Origin of Cosmic Rays

Satio HAYAKAWA, Kensai ITO* and Yoshinosuke TERASHIMA**

Research Institute for Fundamental Physics, Kyoto University

*Faculty of Science, Kyoto University
**Yoshida College, Kyoto University

Abstract

An attempt is made to draw a systematic view on the origin of cosmic rays. On the basis of the composition of primary cosmic rays and the galactic radio emission, arguments are presented that the galactic cosmic rays are stored in the galactic halo of spherical shape for the mean lifetime of about $10^8$ years. The local sources of cosmic rays consist of the following two kinds; one is supernovae at which about $10^{-4}$ of ejected particles are accelerated to cosmic ray energy and the other is supergiant and red giant stars at which the above ratio seems to be $10^{-8} - 10^{-7}$. The chemical composition of cosmic rays from the latter is equal to that of the interstellar matter, while the former sources are responsible for the overabundance of heavy nuclei in cosmic rays. Our general view is summarized in §1 and detailed discussions on individual problems are made in §§2-5, leaving out more specific problems to Apps. I-VIII. Each section is prepared as independently as possible, and its contents are summarized at the beginning of the section.

§1. General view
§2. Cosmic rays in the Galaxy
  2.1. Classification of the components of primary cosmic rays
  2.2. Charge spectrum of primary nuclei
  2.3. Energy spectrum
  2.4. Contaminating components in the primary radiation
§3. Supernova origin
  3.1. Energetics in the Crab nebula
  3.2. Electromagnetic radiation from the Crab nebula
  3.3. Acceleration mechanism and nuclear particles in the Crab nebula
§4. Cosmic rays associated with the matter ejection
  4.1. Cosmic ray outbursts at solar flares
  4.2. Cosmic ray production at active stars
§5. Extragalactic origin
Appendices
  I. Yields of nuclides due to spallation processes
  II. Fragmentation probabilities
III. Radioactive energy source in Type I supernovae
IV. Dynamics of supernova explosion
V. Mechanism of acceleration
VI. Synchrotron radiation
VII. Diffusion problems
VIII. Intensity of solar cosmic rays

§1. General view

Since the discovery of the cosmic radiation various attempts have been made for explaining how and where it is generated (Ha 50). However, they had been more or less speculative until we knew some important features of primary cosmic rays, such as the presence of heavy nuclei (Fr 48), the practical absence of electrons (Hu 48) and the power energy spectrum extending to very high energies (Co 49). Increasing knowledge about primary cosmic rays has been supplemented by various astrophysical considerations, among which the galactic magnetic fields responsible for stirring and accelerating cosmic ray particles (Fe 49) and the synchrotron radiation of electrons in the magnetic fields (Gi 51) may be regarded as of primary importance. The latter has provided a basis for interpreting the strong radio emission from discrete sources, particularly the Crab nebula, and supernovae may be regarded as candidates for cosmic ray sources (Sh 53a). The supernova origin is supported also by the fact that heavy nuclei are overabundant in cosmic rays (Ha 56a). Throughout these studies*) effort is devoted mainly for constructing a reasonable model on the origin of cosmic rays, leaving out the mechanism of acceleration as a forthcoming problem (Mo 57). The most comprehensive work along this line seems to be due to Ginzburg (Gi 56, 57), with whom we share almost the same opinions, in spite of that his and our works have been carried out nearly independently.

In the present paper we attempt a more detailed analysis of the model concerning the galactic origin of cosmic rays, extending our previous work (Ha 56a). Although our model is similar to that adopted by Ginzburg (Gi 57), our analysis is made as quantitative as possible within the limited knowledge about cosmic rays as well as astronomy, and there exist some differences particularly in the following respects. The composition of primary cosmic rays is considered in relation to the evolution of stars and the formation of elements (Ta 56, Bu 57a, b) and the mean thickness of the interstellar matter traversed by cosmic rays is deduced as a few g cm\(^{-2}\) in §2. This leads us to assume that the mean lifetime of cosmic rays in the Galaxy is determined essentially by the escape out of the Galaxy, while

*) Not all of important works are cited in the present paper, and references are limited only to those directly related to our discussions.
Ginzburg assumed it is mainly due to nuclear collisions. An analogous difference is seen with relativistic particles in supernova remnants, such as the Crab nebula; the relativistic electrons leak out of the nebulae in our model, as seen in § 3. In addition to the supernova origin, which is the sole origin assumed by Ginzburg, other possible sources are considered in § 4. On the basis of the solar outburst of cosmic rays associated with violent flares, the efficiency for accelerating particles caused by the activity at stellar surfaces is estimated, and the contribution of the stellar activity, which is likely to be quite frequent at supergiants and red giants, to galactic cosmic rays is discussed. The responsibility of the extragalactic origin is considered in § 5, but nothing definite can be concluded at the present stage of our knowledge.

Summarizing the discussions in § 2.4, we shall describe the general view of our model in what follows. Our model is based on the following assumptions which are convincing in many respects; (i) the galactic magnetic fields stir and store cosmic rays inside the Galaxy, while they may or may not be responsible for acceleration, (ii) the galactic radio emission is mainly due to relativistic electrons in the magnetic fields, (iii) the charge spectrum of primary cosmic rays reflects the relative abundances of elements at their sources, and (iv) cosmic rays are stationary during the nearly entire history of the Galaxy, say, for last $10^{17}$ sec.

The assumption (i) allows us to introduce the total number of cosmic ray particles, $N$, and the mean lifetime for them to survive, $T$, in the Galaxy. These are related to each other through the overall production rate of cosmic rays, $Q$, by

$$N = QT, \quad (1.1)$$

provided that the production rate is constant in time, according to the assumption (iv). $N$ and $Q$ are expressed in terms of the density $n(r)$ and the production rate per unit volume $q(r)$, respectively, as

$$N = \int n(r) dr, \quad Q = \int q(r) dr. \quad (1.2)$$

The assumption (ii) facilitates to know the spatial distribution of cosmic rays from the intensity contour of general galactic radio waves. The latter indicates that the radio sources extend to the galactic halo and their spatial distribution is approximately expressed by $1/r$ for $r \leq R = 5 \times 10^{22}$ cm. Consequently, the spatial distribution of cosmic rays can be assumed as

$$n(r) = n_0 \left( \frac{a}{r} \right) \text{ for } r \leq R, \quad (1.3)$$

where $a = 3 \times 10^{22}$ cm is the distance of the earth from the galactic center
and \( n_0 = 1 \times 10^{-9} \text{ cm}^{-3} \) is the density of cosmic rays in the vicinity of the earth.

The assumption (iii) is important in determining the mean thickness of interstellar matter traversed by cosmic rays. An appreciable abundance of Li, Be and B in primary cosmic rays in comparison with their negligible relative abundance in cosmic elements suggests that these nuclei in cosmic rays are produced as a consequence of the fragmentation of heavier nuclei colliding with the interstellar gas. This interpretation allows us to deduce the mean thickness of interstellar matter traversed by cosmic rays as

\[ X \approx 3 \text{ g cm}^{-2} . \quad (1.4) \]

This is also consistent with other evidences on the charge spectrum. The overabundances of heavy nuclei in cosmic rays compared with cosmic abundances, particularly of iron, indicate that important part of cosmic rays are produced in the last stage of the stellar evolution, probably in supernovae; in their cores heavy elements such as iron can be synthesized under a very high temperature and then these elements can be scattered in the interstellar space due to violent explosions. Indeed, the Crab nebula, a supernova remnant, provides an evidence for the generation of cosmic rays at a rate of

\[ q_i \equiv 1 \times 10^{40} \text{ sec}^{-1} , \quad (1.5) \]

as will be shown in §3.

The assumption (iv) is based on such a view that the stellar evolution is essentially stationary for major part of the history of our Galaxy, while observations of natural radio activity can tell us the cosmic ray intensity to be substantially constant at least for last one million years (Pe 57). If the essentially stationary nature is taken for granted, we are able to define the overall production rate through (1.1) with the aid of the mean lifetime

\[ T \equiv 3 \times 10^{15} \text{ sec} \quad (1.6) \]

as

\[ Q = N / T \equiv 1 \times 10^{48} \text{ sec}^{-1} . \quad (1.7) \]

The mean lifetime \( T \) in (1.6) will be determined in §2.4 by taking into account the intensities of electrons and galactic radio waves as well as of the diffusion due to magnetic clouds, in which a magnetic field of strength about \( 3 \times 10^{-6} \) gauss extends over a dimension of \( 10^{20} \text{ cm} \). Such a model of the Galaxy is able to store cosmic rays of energies up to \( 10^{17} \text{ eV} \), so that we expect a smooth energy spectrum as well as the essential isotropy up to this energy. Beyond \( 10^{17} \text{ eV} \) we would look for an extragalactic origin of cosmic rays, if there were a considerable amount of such extremely high energy particles, although evidences for their existence seem to be an open ques-
Origin of Cosmic Rays

Comparing the overall production rate $Q$ given in (1.7) with the production rate at a supernova and its remnant, we are led to the following point of view. The rate of supernovae Type I, like the supernova having exploded in 1054, is believed to be one per three hundred years, while that of type II to be one per fifty years. If, therefore, we assume that each supernova produces cosmic rays at a rate $q_s$ given in (1.5) for $10^{11}$ sec, as will be seen in §3, and the rate of explosion is $10^{-9}$ sec$^{-1}$, the contribution of all supernovae to the production rate of cosmic rays turns out to be

$$Q_s \approx 1 \times 10^{48} \text{ sec}^{-1}. \quad (1.8)$$

If this is compared with $Q$ in (1.7), one could say that cosmic rays are produced almost exclusively by supernovae, on account of the uncertainty implied in the way of deriving these figures (Gi 57). However, we propose another possibility that about ten percent of cosmic rays could be attributed to supernovae.

