Global and local depletion of ternary complex limits translational elongation

Gong Zhang1, Ivan Fedyunin1,2, Oskar Miekley1, Angelo Valleriani2, Alessandro Moura3,* and Zoya Ignatova1,*

1Biochemistry, Institute of Biochemistry and Biology, University of Potsdam, Karl-Liebknecht-Str. 24-25, 14467 Potsdam, 2Theory and Bio-Systems, Max Planck Institute of Colloids and Interfaces, 14424 Potsdam, Germany and 3Institute of Complex Systems and Mathematical Biology, Dept. of Physics, University of Aberdeen, AB24 3UE Aberdeen, UK

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ABSTRACT

The translation of genetic information according to the sequence of the mRNA template occurs with high accuracy and fidelity. Critical events in each single step of translation are selection of transfer RNA (tRNA), codon reading and tRNA-regeneration for a new cycle. We developed a model that accurately describes the dynamics of single elongation steps, thus providing a systematic insight into the sensitivity of the mRNA translation rate to dynamic environmental conditions. Alterations in the concentration of the aminoacylated tRNA can transiently stall the ribosomes during translation which results, as suggested by the model, in two outcomes: either stress-induced change in the tRNA availability triggers the premature termination of the translation and ribosomal dissociation, or extensive demand for one tRNA species results in a competition between frameshift to an aberrant open-reading frame and ribosomal drop-off. Using the bacterial Escherichia coli system, we experimentally draw parallels between these two possible mechanisms.

INTRODUCTION

Faithful transmission of the genetic information from a messenger RNA (mRNA) to protein is crucial for normal cellular vitality and physiological functions. Ribosomes elongate the nascent peptide chain by sequential addition of amino acids in an mRNA template-directed manner by repetitive selection of aminoacylated-transfer RNA (aa-tRNA). Thereby the tRNA is repeatedly loaded with the cognate amino acid by ubiquitously expressed abundant aminoacyl-tRNA synthetases (aaRS) (1). The charged aa-tRNAs are delivered to the ribosome as a ternary complex (TC, aa-tRNA-EF-GTP) with the elongation factor [EF-Tu in bacteria and eEF1A (eEF1alpha) in eukaryotes] bound to GTP (2). Consequently, the concentration of the functional TCs is crucial for the rate of elongation at each codon.

The full set of 61 sense codons are read by specific isoaccepting tRNAs, which differ substantially in their concentration (3,4). This asymmetric tRNA abundance leads thereby to variations in the translation rate of each codon, and this results in a nonuniform speed of elongation along the mRNA (5): codons pairing to abundant tRNA species are translated faster compared to codons read by rare tRNAs (6). Bias in the usage of codons that pair to highly abundant tRNAs versus minor tRNAs can be a selection force for elongation speed (7,8), translation accuracy (2) or for higher fidelity of processes downstream of translation (5,9). Until now, codon usage bias (i.e. the variations in the frequency of usage of some synonymous codons versus others) has been used assuming that the codon usage patterns directly mirror the copy numbers and consequently the concentration of the cognate tRNA (4,10). Experimental quantification of the full set of tRNA species for two organisms, Escherichia coli and Bacillus subtilis, however, revealed deviations from the ideal proportionality between the tRNA concentration and codon usage (3,11). Moreover, in multicellular organisms, the tRNA concentrations vary among different tissues and stages of differentiation, in spite of the uniform codon usage and equal genomic copy number of the tRNAs in each cell (4,12). Similarly, in exponentially growing prokaryotic cells, the distributions of the tRNA concentrations have the potential to
change very quickly (13) and certain rare codons are found to be unexpectedly translated at higher rates in spite of the low tRNA abundance (14,15).

Changes in the external environment modulate the abundance and concentration of the factors involved in the elongation cycle influencing thereby the dynamics of the elongation process. Limitations in the amino acids supply alter the pattern of the charged tRNA species (16,17). Variations in the macromolecular crowding of the cellular cytoplasm (e.g. in cells exposed to osmotic stress) influence macromolecular diffusion (18–20) thereby changing the rate of diffusion of the components participating in translation. Intriguingly, translational fidelity negatively correlates with the translational demand (21) and alterations in it (i.e. by variations of the protein expression profile) may alter the availability of some TCs.

