 \( \mu m \) radii.](image)

![Fig. 5. Relative amount of total energy emitted that is absorbed within spheres, from tritium uniformly deposited within the spheres.](image)
ENERGY DEPOSITION IN SPHERE DUE TO THE SURROUNGING SOURCE

According to the assumptions the fraction of energy deposition in a sphere due to $^3$H in the region from the surface of the sphere to the maximum range $R_m$ of the $\beta$-particles, $f$, is given by

$$f = \frac{\sum_{i=1}^{18} P_i E_i \frac{I_i}{R_i}}{\sum_{i=1}^{18} P_i E_i}$$

where $I_i$ is a mean distance traversed in the sphere by the $\beta$-particles of energy $E_i$ and putting $B = R_m^2 + 3R_m^2d + 3R_md^2$

$$I_i = \frac{(2AR_i^2d^2 - R_i^4)}{(64B)} \quad \text{for } R_i \leq 2d$$

$$I_i = \frac{(-18d^4 + 20R_i^2d^2 - 9R_i^2d^2 + 3R_i^2d)}{(8B)} \quad \text{for } R_i \geq 2d$$

The calculation method of $I_i$ is shown in Appendix II.

The results are shown in Fig. 6. The values of $f$ are 0.2, 0.4, 0.7 and 0.9% for the spheres of radii of 1, 2, 3 and 4 $\mu$m respectively.

DISCUSSION

The absorbed doses due to $^3$H have been estimated on the basis of different assumptions, but these results show appreciably different values depending on the models and stopping powers used.

Robertson et al. and later Künkel have calculated the absorbed doses per disintegration about a point source with the substantially same method, using Langer's spectrum and the stopping power by differentiating Brown's range-energy relation, and obtained the same results, as shown in Fig. 3. Robertson and Hughes have calculated as follows. The spectrum was divided into eighteen one-keV intervals and the value of the stopping power, multiplied by the frequency of emission, tabulated at 1/2 $\mu$m intervals for each energy group. Finally the value thus obtained was divided by the volume of the 1/2 $\mu$m thick shell in which the energy in each interval is deposited.

If simple and accurate formulations of the stopping power and the transmission probability are achieved, the energy dissipated by an electron of energy $E$ may be calculated by $d(\eta E)/dx$ instead of basing on theory as Spencer calculated,
where \( \eta \) is the transmission probability and the energy \( E \) expressed as the function of the residual range.

In the calculations of Robertson et al. and Künkel the transmission probability is not taken into account, and therefore the dose distribution obtained may be considered to differ fairly from the true one. Bleeken\(^{29}\) has reported that the dose calculation proposed by Robertson et al.\(^{21}\) cannot explain satisfactorily the autoradiographic results.

Robertson et al. have shown that the dose distribution curve of the point source is closely represented by the equation

\[
D(r) = 185\exp(-5.55r) + 15\exp(-1.92r)
\]

where \( D(r) \) is the average dose per disintegration in rads as a function of the distance \( r \), in \( \mu m \), from the point source. Goodheart\(^{22}\) has also given an equation which fits to the curve of Robertson et al.,

\[
D(r) = \frac{3}{(0.01 + r^2)(1 + 0.1r^2)(1 + 0.2r^2)}
\]

Eqn. (2) give 200 rads at \( r=0 \), while Eqn. (3) 300 rads. These equations can be analytically integrated in order to obtain the dose distributions and so on. The total energy emitted per disintegration, \( E_t \), is given by

\[
E_t = \int_0^\infty D(r)4\pi r^2 dr
\]

Eqns. (2) and (3) give \( E_t = 5.02 \) and 4.49 keV with under-estimations of about 12 and 21% to the average energy 5.69 keV, respectively. In present work \( E_t = \sum_{i=1}^{18} E_i P_i = 5.45 \) keV with under-estimation of about 4%.

Bond and Feinendegen\(^{23}\) have reported the values of the self-absorption fractions in spherical sources, \( F \), which were calculated by Robertson. The values are reproduced in Fig. 5.

The values of \( F \) have been also estimated by Apelgot et al.\(^{8}\) and Kühn\(^{9}\) as shown in Fig. 5. Apelgot and Duquesne have calculated the values of \( F \) for radii greater than 2 \( \mu m \) using the equation

\[
F = 0.876 + 0.124 (d-1)^3/d^3
\]

The energy of the \( \beta \)-particles have been assumed to be 5.7 keV and its range estimated to be 1 \( \mu m \) using classical non-relativistic equation of energy loss. Then they have considered that the average length of trajectory in a region \( C \) which is surrounded by the two surfaces of the concentric spheres of radii of \( d \) and \( d-1 \) is 0.62 \( \mu m \) with energy loss of 2.88 keV and 25% of the excess energy 2.82 keV emitted from the sphere of the radius \( d \).