Firstly, the ratio of He/H is normal in cosmic rays, whilst the overabundance of He is suspected from the viewpoint of the supernova origin, because the conversion of hydrogen to helium in the last phase of stars is supposed to be considerable and observations seem to favour this fact. Since the supply of H and He from the fragmentation due to collisions with interstellar matter is small, we have to look for those sources in which He/H is nearly normal.

Secondly, the relativistic electrons ejected directly from sources are probably fewer than nuclear particles, as will be argued in §2.4, whilst those electrons are ejected from the Crab nebula as many as nuclear particles, as will be seen in §3. Hence there may be such sources other than supernovae that eject a negligible amount of electrons.

As a candidate for such sources we assume supergiant and red giant stars which are so unstable as to be mainly responsible for matter ejection (Bu 57a). If one part of $10^7 - 10^8$ particles can be accelerated to cosmic ray energy, as will be argued in §4 referring to the solar outburst of cosmic rays, the matter ejection is associated with the cosmic ray production at a rate of

$$Q_m = 10^{42} - 10^{43} \text{ sec}^{-1}, \quad (1.9)$$

thus being able to account for the overall production rate.

Now we show that these two kinds of sources can explain the overabundance of heavy nuclei in cosmic rays. For the sake of illustration, we assume the relative contributions of the giant stars and supernovae to cosmic rays as 10:1, while those to interstellar elements as 100:1, following Burbidge et al. The relative abundances of ejected elements by mass
from these sources are estimated by Burbidge et al., as was shown in Table XII, 1 in their article (Bu 57a). We simply multiply by a factor 10 the matter abundances due to supernovae and obtain the figures in the third column of Table 1-1, in which the relative abundances by number are shown. In the table the relative abundances due to the giant stars are taken as equal to cosmic abundances, but some of those due to supernovae are unknown, so that these figures in parentheses are based on a guess which seems to be plausible. The resultant abundances are compared to the composition of cosmic rays.

Table 1-1. Composition of cosmic ray nuclei

<table>
<thead>
<tr>
<th>Element</th>
<th>Sources</th>
<th>Cosmic rays</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>giant stars</td>
<td>supernovae</td>
</tr>
<tr>
<td>H</td>
<td>100</td>
<td>(10)</td>
</tr>
<tr>
<td>He</td>
<td>8</td>
<td>(4)</td>
</tr>
<tr>
<td>Li, Be, B</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C, N, O, Ne</td>
<td>0.2</td>
<td>(0.3)</td>
</tr>
<tr>
<td>Mg, Si, S, A</td>
<td>0.01</td>
<td>0.1</td>
</tr>
<tr>
<td>Fe</td>
<td>0</td>
<td>0.03</td>
</tr>
</tbody>
</table>

§2. Cosmic rays in the Galaxy

As discussed in the preceding section, there are strong evidences for the existence of local cosmic ray sources. In order to see the characteristic features of the local sources, we have first to consider the propagation of cosmic rays in our Galaxy. Then various pieces of information on primary cosmic rays, such as their constitution, charge spectrum and energy spectrum, will facilitate to solve the problem on the origin of cosmic rays. In §2.2 we discuss what is implied in the charge spectrum, to see how thick the interstellar matter traversed by cosmic ray nuclei is, by taking account of the nuclear transformations arising from the collisions with interstellar materials. Special attention is paid to the amount of some particular isotopes, because they will be able to give us a rather definite conclusion on our problem.

The energy dependence of the charge spectrum is very important to see whether the Fermi mechanism of acceleration is operative or not in the interstellar space, because it would predict a steeper decline of the energy spectrum, as the nucleus becomes heavier. This problem will be discussed in §2.3.

In §2.4 we consider contaminating components, such as electrons, photons and antiprotons. These components are not necessarily unobserv-
able. We call attention of experimenters to the implication of such observations.

2.1. Classification of the components of primary cosmic rays

It has been established that primary cosmic rays consist almost exclusively of stripped nuclei. Most of them are hydrogen and about 10% of them are helium, if they are compared at the same energy per nucleon in the energy range around several GeV per nucleon. The abundance of nuclei heavier than He constitutes only about 1% of the total abundance. But the precise values of the relative abundances of these nuclei are not conclusive yet, because good statistics can hardly be obtained with present experimental techniques on one hand, and because the reduction from the abundances observed at finite atmospheric depths to the true primary abundances is subject to the ambiguity in the fragmentation of primary nuclei colliding with air nuclei on the other hand. The latter defect can be amended partly by reference to laboratory experiments on high energy nuclear reactions, as shown in App. I. The poorness of statistics is usually overcome by grouping a number of nuclear species. In the present paper we shall also make such grouping with slight modification of the conventional way of grouping.

*Hydrogen*, designated by suffix $p$, may contain not only protons but also deuterons. The abundance of deuterons will be predicted in §2.2. The abundance of tritons is expected as unappreciable, but it would be a very important quantity in interpreting the origin of cosmic rays, if it were measured.

*Helium*, designated by suffix $\alpha$, may contain $\alpha$-particles and $^6$He, the latter playing a similar role to deuterons and tritons.

*Light* nuclei, designated by suffix $L$, consist of Li, Be and B. The cosmic abundances of these elements are so small that most of them are produced from heavier nuclei by collisions with the interstellar matter.

*Medium* nuclei, designated by suffix $M$, contain C, N, O, F and Ne. In contrast to the conventional grouping, we classify Ne into this group, because C, O and Ne are presumed to be built up by a sequence of helium burning processes in giant stars. It is our regret that Ne has been excluded from this group in most of experiments.

*Heavy* nuclei, designated by suffix $H$, contain nuclei of $11 \leq Z \leq 22$. Among them the 4-$\alpha$ nuclei, such as $^{24}$Mg and $^{28}$Si, are more abundant than others, and they are considered to be produced chiefly by $\alpha$-capturing processes starting from Ne. Other nuclei have small cosmic abundances, so

*) In the present paper the abundances are compared at the same energy per nucleon, if not specially mentioned.
that they appear in cosmic rays mainly as fragmentation remnants of heavier nuclei.

Iron group, designated by suffix $Fe$, is particularly regarded as an independent group, because it has an extraordinarily high abundance in cosmic rays on one hand, and because the nuclei belonging to this group are supposed to be produced substantially under the thermal equilibrium in the hottest cores of stars on the other hand. In this group $^{56}Fe$ is most abundant and the nuclei of $23 \leq Z \leq 30$ should be included.

It is almost needless to say that the classification is more or less conventional and the boundary between two groups is not always clear. For example, Ti ($Z=22$) may be included in the iron group, because part of it should be built up under the thermal equilibrium.

The relative abundances of primary nuclei thus grouped, at energies below 10 GeV per nucleon, are listed in Table 2-1. Although there was the well-known disagreement among a number of authors, as to the abundance of the light nuclei, it seems to have converged to the figure in the table within a factor of two. Its precise value is of secondary importance.

<table>
<thead>
<tr>
<th>group</th>
<th>notation</th>
<th>hydrogen</th>
<th>helium</th>
<th>light ($3 \leq Z \leq 5$)</th>
<th>medium ($6 \leq Z \leq 10$)</th>
<th>heavy ($11 \leq Z \leq 22$)</th>
<th>iron ($23 \leq Z \leq 30$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>range of atomic numbers</td>
<td>$Z=1$</td>
<td>$Z=2$</td>
<td>$3 \leq Z \leq 5$</td>
<td>$6 \leq Z \leq 10$</td>
<td>$11 \leq Z \leq 22$</td>
<td>$23 \leq Z \leq 30$</td>
<td></td>
</tr>
<tr>
<td>relative abundance at the top of atmosphere</td>
<td>100</td>
<td>10</td>
<td>0.18</td>
<td>0.64</td>
<td>0.10</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>at local sources</td>
<td>100</td>
<td>12</td>
<td>0</td>
<td>0.83</td>
<td>0.15</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>cosmic ($Su56$)</td>
<td>100</td>
<td>7.7</td>
<td>$\sim 10^{-6}$</td>
<td>0.20</td>
<td>0.03</td>
<td>0.003</td>
<td></td>
</tr>
</tbody>
</table>

The relative abundances of nuclei within a group are much more obscure than those among the groups. For the former we shall mainly refer to a recent observation made by Koshiba et al. (Ko 57) (see Table 2-1), simply because this is the latest and most comprehensive information available to us.$^{*}$.$^{**}$ Their result is plotted as curve A in Fig. 2-1. Other information in disagreement with this will be mentioned in the course of discussions.

$^{*}$ Our thanks are due to these authors for sending us a preprint of this paper promptly. Conversation with Dr. Koshiba was very helpful.

$^{**}$ After we completed this manuscript, a preprint by M. V. K. Apparao, S. Biswas, R. R. Daniel, K. A. Neelakantan and B. Peters has come to our notice. The relative abundances obtained by the latter authors are essentially the same as those in (Ko 57), except for the insignificant abundance of the light nuclei.
It must be mentioned that the abundances referred above are based on the primary cosmic rays in an energy range around several GeV per nucleon. A number of authors have reported a rather strong energy dependence of the relative abundances, while some others have argued an energy independence in a wide range of energy. For the time being we stand on the latter point of view and look for the energy dependence expected from the ionization loss at low energies and from the Fermi mechanism of acceleration at high energies in §2.3.

Components other than stripped nuclei should be expected as arising from interactions with interstellar matter, even if stripped nuclei are only components at local sources. The loss and gain of orbital electrons will be considered in §2.2, since this problem will also give us a piece of information on the propagation in the Galaxy. More important is the secondary products produced by collisions of nuclear particles with interstellar matter.

*Electrons*, designated by suffix \(e\), are not negligible. Their near absence has important bearing on the propagation problem (Ha 53). We shall make a revised estimate on their abundance.