To understand the dynamics and complexity of the elongation process, we developed a general model of translational elongation consisting of deterministic and stochastic parts. Analysis of the model revealed that perturbations in the delivery of the active TC to the ribosomes either by exhaustive demand for TC or diffusion limitations led to profound differences in the elongation rate of a single codon. Experimental analyses of translational profiles confirmed the predictions of our model; we observed that slower diffusion of TC in cells exposed to osmotic stress considerably increased the probability of premature termination of translation. In turn, an extensive demand for one single TC species during the overexpression of proteins with homopolymeric stretches was not compensated by an upregulated transcription of the cognate tRNA. This general model for translational elongation applies to all kingdoms of life, providing a systematic insight into mRNA elongation rate in response to diverse intrinsic (i.e. extensive request for one amino acid) and extrinsic (i.e. environmental changes) factors.

**MATERIALS AND METHODS**

**tRNA quantification**

Exon 1 of Htt with 53 CAG repeats was subeloned downstream of the GST moiety in the pGEX-6P-1 plasmid. The pure exon 1 with pathological repeats of 53Gln is highly aggregation-prone and the N-terminal GST-tag ensures its solubility throughout the expression cycle. Plasmids containing the insert were expressed in E. coli BL21(DE3) cells and cultured in LB medium at 37°C till OD = 0.6. Total tRNA was isolated as described elsewhere (22). The quality of tRNA was verified with the absorbance ratio A_{260nm}/A_{280nm}. cDNA was synthesized from the total tRNA samples using the in vitro reverse transcription kit (Fermentas) with random hexamer primers, and quantified in a real-time PCR cycler (StepOnePlus™ Real-Time PCR system, Applied Biosystems) with tRNA-specific primers. The tRNA concentration was normalized using the 5S rRNA concentration.

**Polysomal profiles**

The E. coli BL21(DE3) cells were cultivated in LB medium at 37°C and 220 r.p.m. till OD = 0.4 then harvested by centrifugation at 6000 r.p.m. for 5 min, resuspended in equal volume of M9 minimal medium and incubated further at 37°C till OD = 0.5. Cells were split into equal aliquots and to one aliquot NaCl was added to a final concentration of 300 mM (0.6 Osm) to exert osmotic stress. Cells were further cultured at 37°C for 20 min and their polysomes were isolated using the Cozzone’s procedure with following modifications (23). The protoplasts from 100 ml culture were resuspended in 0.7 ml freshly prepared lysis buffer with 0.2% tritonX100, 10 U DNase I (RNase-free, Fermentas) and 1 U/µl Ribolock™ RNase inhibitor (Fermentas). The mixture was incubated 5 min at 37°C and clarified by centrifugation at 10000g for 10 min at 4°C. Lysates containing the same amount of nucleic acids were layered onto 15-40% sucrose gradient in lysis buffer, and centrifuged for 2.5 h at 40 000 r.p.m. in SW65 rotor (Beckmann). The gradient was slowly pumped out and the A_{260nm} was monitored by a flow-through UV spectrophotometer.

**Numerical methods and stochastic simulations**

The differential equations were solved in MatLab using non-stiff Runge Kutta method with a relative error tolerance of 10⁻⁷ (24).

The stochastic simulations were performed in a small cubic volume, which was equal to 1/10 of the typical E. coli volume (10⁻¹⁵ l) and contains the same density of ribosomes and TC as in E. coli cells. The numbers of ribosomes and TC in this volume were determined according to their concentrations in E. coli cells. The diffusion of the ribosomes is neglected, a simplification that is permitted because of the much larger size of the ribosomes (~3000 kDa) compared to the TC (~70 kDa). The ribosomes and TC are considered as spheres and are randomly located in this volume. In all three axes, the distance which a TC traverses in a Brownian diffusion can be described with the following probability distribution:

\[ P(d_i,t) = \frac{1}{\sqrt{\pi}Dt} e^{-\frac{d_i^2}{4Dt}} \]  

where \( d_i \) is the distance projected on one axis, \( D \) is the diffusion coefficient and \( t \) is the time interval. We used periodic boundary conditions in the cubic spatial domain where the TC particles follow a Brownian motion and collisions are detected by calculating the distance between the TC and ribosomes.

An amino acid is delivered to the ribosome by an effective collision between TC and ribosomes. A collision event was detected when the distance between the centers of a TC and ribosomes becomes closer than the sum of their radii. However, only a small fraction of collisions was effective as determined by the effectiveness parameter: The collision effectiveness is adapted to the mean value of the translation rate derived from the experimental data in the deterministic model which comprises both structural details (the surface of the A-site entrance from the whole
The distribution of the waiting time of the ribosomes at the ribosome surface) (25) and kinetic details on discrimination and tRNA-induced proofreading (2).

