Supplementary Material to "Prices vs. quantities for international environmental agreements"

By Ulrike Kornek[1] and Robert Marschinski[2]

[1]Mercator Research Institute on Global Commons and Climate Change, EUREF Campus 19, 10829 Berlin, Germany; Potsdam Institute for Climate Impact Research; and Department of Economics of Climate Change, Technische Universität Berlin; e-mail: kornek@mcc-berlin.net

[2]Joint Research Center, European Commission; Potsdam Institute for Climate Impact Research; and Department of Economics of Climate Change, Technische Universität Berlin; The views expressed are purely those of the author and may not in any circumstances be regarded as stating an official position of the European Commission.

1 List of symbols

Table 1: List of symbols with explanation

$\mathrm{E}[\cdot]$	Expectation value operator
i, j	Indices running over all countries
m, nm	Subscript indicating a member and non-member, respectively
TC	Total costs, sum of damages and abatement costs
D	Damages
C	Abatement costs
d_1, d_2	Damage parameters
ε_i, $\mathrm{E}[\varepsilon]$	Country i's uncertain and expected baseline emissions, respectively
σ, ρ	Parameters characterizing uncertainty in baseline emissions: standard deviation and coefficient of correlation between two distinct countries, respectively
N	Total number of countries
$k, k^*, \bar{k}^*$	Size of the coalition: given at second stage, in equilibrium of the game, in equilibrium of the game in the absence of uncertainty, respectively

k_q	For all $k \geq k_q$ the k-coalition implements quantities in equilibrium
e_i	Individual ex-post emissions
e, $e^*(k)$	Level of global emissions and level at optimum for a k-sized coalition, respectively
p_i, p_i^*	Emission tax and optimal emission tax set by regulator, respectively
$\bar{e}_i$, $\bar{e}_i^*$	Emission assignment and optimal emission assignment set by regulator, respectively
$\tilde{e}_k^*$	Optimal total emission allowances of a k-sized coalition in an emissions trading regime
$\mathrm{E}[\cdot]\|_{\bar{e}^*/p^*/\tilde{e}_k}$	Expected value under either instrument
Δ, Δ^k	Difference in expected total costs when switching the instrument from prices to quantities for: a single regulator; a coalition of size k, respectively
ℓ, ℓ_{-i}, ℓ_{-k}	Number of: countries with prices; countries but i with prices; non-members to coalition of size k with prices, respectively
ℓ_∞	Maximum number of countries with prices in the no-agreement equilibrium (and exact number for $N \to \infty$)
ℓ^*	Equilibrium number of countries implementing prices
$\Phi(k)$	Stability function: difference in expected total costs of a non-member in the presence of a $(k-1)$-sized coalition and a member in presence of a (k)-sized coalition
$\bar{\Phi}(k)$	Stability function under certainty
$\frac{d\bar{e}_j}{d\bar{e}_\mathrm{m}}$, $\frac{d\bar{e}_j}{dp_\mathrm{m}}$	Derivative of emission policy level of non-members with respect to emission policy levels of a member
$\frac{dp_j}{d\bar{e}_\mathrm{m}}$, $\frac{dp_j}{dp_\mathrm{m}}$	Derivative of emission tax level of non-members with respect to emission policy levels of a member

2 Equivalence of costs of abatement in Eqs. (1) and (2)

Starting from the expression for abatement costs taken directly from Weitzmann 1974, our Eq. (2):

$$C^{\mathrm{W}}(q,\theta) = a(\theta) + \left(C' + \alpha(\theta)\right)(q-\hat{q}) + \frac{C''}{2}(q-\hat{q})^2, \tag{1}$$

we aggregate the last two summands into one square and subtract the extra term $\left(\frac{C'+\alpha(\theta)}{C''}\right)^2$:

$$C^{\mathrm{W}}(q,\theta) \quad = a(\theta) + \frac{C''}{2}\left(\frac{C'+\alpha(\theta)}{C''} + q - \hat{q}\right)^2 - \left(\frac{C'+\alpha(\theta)}{C''}\right)^2.$$