*Photons*, designated by suffix \(\gamma\), are the most important, since they form the background against the intensive photons expected from local sources. We are particularly interested in line \(\gamma\)-rays, such as those from the positron-negaton annihilation and the neutron capture by the proton, as well as in high energy photons due to the decays of secondary neutral pions.
Antiprotons, designated by suffix \( \vec{p} \), are supposed to be very few. We shall give their expected amount, revising the previous estimate (Ha 53).

2.2. Charge spectrum of primary nuclei

In this subsection we attempt to correlate the primary charge spectrum of stripped nuclei of several GeV with that at local sources. In order to determine the average thickness of the interstellar gas (collisions with dust and stars are negligible) traversed by these nuclei, we solve a set of diffusion equations for the groups of nuclei. From the solution we obtain the average thickness as well as the abundances of the groups at the sources. On the basis of the average thickness thus fixed, we discuss how many such nuclei are expected that are very few at the sources but can be produced as a result of the fragmentation of heavier nuclei. From a comparison of the expected abundances with the observed ones, we can reexamine the value of the average thickness as well as the abundance of elements at the sources.

Let the number of nuclei of group \( i \) at thickness \( x \) from a local source to be \( N_i(x) \). As a result of collisions, these nuclei undergo nuclear transformations that produce lighter nuclei. The diffusion equations for such processes are as follows;

\[
\frac{dN_i(x)}{dx} = -\frac{1}{\lambda_i} N_i(x) + \sum_{j>i} \frac{1}{\lambda_j} P_{ji} N_j(x). \tag{2.1}
\]

Here \( P_{ji} \) is the probability for yielding secondary nuclei of group \( i \) from a primary nuclei of group \( j \), and \( \lambda_j \) is the mean free paths of group \( j \) against collisions with hydrogen. \( \lambda_j \) are given in Table 2.2 where we take as representatives of respective groups those nuclei that have charges 1, 2, 4, 8, 14 and 26 and the mass numbers corresponding to these charges. The collision cross section is assumed to be \( \pi r_0^2 A^{2/3} \) with \( r_0 = 1.2 \times 10^{-13} \text{ cm} \) on account of cosmic rays as well as Cosmotron experiments (Wi 55). The summation is carried out over group \( i \) and heavier ones.

<table>
<thead>
<tr>
<th>group</th>
<th>hydrogen</th>
<th>helium</th>
<th>light</th>
<th>medium</th>
<th>heavy</th>
<th>iron</th>
</tr>
</thead>
<tbody>
<tr>
<td>notation</td>
<td>( \lambda_p )</td>
<td>( \lambda_{\alpha} )</td>
<td>( \lambda_L )</td>
<td>( \lambda_M )</td>
<td>( \lambda_H )</td>
<td>( \lambda_{Fe} )</td>
</tr>
<tr>
<td>collision mean free path (g cm(^{-2}))</td>
<td>37</td>
<td>14.6</td>
<td>8.7</td>
<td>6.0</td>
<td>4.0</td>
<td>2.5</td>
</tr>
</tbody>
</table>

These equations are solved for four groups, assuming that the abundance of the light nuclei is negligible at the source.
Origin of Cosmic Rays

The solutions are:

\[ N_{Fe}(x) = N_{Fe}(0) \exp(-x/\lambda_{Fe}). \]  \hspace{1cm} (2.2a)

\[ N_H(x) = N_H(0) \exp(-x/\lambda_H) + \left( P_{H \rightarrow H} \lambda_{H} / \lambda_{Fe} \right) \left( N_{Fe}(0) \exp(-x/\lambda_{Fe}) - N_{Fe}(x) \right), \]  \hspace{1cm} (2.2b)

\[ N_M(x) = N_M(0) \exp(-x/\lambda_M) + \left( P_{H \rightarrow M} \lambda_{H} / \lambda_{M} \right) \left( N_{M}(0) \exp(-x/\lambda_{M}) - N_{M}(x) \right) \]  
\[ + \left( \lambda_{ML} N_{L}(0) \right) \left( P_{M \rightarrow L} \lambda_{M} / \lambda_{L} \right) \left( N_{L}(0) \exp(-x/\lambda_{L}) - N_{L}(x) \right), \]  \hspace{1cm} (2.2c)

\[ N_L(x) = N_L(0) \exp(-x/\lambda_L) + \left( \lambda_{LM} N_{M}(0) \right) \left( P_{L \rightarrow M} \lambda_{L} / \lambda_{M} \right) \left( N_{M}(0) \exp(-x/\lambda_{M}) - N_{M}(x) \right) \]  
\[ + \left( \lambda_{LL} N_{L}(0) \right) \left( P_{L \rightarrow L} \lambda_{L} / \lambda_{L} \right) \left( N_{L}(0) \exp(-x/\lambda_{L}) - N_{L}(x) \right), \]  \hspace{1cm} (2.2d)

where

\[ P_{\alpha} = P_{\alpha}, \]

\[ \lambda_i^{-1} = (1 - P_i) \lambda_i^{-1}, \quad \lambda_{ij}^{-1} = \lambda_i^{-1} - \lambda_{ij}^{-1} > 0 \quad \text{for} \quad \lambda_i > \lambda_j, \]

\[ N_1 = N_H(0) + \left( P_{Fe \rightarrow H} \lambda_{Fe} / \lambda_{Fe} \right) N_{Fe}(0), \]

and

\[ N_2 = - \left( P_{Fe \rightarrow H} \lambda_{Fe} / \lambda_{Fe} \right) N_{Fe}(0). \]

For fragmentation probabilities \( P_{ij} \), though their accurate values are difficult to deduce, we may be allowed to set their lower and upper limits from experimental and theoretical information, as can be seen in Apps. I and II. The results obtained are given in Table II-1. It is hoped that these values in Table II-1 will become more accurate in near future by means of high energy accelerator experiments as well as of cosmic ray experiments.

First of all, we shall assume that the number of the light nuclei are zero at the time of ejection, since this seems not un plausible on the astrophysical ground; \( N_L(0) \approx 0 \). On account of that \( \lambda_{ij} \) is probably much larger than \( X \), the averages thickness of traversed matter, approximate relations among \( N_H, N_M, N_F \) are obtained from (2.2d):

\[ N_L(X) = \left( \left( P_{Fe \rightarrow H} \lambda_{Fe} \right) N_{Fe}(X) + \left( P_{H \rightarrow H} \lambda_H \right) N_H(X) + P_{ML} \lambda_M N_M(X) \right) X. \]  \hspace{1cm} (2.3)

It must be noticed that other uncertain parameters vanish in this approximation. If we substitute the observed values \( N_L(X) / \left( N_H(X) + N_{Fe}(X) \right) = 1 \) and \( N_L(X) / N_M(X) = 0.3 \) in (2.3), the lower and the upper limits of \( X \) are obtained corresponding to the upper and the lower limits of \( P_{ij} \), res-
pectively. Thus the thickness traversed by cosmic rays is estimated to be
\[2.5 \text{ g cm}^{-2} \leq X \leq 3.5 \text{ g cm}^{-2} .\]  
(2.4)

For later use we fix
\[X = 3 \text{ g cm}^{-2} .\]  
(2.4')

This value is three times as large as the previous value adopted by one
of the authors (Ha 56a). The magnitude of \( X \) is so small that the ap­
proximation leading to (2.3) may be permitted.

Since the value of \( X \) is fixed, we can deduce the element abundances
at the source from the charge spectrum at the top of the atmosphere by
using the quantities given in Table 2.2 and Table II-1.

The relations between \( N_i(X) \) and \( N_i(0) \) for respective groups are
roughly given for \( X = 3 \text{ g cm}^{-2} \) as
\[
N_{Fe}(0) \approx 2.8 N_{Fe}(X) , \quad (2.5a) \\
N_{H}(0) \approx 1.5 N_{H}(X) , \quad (2.5b) \\
N_{M}(0) \approx 1.3 N_{M}(X) , \quad (2.5c) \\
and \quad N_{r}(0) = 0 . \quad (2.5d)
\]

The values of \( N_i(0) \), that is, the abundances at sources, thus obtained
are shown in Table 2-1. By comparing these figures with cosmic abun­
dances, we notice that the relative abundances of the heavy nuclei and
the iron group are significantly larger than those of cosmic abundances.
To make this comparison in more detail, we evaluate the numbers of the
hydrogen and helium nuclei produced from heavier nuclei through their
fragmentation and lost by collisions with interstellar matter.

In order to simplify the calculation, the medium nuclei are taken as
representatives of nuclei heavier than helium, and they are designated by
suffix \( m \). On account of \( |\lambda_{Na}|, |\lambda_{M} | \gg X \), we obtain
\[
N_{a}(0) \simeq \exp(X/\lambda_{a}) \left[ N_{a}(X) - \sum_{m} P_{mm} N_{m}(X) (X/\lambda_{a}) \right] , \quad (2.6a) \\
N_{p}(0) \simeq \exp(X/\lambda_{p}) \left[ N_{p}(X) - \sum_{m} P_{mm} N_{m}(X) (X/\lambda_{p}) - \sum_{m} P_{mm} N_{m}(X) (X/\lambda_{p}) \right] , \quad (2.6b)
\]

where \( N_{m} \) contains all nuclei heavier than He. Since the first terms in
the curly brackets in (2.6a, b) are larger than the other terms respective­
ly, the ambiguity in the fragmentation probabilities hardly affect the values
of \( N_{a}(0) \) and \( N_{p}(0) \). This means that a major part of primary hydrogen
and helium nuclei comes directly from the sources. By employing the
observed abundances.

\[ N_p(X) : N_\alpha(X) : N_m(X) = 1 : 0.1 : 0.01, \quad (2.7) \]

as well as the fragmentation probabilities and the mean free paths given in Table 2-2 and II-1, we obtain

\[ N_p(0) : N_\alpha(0) : N_m(0) = 1 : 0.12 : 0.01. \quad (2.8a) \]

This is compared with the cosmic abundances

\[ N_p : N_\alpha : N_m = 1 : 0.077 : 0.0020. \quad (2.8b) \]

The results obtained through the above analysis are summarized in Table 2-1 and Fig. 2-1. A comparison of these relative abundances at the cosmic ray sources with the cosmic abundances (Su 56) is also made in Table 2-1. This clearly shows that the cosmic ray sources are rather peculiar than general celestial objects; namely, the formers are richer in heavy elements than the latters.