In an effective collision, a TC enters the A-site, therefore it is removed from the simulation volume and is replaced by another TC generated randomly in the volume, so that the TC concentration is maintained constant. To accelerate the simulation while minimizing the missing events, the time interval \( t \) was determined as 95% of the distance moved within one time interval, which was less than the radius of the TC. The simulations were programmed in VB.NET 2008.

A total of 53,661, 27,744 and 23,273 individual events were recorded in the simulations for cells grown in optimal conditions; a medium with high osmolarity and by overexpressing polyGln-containing protein. The fitted parameters showed a significant deviation from the widely assumed exponential distribution \( (a = 0.8393, 95\% \text{ confidential interval is 0.8307–0.8481 for normal conditions}; \ a = 0.8822, 95\% \text{ confidential interval is 0.8695–0.8951 for cells exposed to osmotic upshift}; \ a = 0.8533, 95\% \text{ confidential interval is 0.8400–0.8669 for polyGln expressing cells}) \) as justified by the comparison of the fitting parameters as ascertained by MatLab.

The distribution of the waiting time of the ribosomes at the individual elongation events is fitted to the gamma function:

\[
P(x) = \frac{1}{b^\alpha \Gamma(a)} x^{a-1} e^{-x/b}
\]  

**RESULTS**

**Deterministic model of translation elongation**

The rate of translation of a single codon in the cell is determined by following four subsequent processes (Figure 1): (i) attachment of the amino acid to its cognate tRNA by the corresponding aaRS; (ii) aa-tRNA forms a TC-complex with GTP-bound elongation factor (EF-Tu in prokaryotes and eEF1A in eukaryotes); (iii) TC diffuses to a ribosome requesting it and transfers the amino acid to the growing polypeptide chain, whereby the tRNA is released; and (iv) the GDP-bound elongation factor is regenerated into its GTP-bound state by EF-Ts (prokaryotes) or EF-1B/C (eukaryotes). All isoaccepting tRNAs for one amino acid are charged by their common aaRS with identical kinetic parameters \( (K_m \text{ and } k_{\text{cat}}) \). The concentration of the aaRS is generally in proportion to the total concentration of its set of substrates, i.e. the full set of tRNAs mediating the transfer of one amino acid (26) with, as assumed here, homogeneous distribution of all components of the translational apparatus. Aside from methionine and tryptophan, all other amino acids are encoded by several synonymous triplets; therefore, the common aaRS is in a large excess over each tRNA isoacceptor, so we assume for all aaRS a constant concentration. The rate of formation of TC depends on the balance between the rate of supply (tRNA-charging and formation of the complex with the elongation factor) and demand of TC (consumption by the ribosomes, which is proportional to codon frequencies), and regeneration of EF-GTP (Figure 1). The active concentration of each of these components determines the rate of elongation of a single codon. Under conditions of non-limited amino acid supply, the mass balance equation for charged tRNA (aa-tRNA) concentration can be written as

\[
\frac{df}[\text{tRNA}]}{dt} = \frac{k_{\text{cat}}(1-f)[\text{tRNA}]}{K_m+(1-f)[\text{tRNA}]}[\text{aaRS}]-k_2[\text{EF-GTP}]/[\text{tRNA}]
\]  

Here \([\text{tRNA}]\) and \([\text{aaRS}]\) are the total concentration of a specific tRNA species and aaRS enzyme, respectively; \( f \) is the fraction of charged tRNA, \( k_{\text{cat}} \) and \( K_m \) are the enzymatic parameters of tRNA acylation catalyzed by the aaRS; \( k_2 \) is the rate constant of the complex formation between the charged tRNA with the EF [process (2), Figure 1]. The formation of the complex of EF-GTP with the aa-tRNA is very fast due to the high affinity and extremely high abundance of EF in both prokaryotes and eukaryotes (27). The concentration of TC (TC) depends on both TC formation and TC consumption with a rate constant \( k_i \) from a fraction of the ribosomes \( m \) which requests this tRNA isoacceptor [process (3), Figure 1]:

\[
\frac{d[\text{TC}]}{dt} = k_2[\text{EF-GTP}]/[\text{tRNA}]-k_i[\text{TC}]m
\]  