However, as the average length \( \bar{L}_e \) of the trajectory in the sphere of the particles liberated in \( C \) is given by

\[
\bar{L}_e = \frac{168d^2 - 192d + 65}{64(3d^2 - 3d + 1)}
\]
as known in Appendix III, the values of $F$ calculated by Apelgot and Duquesne are easily understood to have been over-estimated.

Kühn has calculated the values of $F$ using Lea's LET\textsuperscript{24} by the following equation.

$$F = \frac{\sum_{i=1}^{l} W_i E_i + L \sum_{i=l+1}^{n} W_i \text{LET}(E_i)}{\sum_{i=1}^{n} W_i E_i}$$

where $W_i$ is the relative frequency of emission of $i$-th energy interval which the mean energy is $E_i$ if the $\beta$-spectrum is divided into $n$ intervals. He has assumed that the $\beta$-particles are completely absorbed in the sphere if its ranges are equal to or smaller than a mean distance $L = 1.2d$ between all points in the sphere and the surface. Kühn's calculation results in over-estimation because it is expected that $L$ is correctly $3d/4$ as shown in the calculation of self-absorption in spherical source and a part of the $\beta$-particles is always emitted from the sphere even if its range is enough small.

The energy dissipated by a low-monoenergetic electron may be represented by a typical curve shown in Fig. 7 with the solid line\textsuperscript{25}, while the present model gives a curve shown by the dashed line. Therefore, in present work, the point source dose distribution, the fraction of self-absorption of spherical source, the dose distributions inside and outside the sphere, and the fraction of energy deposition in the sphere due to the surrounding source have been analytically estimated with the constant absorption model, but the values may be conjectured to contain the differences with the true ones such as seen in Fig. 7.

The estimation methods of absorbed dose in cell after incorporation of $^3$H have been studied by some workers as compared and discussed now, while these approximate calculations led to the appreciably different values. The discrepancies are considered to be not only dependent on the assumed models but also on the used range-energy relations or stopping powers. For example, it is readily known that all of $F$ values obtained by them would become larger if the range-energy relation of Katz and Penfold is applied. The range-energy relation or the stopping power and the transmission probability of low-energy electrons may be required to be estab-

![Fig. 7. Typical curves of energy absorbed from an electron beam vs. transmission length. The solid line was conjectured on the basis of Huffman et al. The constant absorption model gives the dashed line.](image-url)
lished theoretically and experimentally.

Further study of estimation of absorbed dose due to low-energy electrons would be attempted using modern physical data.

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APPENDIXES

I. Calculation of $G_i$ in a spherical source

The equations to calculate the values of $G_i$ for various cases were given in text, but the derivation of the equations to calculate the dose distribution in a spherical source will be only shown here because the other equations can be also derived by the same method.

The geometry for this case is shown in Fig. 8. As $V$ is the sphere and $dv = r^2 \sin \theta d\theta d\phi$ according to text, $G_i$ at a point in the spherical source, if one disintegration per unit volume occurs, is given by

$$G_i = \frac{1}{4\pi R_i} \int_0^{r_{\max}} \int_{\phi}^{2\pi} \frac{r^2 \sin \theta d\phi drd\theta}{r^2} = \frac{1}{4\pi R_i} \int_0^{r_{\max}} \int_{\theta}^{2\pi} \sin \theta d\phi drd\theta$$

Therefore

$$G_i = \frac{1}{2R_i} \int_0^{\theta_{\max}} \sin \theta d\theta$$

where $r_{\max}$ is the maximum value of $r$.

Now $r_{\max}/R_i$ and the limits of the integration with respect to $\theta$ are given as follows:

1) if $R_i \leq d - x$

$$r_{\max}/R_i = 1$$

for $0 \leq \theta \leq \pi$

2) if $d - x \leq R_i \leq d + x$

$$r_{\max}/R_i = \left\{ \begin{array}{ll}
- b \cos \theta + \sqrt{a^2 - b^2 \sin^2 \theta} & \text{for } 0 \leq \theta \leq \theta_0 \\
1 & \text{for } \theta_0 \leq \theta \leq \pi
\end{array} \right.$$ 

3) if $R_i \geq d + x$

$$r_{\max}/R_i = - b \cos \theta + \sqrt{a^2 - b^2 \sin^2 \theta}$$

for $0 \leq \theta \leq \pi$

where $\cos \theta_0 = (a^2 - b^2 - 1)/(2b)$. Thus the equations for the case A, (a) in text are obtained by brief calculations.