Now we proceed to the discussions on individual nucleides, which are supposed to be rare at the sources, with use of the available theoretical and experimental information (Ki 57, Ru 54, Fr 55, Fi 55a, b, He 56, Ros 56, Na 58).

**Fluorine.** The cosmic abundance of fluorine is much smaller than the abundances of neighboring elements, and there are many reasons for this small abundance. It is well known from laboratory experiments that \(^{19}\text{F}\) is easily destroyed by nuclear bombardments. Hence one would expect a negligible amount of fluorine among primary cosmic rays. However, Koshiba et al. (Ko 57) observed fluorine only slightly fewer than oxygen and neon. This might look surprising at first sight, but such a large amount may be explained by the fragmentation of neon in the upper atmosphere in addition to that in the interstellar space.

Koshiba et al. observed seven F nuclei at an effective depth of 10 gcm\(^{-2}\) under the top of the atmosphere. About a half of them can be attributed to the reaction products of impinging neon nuclei against air nuclei, as will be shown in what follows. If a \(^{20}\text{Ne}\) nucleus is bombarded by a proton, \((p, 2p)\) reaction leads to the production of \(^{19}\text{F}\). This is analogous to the reaction \(^{12}\text{C}(p, pn)^{11}\text{C}\), to which we shall refer below. Since the latter cross section is known to be nearly energy independent and the proton-\(^{20}\text{Ne}\) collision easily leads to the formation of \(^{19}\text{O}\), we estimate the cross section for \(^{20}\text{Ne}(p, 2p)^{19}\text{F}\) or \(^{20}\text{Ne}(n, np)^{19}\text{F}\) as about 30 mb. On account of that approximately six surface nucleons of an N or O nucleus may take this part in the collision under consideration, the cross section of \(^{20}\text{Ne}\) turning into \(^{19}\text{F}\) may be as large as 180 mb. In penetrating through a layer of air of 10 gcm\(^{-2}\), \(^{20}\text{Ne}\) nuclei transform into \(^{19}\text{F}\) with probability of 1/5. This
predicts roughly three $^{19}\text{F}$ nuclei among those observed by Koshiba et al., since the number of neon nuclei they would observe at the top of the atmosphere is about thirty five.

There still remains a finite flux of fluorine in primary cosmic rays. This can be expected again from the collision of neon against interstellar hydrogen. Now both $(p, 2p)$ and $(p, pn)$ reactions take part in the formation of fluorine, and the cross section for them may be as large as 50 mb. Hence the thickness of $3 \text{gcm}^{-2}$ is just as large as to produce three $^{19}\text{F}$ nuclei compared with thirty five $^{20}\text{Ne}$ nuclei.

**Oxygen, nitrogen and carbon.** Since the relative abundances of these elements and the ratio of $^{12}\text{C}$ and $^{13}\text{C}$ change according to the evolutionary stage of stars, detailed investigation of their abundances is very important. Indeed, Koshiba et al. (Ko 57) concluded that hot, young stars could be main cosmic ray sources on the basis of the overabundance of carbon observed by them. For this purpose, a more detailed study of the fragmentation probabilities is needed. Since little information on the fragmentation is available at high energies, we are obliged to use experimental data at low energies, supposing that the relative values of reaction widths obtained by low energy nuclear reactions will govern the main features of the transformations to nearby nuclei even at high energies.

A supply of $^{16}\text{O}$ from heavier nuclei comes mainly from $^{20}\text{Ne}(p, p\alpha)$ reaction, for which the width is about $2/3$ of the total width. $^{16}\text{O}$ transforms into $^{15}\text{N}$ through $^{16}\text{O}(p, 2p)^{15}\text{N}$ and $^{16}\text{O}(p, pn)^{15}\text{O}(\beta^+\nu)^{15}\text{N}$, whose resultant width is estimated as about $1/15$ of the total one. The largest width for proton–$^{16}\text{O}$ collisions is given to $^{16}\text{O}(p, p\alpha)^{12}\text{C}$ and the next one to $^{16}\text{O}(p, \alpha)^{12}\text{C}$. Carbon is further supplied by $^{14}\text{N}(p, 2p)^{13}\text{C}$ and $^{14}\text{N}(p, t)^{13}\text{C}$ with partial widths of $1/5$ and $\sim 1/10$ respectively, in which are included the widths to produce $\beta$ unstable nuclei eventually giving carbon. Thus we assign the fragmentation probabilities as $P_{\text{NeO}} = 0.8$, $P_{\text{O}0} = 0.3$, $P_{\text{ON}} = 0.06$ and $P_{\text{NO}} = 0.2$.

As can be seen from the above discussion, a considerable amount of carbon are supplied by oxygen through the fragmentation. Even so the qualitative result on the overabundance of carbon is not altered. The experimental results obtained by Koshiba et al. together with our considerations on the fragmentation give us the relative abundances at the sources as

$$C : N : O : \text{Ne} = 1.2 : 1 : 1 : 0.5. \quad (2.9a)$$

In comparison with this the cosmic abundances are given by Suess and

---

* See, however, the footnote in the next page.
Origin of Cosmic Rays

Urey (Su 56) as*:

\[ C : N : O : Ne = 0.16 : 0.31 : 1 : 0.40. \]  
\[(2\cdot9b)\]

If the mean traversed thickness were chosen larger than 3 gcm\(^{-2}\), the carbon abundance at the cosmic ray sources could be made smaller. But the neon abundance would turn out to be too large. Thus the difference between (2\cdot9a) and (2\cdot9b) indicates the peculiarity of the cosmic ray sources, in so far as we refer to the experimental result obtained by Koshiba et al.

*Boron, Beryllium and Lithium.* The abundances of these nuclei in primary cosmic rays played an essential role in determining the mean traversed thickness. In order to examine whether the present method is really legitimate or not, we must see if the relative abundances among them can be interpreted in the same term. However, there are difficulties in such a study; the observed relative abundances are not yet agreed among various authors on one hand and the fragmentation probabilities are not well known on the other hand. Keeping these facts in mind, we shall make use of qualitative features of nuclear reactions and refer to the experimental results by Koshiba et al. (Ko 57).

As heavy splinters of rather high energies lighter nuclei may have larger yields, while the fragmentation remnants of the medium nuclei are apt to give larger yields to the nuclei nearer to their parents. These two will make the minimum yield of Be. This tendency is amplified by the fact that beryllium has only one stable isotope. However, one must be cautious that an unstable isotope \(^{10}\text{Be}\) could be regarded as stable, if the mean lifetime of cosmic rays in the Galaxy were not longer than millions of years, since the mean lifetime of \(^{10}\text{Be}\) is about \(4 \times 10^6\) years. It is very important to observe whether \(^{10}\text{Be}\) exists or not in primary cosmic rays, because this observation will determine the relation between the thickness of traversed matter and the actual length passed in the interstellar space and, consequently, the density of matter in which cosmic rays spend most of time.

After this remark we shall discuss individual processes. The cross sections for producing Li and Be as heavy splinters of heavier nuclei are found to be about 10 mb respectively and their difference has been hardly observed. The cross section for producing boron splinters may be low and can be neglected compared with that of other important processes. The dependences of the cross sections upon target nucleus and energy seem to be weak, if the energy of bombarding nucleons is relativistic.

*\(^{*}\) A recent analysis on the relative abundances of these elements of early type stars has shown C:N:O = 0.85:0.46:1.00, in essential agreement with those of cosmic rays, (2\cdot9a). Therefore, there would be no reason to emphasize the "overabundance" of carbon in cosmic rays. (H. C. Aller and J. Jugaku, private communication)
Boron can be produced mainly by the reaction $^{12}\text{C}(p, pp$ and $pn)^{11}\text{B}$ and $^{11}\text{C}$. The formation of $^{11}\text{C}$ is accurately measured in a wide range of energy. From this we estimate the partial cross section of producing $^{11}\text{B}$ and $^{11}\text{C}$, the latter decaying into $^{11}\text{B}$, as about 30% of the total one. An additional supply comes from $^{14}\text{N}(p, p\alpha)^{11}\text{B}$ reactions whose cross section is about 10% of the total one. Contributions from other processes are small and may be evaluated collectively by means of the fragmentation probabilities given in Table II-1. Thus we can give the expected number of $\text{B}$, $N_b$, in relation to the observed numbers of $\text{C}$, $N$, the rest of the medium nuclei and the heavy nuclei, designated respectively as $N_c$, $N_n$, $N_m$, and $N_H$ as

$$N_b(X) = \left( \frac{P_{GB}}{\lambda_c} N_c(X) + \frac{P_{NH}}{\lambda_n} N_n(X) + \frac{P_{MB}}{\lambda_m} N_m(X) \right) \cdot X.$$  \hfill (2.10)

The fragmentation probabilities are taken as $P_{GB}=0.3$, $P_{NH}=0.1$, $P_{MB}=0.1$, $P_{HB}=0.2$, $P_{FeB}=0.0$, and the interaction mean free paths as $\lambda_c = 7.0 \text{ g cm}^{-2}$, $\lambda_n = 6.3 \text{ g cm}^{-2}$, $\lambda_m = 5.5 \text{ g cm}^{-2}$.

For $X=3 \text{ g cm}^{-2}$ we have

$$N_b(X) : N_c(X) : N_n(X) : N_m(X) = 29 : 100 : 65 : 100,$$  \hfill (2.11)

in good agreement with the ratio observed by Koshiba et al.