The mass balance of the free tRNA can be given as

\[
\frac{d[(1-f)[\text{tRNA}]}{dt} = k_i[\text{TC}]m-\frac{k_{\text{cat}}(1-f)[\text{tRNA}]}{K_m+(1-f)[\text{tRNA}]}[\text{aaRS}]
\]  

The elongation factor delivers the aa-tRNA to the ribosomes and after initial stabilization of the complex by interaction of tRNA with mRNA, the GTPase activation and GTP-hydrolysis takes place. Cognate tRNA increases the rate of GTPase activation, whereas the mismatches in the codon–anticodon region with incorrect tRNA (near-cognate or non-cognate) impair this reaction (28). GTP-hydrolysis yields an inorganic phosphate and GDP in an EF-bound form. EF-GTP is regenerated by a second group of elongation factors with high abundance [EF-Ts in prokaryotes and eEF-1B/C in eukaryotes; process (4), Figure 1] and the mass balance is described by

\[
\frac{d[\text{EF-GDP}]}{dt} = k_i[\text{TC}]m-k_4[\text{EF-GDP}][\text{GTP}]
\]
where $k_4$ is the overall rate constant of EF-GTP regeneration. Furthermore, the mass balance of EF-GTP can be introduced as

$$\frac{d[EF-GTP]}{dt} = k_4[EF-GDP][GTP] - k_2[EF-GTP][tRNA]$$

(7)

The general model of translation elongation is valid for any organism. The EF-GTP regeneration is uniform for all TCs with a constant contribution to each single step of elongation independent of the TC species. In eukaryotes the recycling of eEF1A is much slower and shares rate controlling functions with processes (1) and (3), whereas in prokaryotes, the regeneration of EF-Tu-GDP is, compared to all other processes, relatively fast with only a minor contribution to the overall kinetics of elongation and can be therefore neglected (29,30). For prokaryotes the process (2) is much faster than (3) (Figure 1) and consequently the mass balance of TC can be simplified to

$$\frac{d([tRNA])}{dt} = \frac{k_{cat}(1-f)[tRNA]}{K_m+(1-f)[tRNA]}[aaRS] - k_f[tRNA]m$$

(8)

As the elongation factor is in large excess over each tRNA species, the concentration of the TC is directly proportional to the concentration of the free tRNA (26). In turn, the average waiting time of the ribosome for a cognate TC will be

$$\tau = \frac{1}{k_f[tRNA]}$$

(9)

To illustrate the outcomes of Equation (8), we took as an example tRNA\(_{\text{Gln}}\) which in E. coli pairs solely to the CAG codon encoding glutamine (Gln). Under non-limiting Gln supply, the kinetic parameters of the aaRS enzyme are $k_{cat} = 3.20 \pm 0.48 \text{ s}^{-1}$ and $K_m = 0.31 \pm 0.09 \mu\text{M}$ (for tRNA) (31); the concentrations of the tRNA and glutamyl-tRNA synthetase (GlnRS) are $1.46 \times 10^{-6} \text{ M}$ and $2.79 \times 10^{-6} \text{ M}$, respectively (3,26). The maximal concentration of the ribosomes which could simultaneously request the CAG codon can be estimated from the CAG codon usage and is calculated to be $6.88 \times 10^{-7} \text{ M}$. At steady state, where 80% of the tRNA is charged ($f = 0.8$) (16), Equation (8) can be then solved for $k_i$; the resulting value is $5.39 \times 10^6 \text{ s}^{-1} \text{ mol}^{-1} \text{ l}$. Consequently, the translation rate ($r_t$) is predicted to be: $r_t = k_i f[tRNA]m = 4.33 \times 10^{-6} \text{ mol}^{-1} \text{s}^{-1}$, or 6.3 CAG codons/ribosome/s. This result is consistent with the experimentally measured translation rate in E. coli (4.2–21.6 codons/s) (32).