This equation can be expressed in terms of emissions by carrying out a change of variable $q_i = \varepsilon_i - e_i$:

$$\begin{aligned} C^{\mathrm{W}}(q,\theta) \quad & \overset{q_i=\varepsilon_i-e_i}{\rightarrow} C^{\mathrm{W}}(e_i,\theta) \\ & = \frac{C''}{2}\left(\frac{C'+\alpha(\theta)}{C''} - \hat{q} + \varepsilon_i - e_i\right)^2 + a(\theta) - \left(\frac{C'+\alpha(\theta)}{C''}\right)^2. \end{aligned}$$

This expression can be further simplified to

$$C^{\mathrm{W}} \quad \overset{\text{dropping constants}}{\rightarrow} \frac{C''}{2}\left(A_i(\theta) - e_i\right)^2$$

by dropping all constant terms from the abatement cost function, given that they do not influence the relative ranking of instruments, neither the formation of coalitions. In addition, we aggregated terms by introducing $A_i(\theta) \equiv \frac{C'+\alpha(\theta)}{C''} - \hat{q} + \varepsilon_i$. In this expression the effect of having uncertainty on θ or on baseline emissions ε_i is equivalent.

3 Numerical algorithm and extended Table 1

This appendix first describes the algorithm used to compute the numerical equilibria shown in Table 1, and then provides an extended version of the table which includes values for coalition members' and global emissions.

Parameters d_2 and ρ are sufficient to determine ℓ_∞, as by Eq. (20). The algorithm proceeds by computing for each coalition size $0 < k \leq N$ the stability function of Eq. (21), by performing – according to the desired type of equilibrium – the following steps:

1. Equilibrium of stage 3: Emissions of members and non-members are determined by the Eqs. in Footnote 7, which are independent of the outcome of the second stage.

2. a. Equilibrium of stage 2, endogenous instrument choice: For $k \geq 2$, the expected total costs of the coalition for different emission policies are determined with Eqs. (12), (13) and (14). First, we assume a price-based treaty, i.e. that all k members would do prices and $\max(\min(\ell_\infty - k, N-k), 0)$ of the non-members optimally choose prices as well, and compare the expected total costs of the coalition to the case when the k members switch to quantities keeping the number of non-members with a price-based policy at $\max(\min(\ell_\infty - k, N-k), 0)$. Second, we compare the expected total costs of the coalition under the quantity-treaty, i.e. when the k members do quantities and $\min(\ell_\infty, N-k)$ non-members optimally choose prices, to the case when the coalition would switch to prices and the non-members stick to their emission policies. If expected total costs of the coalition are lower with quantities in both cases, a quantity-based treaty and $\min(\ell_\infty, N-k)$ non-members with prices is the equilibrium of the second stage. If expected total costs of the coalition are lower with a price-based treaty in both cases, the second stage equilibrium consists of k members and $\max(\min(\ell_\infty - k, N-k), 0)$ non-members with prices. In the numerical examples, we do not find cases of mixed-equilibria in instrument

choices for the coalition. For $k \in \{0,1\}$ we set the total number of countries with a price-based policy equal to $\min(\ell_\infty, N)$.

b. Equilibrium of stage 2, restriction to a price-based treaty: We fix the instrument of coalition members to prices and then set the number of non-members with price-based policy to $\max(\min(\ell_\infty - k, N - k), 0)$.

c. Equilibrium of stage 2, tradable quantities: As in point a., but changing the coalition's abatement costs under a quantity agreement from Eq. (12) to Eq. (22).

3. With the outcome of stage 2, we can compute the expected total costs of members and non-members for any given coalition size k, using Eqs. (12), (13) (respectively (22) for the case of tradable quantities) and (14). In those cases where the instrument choice of non-members is not unique – because stage 2 only determines the total *number* of countries with price-based policy – and for the case of $k \in \{0,1\}$, we calculate expected abatement costs by assuming that each country expects to implement prices with the same probability.