For the production of Be and Li, no particular reactions seem to exist as in the case of B, because large reaction widths are attributed to the break up into He and lighter nuclei. Hence the expected number of Li is given by

$$N_L(X) = \left( \frac{P_{BL}}{\lambda_B} N_B(X) + \frac{P_{ML}}{\lambda_M} N_M(X) \right) \cdot X.$$  \hfill (2.12)

Thus we expect the following relative abundances

$$N_L(X) : N_B(X) : N_M(X) = 19 : 29 : 265,$$  \hfill (2.13)

also in qualitative agreement with the referred observation. The number of Be would be supposed to be approximately the same as $N_L$, if a substantial part of $^{10}\text{Be}$ could survive.
The expected ratio of $^9$Be and $^{10}$Be may be evaluated by assuming that both the fragmentation probabilities and the interaction mean free paths are the same for them. This is expressed as a function of the average density of matter $\rho$ traversed by cosmic rays as

$$\frac{N_{^9Be}}{N_{^{10}Be}} = \frac{4 \times 10^{24}\rho}{X} \left\{1 - \exp\left(-\frac{X}{4 \times 10^{24}\rho}\right)\right\}. \quad (2.14)$$

The modification of the abundance of $^{10}$B due to this cause is unappreciable, but this ratio itself could be measured by improving experimental techniques.

*Isotopes of helium and hydrogen.* Measurement of $^3$H and $^3$He is more feasible than that of $^9$Be and $^{10}$Be. Tritons can scarcely exist, because of their short lifetime, but they turn into $^3$He. If tritons were observed, cosmic rays would have travelled through a particular region of very high material density near the earth.

The cross sections for producing these isotopes are rather well investigated. The fragmentation probabilities are given as

$$P_{^3He} = 0.9, \quad P_{^2H} = 1.0, \quad P_{^3He} = 1.0, \quad P_{^3He} = 0.3 \quad \text{and} \quad P_{^3He} = 0.3.$$  

Thus we expect the ratios of $^3$He and $^2$H to $^3$He and $^4$He respectively as

$$\frac{N_{^3He}}{N_{^3He}} \lesssim 5 \times 10^{-2}X \quad \text{and} \quad \frac{N_{^2H}}{N_{^3He}} \lesssim 3 \times 10^{-3}X. \quad (2.15)$$

Now we investigate how cosmic ray particles lose their orbital electrons, if they are not completely ionized at the time of injection. Even if a nucleus retains one or two orbital electrons (the K-shell electrons), these electrons will be easily lost in the course of its propagation. Main processes of the loss of orbital electrons are the collisions with interstellar matter and the photo-ionization. Inverse processes are the rearrangement collision with interstellar neutral particles and the free electron capture. We present the mean free paths for these processes in the following.

Let us consider a nucleus with atomic number $Z$, which has only one K-shell electron, and assume the interstellar matter to be mainly composed of hydrogen. The mean free path for the electron loss due to collisions with interstellar hydrogen is defined by

$$\lambda_e(Z) = \frac{M}{\sigma_e(\alpha + 2(1 - \alpha))} \text{gcm}^{-2}, \quad (2.16)$$

where $M$ is the nucleon mass and $\alpha$ the degree of ionization of hydrogen gas. The cross section is given for the non-relativistic case by (Mo 49)

$$\sigma_e(Z) = 1.12 \pi (h/mc)^2 (c/v)^2 Z^{-2} \ln (20m_e v^2 / I_0 Z^2)$$

*) The formation of deuterons through the reaction $p + p \rightarrow d + \pi^+$ is expected as unappreciable, because the cross section is small and has a peak only near the threshold.
where \( v \) is the velocity of the incident nucleus, \( E_k \) its kinetic energy per nucleon in eV and \( I_0 \) the ionization potential for hydrogen. For the relativistic case we calculate \( \sigma_s \) with the Williams-Weizsäcker method (He 54) as

\[
\sigma_s(Z) = 5.86 \left( \frac{\hbar}{m_c} \right)^2 Z^{-2} \left\{ \ln \left( \frac{E}{I_0 Z^2} \right) - 0.38 \right\} \text{cm}^2, \tag{2·17b}
\]

where \( E \) is the total energy per nucleon of the incident nucleus in eV.

Taking \( \alpha = 50\% \), we give the values of \( \lambda_s \), for example, at energies of \( E_k = 10^8 \text{ eV/nucleon} \) and \( E = 10^{10} \text{ eV/nucleon} \) as shown in Table 2·3.

### Table 2-3.
Mean free paths for the K-shell electron loss of nuclei by collisions with interstellar hydrogens

<table>
<thead>
<tr>
<th>( Z )</th>
<th>At the energy of ( 10^8 \text{eV/nucleon} )</th>
<th>At the energy of ( 10^{10} \text{eV/nucleon} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 3.7 \times 10^{-4} )</td>
<td>( 5.6 \times 10^{-6} )</td>
</tr>
<tr>
<td>2</td>
<td>( 1.7 \times 10^{-4} )</td>
<td>( 2.4 \times 10^{-4} )</td>
</tr>
<tr>
<td>4</td>
<td>( 7.8 \times 10^{-5} )</td>
<td>( 1.0 \times 10^{-4} )</td>
</tr>
<tr>
<td>8</td>
<td>( 3.7 \times 10^{-4} )</td>
<td>( 4.5 \times 10^{-4} )</td>
</tr>
<tr>
<td>14</td>
<td>( 1.3 \times 10^{-3} )</td>
<td>( 1.4 \times 10^{-3} )</td>
</tr>
<tr>
<td>26</td>
<td>( 5.6 \times 10^{-3} )</td>
<td>( 6.5 \times 10^{-3} )</td>
</tr>
</tbody>
</table>

For the collision loss of one electron in a higher state with principal quantum number \( n \), in which \( Z_n \) electrons are present, the mean free path is equal approximately, at low energies, to \( \lambda / n^2 Z_n \). When a nucleus has two K-electrons, the mean free path for the complete ionization is 1.5 times of that given above. (This remark will be omitted in the following.)

For the photo-ionization of a K-shell electron, the mean free path is found to be (He 54)

for N.R.,

\[
\lambda_\gamma = \left( 1.23 \pi^3 / 2^9 \right) \left( \hbar / m_c c \right)^{-2} \left( e^2 / \hbar c \right) \left( v / c \right) \left( kT / I_0 \right)^2 \exp \left( I_0 Z^2 / kT \right) \cdot Z^{-2} n_\gamma^{-1} \\
= 3.66 \times 10^{17} \left( v / c \right) \left( kT / I_0 \right)^2 \exp \left( I_0 Z^2 / kT \right) \cdot Z^{-2} n_\gamma^{-1} \text{ cm}, \tag{2·18a}
\]

for \( I_0 Z^2 / kT \gg \tau = E / Me^2 \gg 1 \),

\[
\lambda_\gamma = \left( 3.69 \pi^3 / 2^9 \right) \left( \hbar / m_c c \right)^{-2} \left( e^2 / \hbar c \right) \left( kT / I_0 \right)^2 \exp \left( I_0 Z^2 / 2 \tau kT \right) \cdot \tau^{-2} Z^{-2} n_\gamma^{-1} \\
= 1.10 \times 10^{18} \left( kT / I_0 \right)^2 \exp \left( I_0 Z^2 / 2 \tau kT \right) \cdot \tau^{-2} Z^{-2} n_\gamma^{-1} \text{ cm}, \tag{2·18b}
\]

and
for $I_0 Z^2 / kT \ll \gamma = E / M c^2$,

$$\lambda_\gamma = (3.69 n^3 / 2^9) (\hbar / m c)^{-3} (e^2 / \hbar c) (kT / I_0)^2 \gamma^2 Z^{-2} n^{-1}_r$$

$$= 2.20 \times 10^{18} (kT / I_0)^2 \gamma^2 Z^{-2} n^{-1}_r \text{cm}, \quad (2.18c)$$

where $n_r$ is the photon number density and $T$ the radiation temperature.

For the interstellar space,

$$n_r = 0.5 \text{ cm}^{-3} \quad \text{and} \quad I_0 / kT = 30;$$

we estimate $\lambda_\gamma$ at an energy of $10^8 \text{eV/nucleon}$ as

$$\lambda_\gamma (F_e) \gg \lambda_\gamma (Z = 1) = 2 \times 10^8 \text{cm}. \quad (2.19)$$

Taking into account that the average density of interstellar matter is of the order of $10^{-26} \text{gcm}^{-3}$, this is much larger than $\lambda_\gamma$ given in $(2.16)$ with $(2.17a)$. Also we find $\lambda_\gamma$ greatly exceeds $\lambda_s$ at relativistic energies.

On the other hand the electron capture due to the rearrangement collision of a naked nucleus with neutral hydrogen has a small probability at high energies. The mean free path for a K-electron transition is given in the low energy region by (Mo 49)

$$\lambda_s = M / \sigma_c (1 - \alpha) \text{gcm}^{-2}, \quad (2.20)$$

with

$$\sigma_c = 1.13 \times 10^{-15} E_0^2 Z^6 / \{E_0 + (Z + 1)^2 / 4\}^5 \{E_0 + (Z - 1)^2 / 4\}^5 \text{cm}^2, \quad (2.21)$$

where $E_0$ is the kinetic energy per nucleon of the incident nucleus in $10^8 \text{eV}$. At $10^8 \text{eV/nucleon}$, $\sigma_c$ is as large as

$$\lambda_s (Z = 1) \gg \lambda_c (F_e) = 10^8 \text{gcm}^{-2}. \quad (2.22)$$

The mean free path for the free-electron pick-up of a naked nucleus into the K-orbit is calculated as,

for N.R.,

$$\lambda_s = (3/2^8 \pi) (\hbar / m c)^{-3} (e^2 / \hbar c) (m c^2 / I_0) (m c E_k / M I_0 Z^2)^{5/2} \alpha^{-1} \text{M} \quad (2.23a)$$

and for E.R.,

$$\lambda_s = (1/4 \pi) (\hbar c / e^2)^4 (e^2 / m c^2)^{-3} Z^{-5} (E / M c^2) \alpha^{-1} \text{M} \quad (2.23b)$$

$\lambda_s$ increases rapidly as the energy of an incident nucleus increases in the non-relativistic region, and in the relativistic region it is so large values
From the above calculation we know that the fraction of nuclei retaining their orbital electrons to naked nuclei at the earth is negligibly small even for Fe as

\[ \frac{N_{Fe} \text{ (with electrons)}}{N_{Fe} \text{ (without electrons)}} = e^{-x/\bar{\lambda}} = e^{-5 \times 10^2}, \] (2.25)

where \( \bar{\lambda} \) is the average value of \( \lambda_i \). Also we substitute for \( X \) its value given in (2.4').