**TC can be globally exhausted by excessive demand**

By expression of proteins with homopolymeric amino acid runs or repetitive sequences, or by heterologous and homologous overexpression of proteins, the translation demand of specific TC species may significantly increase, thereby increasing the waiting time of the ribosome for the cognate TC. The presence of additional homopolymeric codon stretches (here exemplified by CAG runs) can be modeled by including an additional term in Equation (8):

$$\frac{d([tRNA])}{dt} = \frac{k_{cat}(1-f)[tRNA]}{K_m+(1-f)[tRNA]}[aaRS] - k_f[tRNA]m$$

(10)

where the $m_{\text{CAG}}$ is the concentration of the fraction from the ribosome pool involved in the translation of proteins with homopolymeric Gln stretches or of overexpressed proteins rich in CAG codons. Solving this equation numerically using the Runge–Kutta integration method (exemplified again for E. coli tRNA\(_{\text{Gln}}\)) revealed that because of the excessive demand, the concentration of TC-tRNA\(_{\text{Gln}}\) decreased significantly within <0.5 s (Figure 2A). This transition time corresponded to the time needed to translate 3–5 consecutive codons. The waiting time of the ribosomes increased parallel to the number of ribosomes simultaneously requesting the same TC (Figure 2B). The effect is inversely correlated.

![Figure 2](image-url)
to the codon usage: the depletion and the new steady state is established faster for tRNAs pairing to rare codons than to frequently used ones (Figure S1, Table S1). Essential proteins for growth are highly abundant and their mRNAs are enriched in codons that pair to major tRNAs and are thus optimized for fast translation speed (3,33). The slower depletion of the major tRNAs will ensure a sufficient level of expression of essential genes even in a conditions of high demand of the particular cognate tRNA.

Here, we assumed that the tRNA concentration of each species remains constant. However, the cell can modulate tRNA abundance according to the demand and specificity of the protein pattern (12,34). To investigate whether the tRNA\textsubscript{Gln}\textsuperscript{2} is upregulated when it is highly requested in translation, we measured its cellular concentration in cells overexpressing model proteins with a consecutive polyglutamine (polyGln) stretch built of CAG-codons only. We used the GST-fusion of Huntingtin exon 1 with 53 Gln (GST-Htt53Gln) which is implicated in Huntington’s disease. To experimentally simulate a large demand for tRNAGln\textsuperscript{2}, we expressed the GST-Htt53Gln on a plasmid with a high-copy number which produced >2500 mRNA copies simultaneously. Using quantitative RT-PCR under a confidence level of 95%, we could not detect an upregulation of the tRNA\textsubscript{Gln}\textsuperscript{2} (Figure 3), suggesting that an increased demand for one TC is not compensated by a transcriptional upregulation of the corresponding tRNA species; the level of the tRNA in the cell remained unchanged which in turn confirmed our assumption of constant concentration of tRNA\textsubscript{Gln}\textsuperscript{2}.

**TC is locally depleted in the vicinity of the ribosomes translating homopolymeric stretches**

To address the question of whether there is a concentration gradient built in the vicinity of a ribosome repeatedly translating one codon (i.e. by translation of homopolymeric stretches), we extended the model with the spatial diffusion of the TC complexes around ribosomes translating repetitive sequences. The TC is delivered to the ribosomes solely by passive diffusion, which we assumed here to follow Fick’s law. Even though the cytosol is highly crowded, both proteins and RNA particles diffuse in a typical Brownian manner albeit with a slower diffusion coefficients (20,35). We also assumed that each TC is generated away from the aa-tRNA-binding site, A-site, of the ribosome and the diffusion is isotropic. One evidence for this assumption is that aaRS have not been co-isolated or identified in the close proximity of the A-site and even though they might be in a close proximity to the ribosomes (36) there is still a diffusion path for the TC to the A-site. Approximating the ribosome as a sphere, the diffusion of the TC in its vicinity can be described as

$$\frac{\partial c}{\partial t} = D \nabla^2 c = D \frac{\partial}{\partial r} \left( r^2 \frac{\partial c}{\partial r} \right)$$

(11)

where $D$ is the diffusion coefficient, $c = c(r,t)$ is the concentration of TC as a function of position and time and $r$ is the distance to the center of the ribosome. We used a spherical coordinate system with the ribosome at the origin. At the radius where the TC collides with ribosome $r = a$ and $c = c_0$, whereas far away from the ribosome $r = \infty$, $c = c_\infty$. We solved Equation (11) in the case of *E. coli* with the following setup: $a = 14$ nm, the diffusion coefficient of TC is $2.569 \times 10^{-12}$ m\textsuperscript{2}/s (37). At steady state, $\partial c/\partial t = 0$. Solving the Equation (11) reveals the radial profile of TC concentration (Figure 4A):

$$c(r) = c_\infty - \frac{1}{r} \phi (c_\infty - c_0)$$

(12)