After these steps the expected total costs of members and non-members for each possible k are known, allowing to determine straightforwardly the stability function in Eq. (21). As the very last step, we identify the size $k^* < N$, for which the stability function is non-negative for k^* and negative for $k^* + 1$. In case the stability function is positive for $k = N$, the grand coalition comprising all countries is stable. This algorithm has so far only produced unique values k^* for each parameter set, justifying its identification as the stable equilibrium.

The following table replicates the results of Table 1 and in addition shows the corresponding values found for the expected emissions of members and total expected emissions.

Table 2: Numerical solutions of the game specified in Section 2, in which the coalition acts as a Stackelberg leader in the last stage. $N = 100$, $d_1 = 0.1$, $E[\varepsilon] = 0.2$ and values for d_2, σ and ρ as indicated. ℓ_∞ as defined in Eq. (20), k^* is the unique size of the stable coalition, $\mathrm{E}[TC_\mathrm{m}]$ and $\mathrm{E}[e_\mathrm{m}]$ the expected total costs and emissions of the members and $\sum_{i=1}^N \mathrm{E}[TC_i]$ and $\sum_{i=1}^N \mathrm{E}[e_i]$ the global expected total costs and emissions, *Source:* Authors' calculations

	Endogenous choice (always quantities)					Restriction to prices					Endogenous choice (always tradable quantities)				
σ	k^*	$\mathrm{E}[TC_\mathrm{m}]$	$\mathrm{E}[e_\mathrm{m}]$	$\sum_{i=1}^N \mathrm{E}[TC_i]$	$\sum_{i=1}^N \mathrm{E}[e_i]$	k^*	$\mathrm{E}[TC_\mathrm{m}]$	$\mathrm{E}[e_\mathrm{m}]$	$\sum_{i=1}^N \mathrm{E}[TC_i]$	$\sum_{i=1}^N \mathrm{E}[e_i]$	k^*	$\mathrm{E}[TC_\mathrm{m}]$	$\mathrm{E}[e_\mathrm{m}]$	$\sum_{i=1}^N \mathrm{E}[TC_i]$	$\sum_{i=1}^N \mathrm{E}[e_i]$
$d_2 = 0.200$, $\rho = 0.00 \Rightarrow \ell_\infty = Inf$															
0.00	18	0.09	-0.001	8.83	0.47	18	0.09	-0.001	8.83	0.47	18	0.09	-0.001	8.83	0.47
0.15	14	0.29	0.045	29.06	0.51	18	0.31	-0.001	31.33	0.47	19	0.27	-0.012	26.83	0.46
0.30	5	0.98	0.149	95.43	0.51	18	0.99	-0.001	98.83	0.47	21	0.80	-0.032	79.22	0.43
$d_2 = 0.077$, $\rho = 0.20 \Rightarrow \ell_\infty = 30.00$															
0.00	9	0.18	-0.010	17.92	1.13	9	0.18	-0.010	17.92	1.13	9	0.18	-0.010	17.92	1.13
0.15	8	0.37	0.013	36.85	1.15	11	0.36	-0.054	35.16	1.06	10	0.36	-0.032	35.73	1.09
0.30	3	0.93	0.132	93.02	1.19	15	0.88	-0.131	87.38	0.87	12	0.90	-0.075	89.42	1.02
$d_2 = 0.077$, $\rho = 1.00 \Rightarrow \ell_\infty = 6.00$															
0.00	9	0.18	-0.010	17.92	1.13	9	0.18	-0.010	17.92	1.13	9	0.18	-0.010	17.92	1.13
0.15	8	0.23	0.013	22.57	1.15	8	0.24	0.013	24.97	1.15	8	0.23	0.013	22.57	1.15
0.30	8	0.35	0.013	35.10	1.15	7	0.35	0.037	39.93	1.17	8	0.35	0.013	35.10	1.15