2.3. Energy spectrum

In his original theory (Fe 49), Fermi assumed that the cosmic ray particles are accelerated primarily by collisions against moving magnetic fields in the interstellar space and lose most of its energy by the nuclear collisions with interstellar hydrogen. Under this theory, the energy spectrum computed for the heavier nuclei falls off much more steeply than that of protons at high energies, since their mean free path for collisions with interstellar hydrogen is shorter than that of protons. This contradicts the experimental results in which the energy spectra for all nuclei seem to be quite similar to each other up to about \( 10^{13} \text{ eV per nucleon} \). In order to overcome this difficulty, Morrison et al. (Mo 54) and Fermi (Fe 54) suggested that cosmic-ray particles are eliminated by their leakage from the Galaxy. Let \( T_e \) be the mean lifetime of cosmic-ray particles for collisions with hydrogen, and let \( T_s \) be that for escaping from the Galaxy, then the mean lifetime for cosmic-ray particles in the Galaxy, \( T \), is given by

\[ \frac{1}{T} = \frac{1}{T_s} + \frac{1}{T_e}, \] (2.26)

where \( T_s \) depends slightly on energy, strictly speaking on rigidity, and \( T_e \) is roughly independent of energy. The average value of \( T_s \) is related to \( X \) in (2.4') and the mean density of interstellar matter \( \rho \) as

\[ T = X/\rho c. \] (2.27)

According to the acceleration mechanism proposed by Fermi a charged particle gains its energy with a constant rate, \( \alpha \), per collision with magnetic clouds. (See App. V (i).) Hence, a particle starting with \( E_0 \) will attain, after \( N \) collisions, an energy

\[ E = E_0 \exp(\alpha N). \]

If we call \( \tau \) the mean time between two successive collisions, its age \( t \) is
connected with the energy $E$ by the following relation:

$$t = (\tau/\alpha) \ln(E/E_0).$$

Thus the integral spectrum of the cosmic-ray particles is given by

$$N(>E) = N_0 \exp(-t/\tau) = N_0 (E/E_0)^{\tau/\alpha T},$$

where $N_0$ the number of particles injected at $t=0$. Such a power spectrum is in fact observed; this would be an argument for the Fermi mechanism, if

$$\tau/\alpha T = (1 \sim 2).$$

However, it is a question whether or not astrophysical conditions permit this value of $\tau/\alpha T$. This question will be answered, if the energy spectra of heavy nuclei are observed.

For heavy nuclei the cross section for collisions with interstellar hydrogen is larger than that for protons, and consequently the mean lifetime for the collisions decreases as

$$1/T_\epsilon(A) = A^{\alpha/3}/T_\epsilon(1),$$

where $A$ is the mass number of a heavy nucleus. Then

$$\tau/\alpha T = \tau/\alpha T_a + A^{\alpha/3 T/\alpha T_\epsilon(1)}. \quad (2\cdot29)$$

The mean traversed thickness obtained in the preceding section tells us $T_d/T_\epsilon(1) = 1/12$. Hence an observation of the charge spectrum at about $10^{18}$ eV per nucleon will easily answer the question, if the Fermi mechanism is operative in the Galaxy.

A feasible method of observing the heavy nuclei in the primary cosmic radiation may be the use of photographic plates near the top of the atmosphere. Scanty data thus far obtained tell us indication that the charge spectrum at about $10^{13}$ eV seems to be identical with that at low energies. But the conclusion should be reserved until more accurate experiments are performed.

As a method of detecting heavy primaries with very high energies, Zatsepin (Za 51) suggested to observe multiple cores of an extensive air shower initiated by several nuclear particles due to the photo-disintegration of heavy nuclei by solar photons. As a result of the collisions with solar photons, a heavy nucleus with an energy of more than $10^{18}$ eV can disintegrate and its fragments reach the earth with narrow angles. The probability for this photo-disintegration is calculated by Zatsepin (Za 55) for various nuclei. As a competitive process with the photo-disintegration, the

---

* The feasibility of this experiment has been suggested in 1955 by Professor Y. Fujimoto on the basis of the discussions described above.
photo-pi production occurs for energies larger than $10^{17}$ eV.

However, the probability of observing such events by means of extensive air showers is so small that the abundance of the heavy nuclei at high energies is not determined yet.

Observations of parallel mu-meson showers (Hi 56, 57) may provide another possible method. From the analysis of the observations, however, the majority of the multiple penetrating showers cannot be attributed to the heavy primaries (Ha 57b). This suggests that the intensity of heavy primaries of energies around $10^{15}$ eV is much smaller than that extrapolated from low energies by employing the power spectrum of protons. This may be indication favourable to the Fermi mechanism, but may also be attributed to the acceleration at local sources.

The low-rigidity (or low energy) cutoff of the primary spectrum is also of great interest in understanding the origin of cosmic radiation. From the measurements in the last decade by Forbush (Fo 54), by Neher (Ne 56) and by Meyer and Simpson (Me 57), it has become clear that the change in the low rigidity cutoff with time is a phenomenon connected with the eleven year solar cycle. They have shown that this is accompanied by the change in the total cosmic-ray intensity. Thus it can be expected that these changes in the primary spectrum have their origin in a mechanism controlled by the solar activity.

According to the above authors, cosmic rays at the minimum solar activity are least affected by solar effects. Even at this moment the energy spectrum has a maximum at an energy a little lower than one GeV. The position of the maximum in the rigidity spectrum and its dependence on the species of particles will give us a clue on the acceleration mechanism.*

It is almost certain that the ionization loss in the interstellar space does not play a major role, because even an iron nucleus of relativistic energy loses only 300 MeV per nucleon in the course of its passage of 3 g cm$^{-2}$.

2.4. Contaminating components in the primary radiation

In discriminating various possibilities on the origin of the cosmic radiation, it is important to consider the intensities of secondary particles produced by the collisions of cosmic rays with interstellar hydrogen. Since most of cosmic ray particles are protons, we consider secondaries produced by proton-proton collisions alone and the cascade development of protons is discarded on account of that the interstellar matter traversed is thin. Then the energy spectrum of protons changes so little in space-time that we may assume the spectrum everywhere to be identical with that observed

---

* On this problem we enjoyed fruitful discussions with Dr. M. Koshiba who expressed an idea that the maximum could be attributed to magnetic acceleration operative in localized space regions.
at the top of the atmosphere. This is assumed in the energy region of our interest as

\[ n_p(E) dE = n_p(E) E^{-2} dE, \tag{2.30} \]

where \( E \) is the total energy of a proton.

**Electrons** (neutrons and positrons). We can calculate the amount of electrons produced through the decay process \( \pi^+ \rightarrow \mu^+ \rightarrow e^+ \), pions being produced by collisions of protons with interstellar hydrogen. These electrons are stirred by magnetic fields and their energies are lost by the inverse Compton effect with galactic photons and the synchrotron radiation.

Let the number of electrons with energies between \( E \) and \( E + dE \) at time \( t \) be \( n_e(E, t) \). Then the diffusion equation for such electrons is

\[ \frac{\partial n_e(E, t)}{\partial t} = \frac{\rho c}{\lambda_p} S(E) + \frac{\partial}{\partial E} \left( b E^2 n_e(E, t) \right) - \frac{1}{T_e} n_e(E, t), \tag{2.31} \]

here the first term in the right-hand side shows the source of electrons, in which \( \lambda_p \) is the collision mean free path of protons against hydrogen (see Table 2-2) and \( \rho \) is the mean density of interstellar matter. The source \( S(E) \) is calculated by reference to the pion production by proton-proton collisions at high energies and \( \pi - \mu - e \) decays. According to the analysis carried out by Kaneko and Okazaki (Ka 58), the average energy of produced mesons is nearly constant in the center of mass system independently of the primary energy up to about 100 GeV (in the laboratory system), so that the multiplicity of charged mesons \( n_\pm \) increases sharply with the primary energy \( E \) as

\[ n_\pm(E) = \int_{m_\pi c^2}^{E^\pi(E/2M c^2)^{1/2}} P(E, E_\pi) dE_\pi = 2.5K(E/Mc^2)^{1/2}, \tag{2.32} \]

where \( P(E, E_\pi) dE_\pi \) is the production spectrum of charged mesons, \( K \) a parameter being taken as the order of unity and \( E_\pi^\ast \) the energy of charged mesons in the center of mass system,

\[ E_\pi^\ast = 0.32 \pm 0.05 \text{ GeV}. \tag{2.33} \]

Thus

\[ S(E) dE = \int dE_\mu 3\delta(E - 3E_\mu) \cdot \int dE_\pi \frac{m_\pi}{m_\mu} \delta(E_\mu - \frac{m_\pi}{m_\mu} E_\pi) \]

\[ \times \int dE' n_p(E') P(E', E_\pi) dE' = 0.15 n_p(E) dE, \tag{2.34} \]

in which the energy spectra of electrons in the \( \mu-e \) decays and of \( \mu \)-mesons in the \( \pi-\mu \) decays are assumed to be a \( \delta \)-functional spectrum, since this
approximation is good enough for the present use.