The supply rate of TC by diffusion is given by

$$I = \frac{\partial c}{\partial r}\bigg|_{r=a} = 4 \pi a^2 \phi = 4 \pi a (c_\infty - c_0)$$

(13)

and the consumption rate of TC in translation is

$$I = \frac{k_j [tRNA]}{N_A} = \frac{k_1 c_0}{N_A}$$

(14)

where $N_A$ is Avogadro’s constant. At steady state, the TC consumption rate is equal to the supply rate. Solving the equations reveals the local depletion factor ($L$):

$$L = \frac{c_0}{c_\infty} = \frac{1}{(k_j/4 \pi a DN_A)+1}$$

(15)

The outcome of this analysis shows that tRNA\textsubscript{Gln}\textsuperscript{2} can be locally depleted due to diffusion (Figure 4A), which limits additionally the rate of translation of consecutive stretches (here polyGln). Importantly, local depletion may also occur in the vicinity of ribosomes translating isolated CAG codons; however, the effect is much stronger for ribosomes involved in the translation of consecutive polyGln stretches. Considering the effect of the local depletion on the ribosomes translating repetitive stretches, the Equation (10) is modified by a factor $Lm_{CAG}$:

$$\frac{df([tRNA])}{dr} = \frac{k_{cat} (1-f) [tRNA]}{K_{cat} + (1-f) [tRNA]} [aaRS]$$

$$- k_j [tRNA] (m+Lm_{CAG})$$

(16)
Consequently, local depletion will affect the waiting time [Equation (9)]:

$$r_L = \frac{\tau}{L} = \frac{(k_t/4\pi aDNA)+1}{k_f}[tRNA]$$

(17)

Under optimal growth conditions, the local depletion has minor contributions to the extension of the waiting time of the ribosomes ($L = 0.98$) (Figure 4B).

External stress generally leads to changes in the cell volume and solute composition in the cytosol (38) and alters the macromolecular crowding (20). In the case of osmotic upshift in the cellular environment, the primary response of the cell is a loss of large amounts of water, which in turn causes an increase in the macromolecular concentration in the cytosol and consequently alters the diffusion properties of proteins (18,19). To address the effect of perturbed diffusion of TC on translation elongation, we next performed the simulations in conditions of osmotic stress (Figure 4C). We chose mild osmotic stress (0.6 Osm) in which diffusion is still permitted albeit much slower; the diffusion coefficient of the $\sim 70$ kDa TC complex decreased from $2.569 \times 10^{-12}$ m$^2$ s$^{-1}$ (37) to $0.32 \times 10^{-12}$ m$^2$ s$^{-1}$ on osmotic upshift to 0.6 Osm imposed by NaCl (18,19). The analysis suggests that even mild osmotic upshift and moderately enhanced macromolecular crowding has a tremendous effect on the waiting time of ribosomes for the cognate TC (Figure 4C) and will considerably delay the translation of a single codon because of an increased global depletion of TC ($L = 0.98$ at 0 Osm, $L = 0.86$ at 0.6 Osm and $L = 0.41$ at 1.45 Osm).

**Stochastic distribution of the individual elongation events**

Intracellular reactions occur far from thermodynamic equilibrium and the relatively low total number of each molecular species in the cell suggests that fluctuations can be significant and are likely to play an important role in the elongation dynamics. To address this, we propose a stochastic model to describe the individual translation elongation events. For simulation of the movement of passively diffusing TC we used Brownian random-walker model (Supplementary Methods in Supplementary Data) that allowed recording of the waiting time of ribosomes in individual elongation events (Figure 5A). Intriguingly, the best fit of the distribution of the individual ribosomal waiting times is given by the gamma function rather than the exponential function (Figure 5A, Supplementary Methods in Supplementary Data). The interactions between two of the ribosomal proteins (L7/L12) with the elongation factor (EF-Tu) promote
ternary complex binding to the ribosome (39) and are most likely the reason for the observed gamma distribution. Additionally, we performed a series of simulations with different parameters to address their impact on the waiting time: ribosomal concentration has no influence on the distribution, whereas the scale (parameter b) shows an inverse correlation to the tRNA concentration and the diffusion coefficient (Figure S2). In turn, the probability