II. Calculation of $l_i$

The geometry to calculate $l_i$ is shown in Fig. 9. According to text $l_i$ may be given by the following equation if $l_i$ is a distance traversed in a sphere by the $\beta$. 
particles emitted from the surrounding source.

\[
I_i = \int x \int \int \int l_i \frac{\sin \theta d\phi d\theta}{4\pi} \frac{4\pi x^2 dx}{4\pi x^2 dx} = \int \int \int \int l_i x^2 \sin \theta d\phi d\theta dx \frac{\int d \cdot R_m}{4\pi x^2 dx}
\]

Therefore

\[
I_i = \frac{3}{2B} \int x \int \theta l_i x^2 \sin \theta d\phi dx
\]

Now putting \( I_i = R_i - x \cdot \cos \theta + \sqrt{d^2 - x^2 \sin^2 \theta} \), \( l_i = 2\sqrt{d^2 - x^2 \sin^2 \theta} \), \( \cos \theta = (R_i^2 + x^2 - d^2)/(2R_i x) \) and \( \cos \theta = \sqrt{x^2 - d^2}/x \), the conditions to perform the integration are given as follows:

1) if \( R_i \leq 2d \)
   \[
   l_i = l_1 \text{ for } d \leq x \leq \sqrt{R_i^2 + d^2} \text{ and } 0 \leq \theta \leq \theta_0
   \]
   \[
   l_i = l_2 \text{ for } d \leq x \leq \sqrt{R_i^2 + d^2} \text{ and } \theta_0 \leq \theta \leq \theta_1
   \]
   \[
   l_i = l_1 \text{ for } \sqrt{R_i^2 + d^2} \leq x \leq R_i + d \text{ and } 0 \leq \theta \leq \theta_0
   \]

2) if \( R_i \geq 2d \)
   \[
   l_i = l_2 \text{ for } d \leq x \leq R_i - d \text{ and } 0 \leq \theta \leq \theta_1
   \]
   \[
   l_i = l_1 \text{ for } R_i - d \leq x \leq \sqrt{R_i^2 + d^2} \text{ and } 0 \leq \theta \leq \theta_0
   \]
   \[
   l_i = l_2 \text{ for } R_i - d \leq x \leq \sqrt{R_i^2 + d^2} \text{ and } \theta_0 \leq \theta \leq \theta_1
   \]
   \[
   l_i = l_1 \text{ for } \sqrt{R_i^2 + d^2} \leq x \leq R_i + d \text{ and } 0 \leq \theta \leq \theta_0
   \]

Thus the equations of \( I_i \) shown in text are obtained by brief calculations.

III. Calculation of \( L_\varepsilon \)

The geometry to calculate \( L_\varepsilon \) is shown in Fig. 10. If \( L_\varepsilon \) is a distance traversed in a sphere by the \( \beta \)-particles emitted in the region \( C \), \( L_\varepsilon \) may be given by

\[
L_\varepsilon = \int \int \int l_\varepsilon \sin \theta d\phi d\theta \frac{4\pi x^2 dx}{4\pi x^2 dx} = \int 4\pi x^2 dx
\]

\[
= \int_{d-1}^d \int \int_{d-1}^{2\pi} l_\varepsilon x^2 \sin \theta d\phi d\theta dx \frac{\int d \cdot R_m}{4\pi x^2 dx}
\]

Therefore

\[
L_\varepsilon = \frac{3}{2} \int_{d-1}^d \int \int_{d-1}^{2\pi} l_\varepsilon x^2 \sin \theta d\phi dx \frac{3d^2 - 3d + 1}{3d^2 - 3d + 1}
\]
Now putting $\cos \theta = (d^2 - x^2 - 1)/(2x)$, $L_c$ is represented by

$$L_c = -xcos\theta + \sqrt{d^2 - x^2 \sin^2 \theta}$$ for $0 \leq \theta \leq \theta_0$$

$$L_c = 1$$ for $\theta_0 \leq \theta \leq \pi$

according to Apelgot et al. Therefore

$$L_c = \frac{1}{64} \frac{168d^2 - 192d + 65}{3d^2 - 3d + 1}$$

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