The second term in the right-hand side of $(2\cdot31)$ denotes the energy loss due to the synchrotron radiation and the inverse Compton effect. From $(3\cdot15)$ and $(61\cdot1')$ with $\langle H_i^2 \rangle \approx 3H_i^2 / 2$, $b$ is given by

$$b = 2.6 \times 10^{-8} (H^2 + 0.38 \times 10^{-10} W_r) \text{ Gev}^{-1} \text{ sec}^{-1},$$

where $H$ is the average strength of the magnetic field in gauss and $W_r$ the energy density of galactic photons in eV cm$^{-3}$. The energy loss by the synchrotron radiation is dominant over that by the inverse Compton effect, so far as $H \gtrsim 5 \times 10^{-6}$ gauss.

The last term shows the escape of electrons and $T_e$ is the mean lifetime, which is nearly equal to that of cosmic rays defined in $(2\cdot27)$. We take

$$T_e = T = X / \rho c.$$  \hspace{1cm} (2\cdot36)

When the equilibrium between primaries and secondaries is reached for large $t$, the diffusion equation $(2\cdot31)$ is easily solved as

$$n_s(E) dE \approx \frac{\rho c}{\lambda_p} \frac{dE}{bE^2} \int_{E'}^{\infty} S(E') dE' = \frac{\rho c}{\lambda_p} \frac{S(E)}{bE} dE \quad \text{for} \quad E \gtrsim 1 / bT,$$

$$\approx T \cdot \frac{\rho c}{\lambda_p} S(E) dE \quad \text{for} \quad E \ll 1 / bT.$$  \hspace{1cm} (2\cdot37a)

The integral energy spectrum of electrons thus expected is given by

$$N_{se}(>E) \equiv N_p(>E) \cdot 2.9 \times 10^4 \rho c / (H^2 + 0.38 \times 10^{-10} W_r) \lambda_p E$$

for $E \gtrsim 1 / bT,$

$$\approx N_p(>E) \cdot 0.15 \rho c T / \lambda_p \quad \text{for} \quad E \ll 1 / bT.$$  \hspace{1cm} (2\cdot37b)

where $N_p(>E)$ is the integral spectrum of protons, and $E$ is measured in unit of GeV hereafter. This spectrum can be extended to electron energies lower than $Mc^2$ on account of that the energy of a proton is divided into small pieces; in this case $N_p(>E)$ should be understood as $N_p(>Mc^2)$.

Further, in addition to such secondary electrons, there may arise considerable electrons from local sources such as the Crab nebula. As will be seen in §3, the intensity of electrons emitted from the Crab nebula is supposed to be as large as that of nuclear particles for energies lower than 100 GeV. We take the same intensity for electrons as nuclear particles at supernovae and assume that no electrons are produced at other sources, because few electrons seem to be generated in the solar flare effect, the...
direct electron flux being a fraction \( f \) of the total flux of cosmic ray particles. Thus, if the density of primary cosmic rays is practically uniform in the Galaxy, the rate of generation of electrons will be

\[
q_e(E) \, dE = f n_p(E) \, dE / T.
\]

Substituting \( q_e \) in the first term in the right-hand side in (2.31), the intensity of such direct electrons at the earth is obtained:

\[
N_{de}(>E) = \int_n \, n_{de}(E) \, dE \approx \frac{1.9 \times 10^5}{(H^2 + 0.38 \times 10^{-10} \, W_T)} \cdot \frac{f}{T} \cdot N_p(>E)
\]

for \( E > 1/bT \),

\[
\approx f N_p(>E) \quad \text{for} \quad E \leq 1/bT.
\]

This is larger than \( N_{se}(>E) \) estimated above, provided \( f > 0.15 \times /p \).

Thus we have the ratio of the electron component to the proton component in the primary radiation as

\[
\frac{N_e(>E)}{N_p(>E)} = \frac{N_{se}(>E) + N_{de}(>E)}{N_p(>E)}
\]

\[
= 1.9 \times 10^5 \frac{f + 0.15 \times /p^{-1}}{H^2 + 0.38 \times 10^{-10} \, W_T} \cdot \frac{1}{E} \cdot \frac{1}{T}.
\]

An observation to be compared with this has been carried out at the cutoff energy of 1.8 GeV for electrons. It shows that the electron component is less than 1% of the primary cosmic radiation at the earth (Cr 50),

\[
N_e(E > 1.8) / N_p(E > 1.8) \leq 10^{-2}.
\]

Hence

\[
\frac{f + 0.15 \times /p^{-1}}{H^2 + 0.38 \times 10^{-10} \, W_T} \cdot \frac{1}{T} \leq 10^{-7}.
\]

This relation limits the possible values of \( f, H \) and \( T \).

Further information on these quantities can be deduced by reference to the general galactic radio intensity, provided that the radio emission is due to the synchrotron radiation of such electrons as estimated above (Fa 56). The spectrum of the radio waves is approximately represented by the inverse law and the spatial distribution of their sources is found to decrease roughly inversely proportional to the distance from the galactic center (Pa 55, Pi 56).

The former fact indicates that the energy spectrum of electrons responsible for the radio emission is \( dE/E^\alpha \) in the energy range around 1

---

\(*\) The convention in the extrapolation to low energies adopted in (2.38) is not permitted for this spectrum.
GeV, on account of that the radio spectrum is observed between 20 Mc/s and 600 Mc/s and the magnetic field strength is supposed to be about $10^{-5}$ gauss. This is consistent with the synchrotron radiation of the secondary electrons, if there holds $E_b T > 1$. This condition is expressed, with the aid of (VI·7') and (2·35) as

$$\left( H^2 + 0.38 \times 10^{-10} W_T \right) T / H^{1/2} > 0.89 \times 10^{12} / \nu^{1/2},$$

(2·44)

where $\nu$ is the radio frequency in c/s. On the other hand, the direct electrons are presumed to have a flatter energy spectrum, since the rigidity spectrum of cosmic rays becomes flat below 2 GV and goes over the maximum, as rigidity decreases. The energy spectrum at sources could be flatter than the one which one would expect from the cosmic ray spectrum observed, although we have assumed the same shape at energies above 1.8 GeV in evaluating the expected number of electrons. Therefore, the intensity of the direct electron in the energy range under consideration may be smaller by one order than that one would obtain by extending the $dE/E^3$ spectrum to lower energies. Thus we may set the upper limit of $f$ as

$$f \leq X / \lambda_p = 0.1$$

(2·45)

and account for the radio emission merely in terms of the secondary electrons.

Now we calculate the radio intensity due to the secondary electrons filling a sphere of radius $R$ with the density distribution inversely proportional to the distance from the center. If an observer locates himself at the distance $a$ from the center and looks at the direction of zenith angle $\theta$ and azimuth angle $\phi$ measured from the direction of the center, the intensity is given, by employing Eqs. (IV·3, 18) and (2·38a), as

$$j(\nu, \theta, \phi) d\nu = 5.5 \times 10^{-16} \frac{H^2}{H^2 + 0.38 \times 10^{-10} W_T} \frac{\rho c}{\lambda_p} R \frac{\ln \left( \frac{R + \nu \sqrt{R^2 - a^2 + a^2 \cos^2 \theta \cos^2 \phi}}{a (1 - \cos \theta \cos \phi)} \right)}{\nu} \frac{d\nu}{\nu} \text{ erg cm}^{-2} \text{ sec}^{-1} \text{ sterad}^{-1} (\text{c/s})^{-1}.$$ 

(2·46)

This agrees reasonably well with the observed one (Pa 55, Pi 56), both for the spectrum and the intensity contour (see Figs. 2-2 and 2-3). The absolute intensity is compared for $\theta = 60^\circ$ and $\phi = 0^\circ$ at $\nu = 10^9$ c/s:

(*) The energy spectrum of electrons in the Crab nebula is less steep than $dE/E^2$, as will be deduced in §3, whereas the radio spectrum of B Cas, which is believed to be a supernova remnant, seems to predict a steeper energy spectrum of electrons, though their energy region is not known, information on magnetic fields being unavailable.
$j(10^4, 60^\circ, 0^\circ) \approx 1.8 \times 10^{-18} \text{erg cm}^{-2} \text{sec}^{-1} \text{sterad}^{-1} \text{c/s}^{-1}$. \hfill (2.47)

On account of $R/a=5/3$, we obtain within the limit of ambiguities of the present estimation,

$$\frac{H^2}{H^2 + 0.38 \times 10^{-10} W_\gamma} \cdot \frac{\rho c}{\lambda_p} \lesssim 1 \times 10^{-10} \text{sec}^{-1}. \hfill (2.48)$$

This inequality suggests us to take into account possible contributions from the direct electrons and other causes.

![Graph 1](image1.png)

**Fig. 2-2.** The observed cosmic radio spectrum from the coldest parts of the sky.

![Graph 2](image2.png)

**Fig. 2-3.** The cosmic radio intensity in the direction of the galactic latitude 0° and the galactic longitude 330°. The solid line is the calculated contour. Points × correspond to the observed values of 6.4 x 10^7 c/s, and points ○ to those of 10^8 c/s.

Now we have four relations, (2.43), (2.44), (2.45) and (2.48), in which we may fix

$$W_\gamma = 0.8 \text{ eV cm}^{-2}, \quad \lambda_p = 37 \text{ g cm}^{-2}, \quad X = 3 \text{ g cm}^{-2}, \hfill (2.49)$$

the last one having been obtained in §2.2.