**Figure 5.** The probability of aberrant translation increased by extensive demand of one TC species or global alterations in the TC concentration. (A) Gamma fit of the distribution of the waiting time of ribosomes under optimal growth conditions (black), osmotic stress (red) and overexpressed polyGln protein with $m_{CAG} = 1000$ (blue). (B) Large number of ribosomes simultaneously translating polyGln stretches enhances the probability of ribosomal disassembly. (C) Polysomal profile of *E. coli* cells growing in normal balanced medium (black) and overexpressing Htt exon 1 with 53Gln (blue). To exclude any effect of the plasmid backbone, the control cells were transformed with the empty vector. The peaks corresponding to single ribosomal subunits are designated 30S and 50S, monosomes as M, and peaks representing mRNA occupied by two and more ribosomes are marked as polysomes. (D) Increased molecular crowding by an external osmotic upshift decreases the diffusion coefficient of the TC and subsequently increases the ribosomal drop-off. The macromolecular concentration and its influence on the diffusion properties on the macromolecules was calculated as described (20). Note that in the simulations in (B) and (D), the competition of the release factor for the A-site of the transiently paused ribosomes is considered. (E) Polysomal profile of *E. coli* cells grown in normal balanced medium (black) and in medium with osmolarity of 0.6 Osm (red). 30S and 50S denote the peaks corresponding to dissociated small and large subunits, respectively.
density function (parameter $a$) positively correlates with the diffusion coefficient, and increases slightly only at a very low TC concentration (Supplementary Figure S2). In all cases, $a < 1$ suggesting a significant deviation from the exponential distribution exists independent of the simulation setup.

Our predictions clearly suggest that both global and local depletion of TC extend the average residence time of the ribosomes at a single codon (Figure 5A). Two outcomes are possible in the case of an aberrant ribosomal stalling in the cell: (i) frameshift to a new open-reading frame (40) or (ii) premature termination of synthesis (41). There is no clear, experimentally determined cutoff in the residence time of the ribosomes to distinguish between the two processes. Radioactive measurements of the drop-off frequency at leucine-encoding codons (42) suggest an average time of 1.17 s; however, it cannot be differentiated among the five tRNA species pairing to the six Leu-encoding codons. The frameshifting at evolutionary non-programmed sites occurs infrequently with a quite low rate below $3 \times 10^{-5}$ per codon, which is two orders of magnitude lower than the natural ribosomal drop-off (43), suggesting that concomitantly to the ribosomal dissociation frameshift will occur albeit with significantly lower frequency. The frameshift probability is non-uniform among the codons, preferably occurring at codons read by minor tRNAs (44), whereas equal selection of all tRNAs at the ribosomes (45) will favor the uniform probability of ribosomal dissociation independent of the codon. Therefore, hereafter for simplicity we refer to any aberrant outcome of the elongation as ribosomal dissociation, which for some intrinsically frameshifting-prone codons might equal frameshift probability. The probability of premature termination of the elongation ($P_d$) can then be expressed as

$$P_d = \int_{t_i}^{\infty} P(t)dt$$  \hspace{1cm} (18)

Under optimal growth conditions, the global depletion caused by excessive demand of tRNA$^{Gln}_{54}$ is significantly influenced by the number of ribosomes simultaneously requesting the same TC (Figure 5A and B), whereas under osmotic stress, altered diffusion properties of TC (Figure 5A and D) add to this effect and the average waiting time of the ribosomes increases. The ribosomes stalled at a nonstop codon can be rescued in two ways in bacteria: (i) transfer-messenger RNA (tmRNA or SsrA) releases the stalled ribosomes by trans-translation tagging of the nascent chain by 10-amino-acid tag and targeting it to degradation (46), or (ii) release factors coordinate the hydrolysis of the stalled nascent chain (47). As the tmRNA is complexed with the elongation factor its diffusion will be similarly influenced as all TCs by the increased macromolecular crowding in the cell. The release factor (~40 kDa), however, is smaller than the TC (~70 kDa) and its diffusion mobility will be affected to a lower extent in the more crowded environment (20); the higher mobility of release factor in osmotically stressed cells will increase the probability to encounter stalled ribosomes. Therefore, in the stochastic simulations, we considered the competition between the release factor and the cognate TC for the A-site of the aberrantly stalled ribosomes (Figure 5D).

To examine whether the frequency of premature translational termination is influenced by an extensive demand of tRNA$^{Gln}_{54}$ or altered macromolecular diffusion, we compared the translational activity by examining the polysomal profiles of exponentially growing E. coli cells with cells overexpressing GST-Htt53Gln (Figure 5C) and E. coli cells subjected to osmotic upshift (Figure 5E). mRNA is actively translated by multiple ribosomes, i.e. polysomes, that can be separated on sucrose density gradients. The translation profiles of polyGln protein expressing cells are dominated by the polysome fraction similar to the profiles of the cells cultured in balanced medium (Figure 5C). The presence of peaks of single ribosomal subunits account for a small fraction of dissociated ribosomes (Figure 5C) as also predicted by the model (Figure 5B). In contrast, there was a dramatic shift of ribosomes from the polysomal region into single dissociated subunits or monosome following osmotic upshift (Figure 5E). The peaks of the single subunits account for the translational drop-off, whereas the enriched monosomal peak corresponds to ribosomal run-off (Figure S3); osmotic stress inhibits translational initiation resulting in ribosomal run-off (48,49).

**DISCUSSION**

Our model provides a general theoretical framework to predict the global elongation rate composed of single elongation steps under dynamic circumstances, e.g. including changes in cellular environment under stress conditions, exhaustive demand for one amino acid, tRNA availability by using both deterministic and stochastic approaches. It sheds light on the kinetics of the single elongation steps, which are by their transient nature difficult to track experimentally. Here, we present an example for translation in prokaryotes; however, the general model (Figure 1) can be applied to any organism and cell type provided that an adequate experimental data set for the system available, e.g. tRNA and elongation factor concentrations, kinetic properties of the cognate aaRS. Currently, the lack of quantitative data sets of eukaryotic tRNAs hinders the predictions for eukaryotic systems, even though the kinetic data of EF-GTP regeneration is currently available (29). The suggestion of structural organization of the eukaryotic protein synthesis machinery in large complexes which are spatially localized by linkage with the cytoskeleton and endoplasmic reticulum (36) would require a change in the parameters in the stochastic simulation and in turn would demand extremely high computational power. However, there is still a distance between aaRS and the A-site of the ribosome, which each TC complex has to pass in a passive Brownian diffusion. Furthermore, the model is not limited to external stress types (osmotic, temperature and oxidative) that change the solute composition and consequently alter the diffusion properties of the ternary complex. When cells are starved, the restricted supply of
an amino acid severely limits the availability of the cognate ternary complex and the charging of the tRNA isoacceptors [process (i), Figure 1] becomes rate limiting. The charging pattern within one set of isoacceptors for the same amino acid differs: the charged level of some tRNAs approaches zero and remains high for others (17). tRNAs that retain the charging pattern are enriched in genes involved in the amino acid biosynthesis suggested to be a regulatory loop to counteract the starvation (17). The differential charging pattern within one isoacceptor family will result in variations in the f-value for each tRNA species increasing the complexity of the Equation (1); however, the principle remains unaltered.

A crucial feature of the model is that it can predict a ribosomal stalling at single codons and a probability of aberrant translational outcome (i.e. frameshift and premature termination of the translational elongation). Both global and local depletion of TC influence the residence time of the ribosomes at a single codon. In the case of a local depletion, the flow of a TC is altered owing to the repeated request of a single TC (i.e. by translation of homopolymorphic stretches) in the vicinity of the ribosome which leads to an unexpected, non-naturally programmed pause in translation. As the diffusion properties and concentrations of all other TCs are unaltered, there is a high probability that an incorrect TC (near-cognate or non-cognate) can compete for binding to the A-site. During such pause, the alignment of the near-cognate and non-cognate TC to at least two nucleotides from the mRNA would cause a shift in the open-reading frame creating thereby a completely new amino acid sequence (40). Interestingly, many homopolymorphic stretches show a high tendency to frameshift in vivo resulting in a more severe disease phenotype: in one cell a small fraction CAG repeats encoding polyGln proteins frameshift to non-cognate) can compete for binding to the A-site. The finding that low TC availability can lead to two quite different outcomes, frameshifting or premature termination, suggests the feasibility of investigating the dynamics of translational elongation using our predictions. Regulation of the elongation cycles during translation plays an important role in the cell to control the cell cycle and to adjust proteome composition to acute needs. As our model can adequately predict the elongation kinetics under dynamic conditions, it bears the potential to understand the mechanisms of translational elongation, their dynamics and sensitivity to environmental factors in a systematic way.

SUPPLEMENTARY DATA
Supplementary Data are available at NAR Online.

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