The relation (2.48) is satisfied if $\rho c / \lambda_p \lesssim 1 \times 10^{-16}$, or

$$\rho \lesssim 1 \times 10^{-25} \text{ g cm}^{-3} \quad T \gtrsim 1 \times 10^{15} \text{ sec}. \hfill (2.50)$$

If this is not the case, (2.48) requires

$$H^2 \lesssim 0.38 \times 10^{-10} W_\gamma / ((1 \times 10^{10} X / \lambda_p T) - 1), \quad \text{or}$$

$$H \lesssim 0.6 \times 10^{-5} / (10^{15} T^{-1} - 1)^{1/2} \text{ gauss.}$$
In this case, however, (2.43) results in
\[ H \geq (f + 0.15X/2_p - 0.38 \times 10^{-5} W_r)^{1/2} \times 10^{-4} \geq 10^{-5} \text{ gauss.} \]

Both of the above conditions cannot be satisfied, unless \( T \) is very close to \( 10^{15} \text{ sec} \). Moreover, \( T \equiv 10^{13} \text{ sec} \) is consistent with (2.44), only if \( H \leq 10^{-7} \text{ gauss} \) or \( H \geq 3 \times 10^{-5} \text{ gauss} \). Therefore \( T \leq 10^{15} \text{ sec} \) is very unlikely, and we adopt the condition (2.50). This means that cosmic rays are stored not only in the galactic disk, in which the average matter density is as large as \( 10^{-24} \text{ g cm}^{-3} \), but in the galactic halo, thus it being consistent with the radio evidence.

The mean lifetime \( T \) may be identified with the diffusion time in the spherical galaxy of radius \( R \):
\[ T = 3R^2/4cl, \tag{2.51} \]
where \( l \) is the mean distance traversed between two successive collisions with magnetic clouds. Since most of particles of rigidity \( 10^{17} \text{ V} \) are supposed to be subject to the diffusion in the Galaxy, \( l \) is restricted by
\[ HI \geq 3 \times 10^{14} \text{ gauss cm}, \tag{2.52} \]
assuming that \( l \) is as large as the average size of the magnetic clouds. Introducing this into (2.51), we have
\[ T \leq 3R^2H/10^{15}c = 0.3 \times 10^{21}H. \tag{2.53} \]
On account of \( T \geq 10^{15} \text{ sec} \) in (2.50), the lower limit of \( H \) is obtained as
\[ H \geq 3 \times 10^{-6} \text{ gauss}. \tag{2.54} \]
This together with (2.52) gives \( l \geq 10^{20} \text{ cm} \). It seems to be reasonable that we take \( l \) to be about \( 10^{20} \text{ cm} \), the same order of magnitude as the size of a globular cluster. As \( H \) may not be larger than the strength of the spiral arm field, about \( 10^{-5} \text{ gauss} \), a plausible choice would be \( 3 \times 10^{-6} \text{ gauss} \).

The above considerations lead us to fix the values of the parameters as
\[ T \approx 3 \times 10^{15} \text{ sec} \ (10^8 y), \rho \approx 3 \times 10^{-25} \text{ g cm}^{-2}, \ H \approx 3 \times 10^{-6} \text{ gauss} \text{ and } f \approx 0.1. \tag{2.55} \]

Needless to say, these figures are subject to uncertainty of factor three or so.

*Photons.* The high energy \( \gamma \)-ray scomming to the earth are almost all due to the decay of neutral pions produced by the nuclear interactions of protons. An estimate of the photon component is made in an analogous way. Also we expect the line \( \gamma \)-rays from the positon-negaton annihilations. The intensity of the latter is of the same order of magnitude as that of
Origin of Cosmic Rays

the former, since most of positons (a little more than a half of secondary electrons estimated above) are degraded into low energies by the synchrotron radiation loss and the inverse Compton effect.

Let $T_\gamma = 10^{12}$ sec be the mean lifetime for escaping of photons out of the Galaxy. Then the fractional intensity of such photons to protons at the earth is given from (2·34) and (2·49) by

\[
\frac{N(\pi^0 \to \gamma)}{N_p} = \frac{N(e^+e^- \to \gamma)}{N_p} = 0.15 \cdot \frac{X}{\lambda_p} \cdot \frac{T_\gamma}{T} = 5 \times 10^{-8}, \tag{2·56}
\]

this being too small to be detectable. If the observation of these photons could be made, we would know the density distributions of cosmic rays and the interstellar matter in the Galaxy.

Antiprotons. At high energy collisions of cosmic ray particles with interstellar matter, anti-nucleons are also produced. Though the available information about the production cross section, both experimental and theoretical, is rather meager, we make the order-of-magnitude estimate on the antiproton component. We assume the constant production ratio of anti-nucleons to pions (Kob 57), and the multiplicity of pions adopted previously. Taking into account that the number of antiprotons near threshold increases as $E^{3·8}$ ($E$ being the primary energy minus the threshold energy) and assuming the production spectrum $\sim dE_\pi/E_\pi^2$ (Ni 58), we get the ratio of antiprotons to protons at the earth

\[
N_\pi/N_p = 1 \times 10^{-8}; \tag{2·57}
\]

the degree of contamination of antiprotons is 0.1% at most.

§3. Supernova origin

A possibility of the local origin of cosmic rays was argued in the foregoing discussions from a number of view-points. Since most of stars like the sun are rather poor producers of cosmic rays, we must seek for some objects in which disturbances of large scale take place. The stellar activities giving rise to the generation of cosmic rays may be associated with the emission of non-thermal radio waves. A supernova is known to be a strong source of radio emission and has been presumed as a possible source of cosmic rays in connection with the enormous energy production at an explosion (Ha 50). The energy output for radio waves has been estimated to be of the same order of magnitude as the energy output for cosmic rays (Un 51). Ginzburg and Shklovskij have developed the idea of the supernova origin based on the interpretation of the radio emission in terms of the synchrotron radiation of electrons in the magnetic field produced in the remnants of supernovae (Sh 53b, 54, Gi 51, 56, 57). One of
the authors (S.H.) has developed also the supernova origin theory laying much stress on the meaning of heavy cosmic ray nuclei (Ha 56a).

The frequency of supernova explosions of Type I is usually assumed as once in every three hundred years in the Galaxy, while that of Type II is estimated as once in every fifty years. Three supernovae of Type I are recorded in our history; the so-called Chinese Nova in Taurus in 1054, whose remnant is identified to be the Crab nebula, B Cassiopeiae in 1572 and Nova Ophiuchi in 1604. The detailed observation is available only for the Crab nebula, one of the strongest radio sources; hence we confine ourselves to the discussions about the Crab. We first treat in §3.1 the general energetics with reference to the efficiency of acceleration of cosmic rays, in §3.2 the electromagnetic radiation in terms of the synchrotron radiation of electrons and in §3.3 the possible mechanism of generation of cosmic rays. Important conclusions obtained in this section are summarized in §3.1.

3.1. Energetics in the Crab nebula

In view of the supernova origin of cosmic rays the high efficiency of converting thermal energy into the kinetic energy of relativistic particles could be realized in the supernovae. It is really seen in the Crab nebula, the remnant of a supernova in 1054.

Our arguments are based on the hypotheses, which seem to be plausible, that (a) the electromagnetic radiation from the Crab nebula is caused by the synchrotron radiation of relativistic electrons (Sh 53a, Oo 56) and (b) the light curve of supernovae of Type I in early days after explosion is due mainly to the spontaneous decay of $^{254}$Cf, so that the synthesis of transuranic elements took place in the explosion of the supernovae (Bu 56, 57a). In this subsection we compare the radioactive energy with energy of other modes, revising the result of our previous note (Ha 57a) with use of the recent determination of half-lives of $^{254}$Cf and $^{250}$Cm. The hypothesis (a) will be discussed in the next subsection and here we quote only the results. At the present epoch the total intensity of the electromagnetic radiation from the Crab nebula amounts to about $6 \times 10^{36}$erg sec$^{-1}$. The rate of energy consumption in the electrons is evaluated as about $10^{38}$erg sec$^{-1}$. If the escape of the electrons from the nebula through diffusion were insignificant, the rate of energy consumption would decrease to $10^{37}$erg sec$^{-1}$, this being the lower limit.

The hypothesis (b) is discussed in App. III, and in connection with it the dynamics of supernova explosion is treated also in App. IV. Our conclusions are as follows. For about a year after its outburst a supernova is in a radiative equilibrium and the visible luminosity should be proportional to the total radioactive energy. After that the light curve of a
supernova gradually deviates from the decay curve of the radioactivity. Within the arbitrariness of the mass determination of the Crab nebula as estimated in (III·6), the present rate of nuclear energy generation is estimated to lie between \((1 - 9) \times 10^{36}\) erg sec\(^{-1}\) as known from Fig. III·1.

The balance sheet with regard to the outcome and income of energy, applicable to the present Crab nebula is shown in Table 3-1.

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Supply</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electromagnetic radiation</td>
<td>(6 \times 10^{36})</td>
</tr>
<tr>
<td>Electrons</td>
<td>(10^{37} - 10^{38})</td>
</tr>
</tbody>
</table>

Table 3-1 shows that the energy supply from the radioactivity may be insufficient to make up the consumption due to high energy particles, or just enough, if more than one percent of the supernova mass could be converted in the synthesis of heavy elements. The situation is more serious, if one compares the energies per particle, of the order of MeV for the radioactive decay products and on the average tens of GeV for the high energy particles. The above comparison suggests us that the energy sources other than the radioactivity could be responsible for the acceleration of the high energy particles, electrons and nuclear particles; the latter ones turn into cosmic rays and the rate of energy consumption by them may be as large as that of electrons.

The largest amount of energy is contained in the expanding motion with velocity of about \(1.3 \times 10^{8}\) cm sec\(^{-1}\). The \(^{254}\)Cf hypothesis for a preferable case gives the mass between \((2 - 6) \, m_\odot\) as shown in (III·6); then the mass of the expanding nebula is about \((1 - 3) \, m_\odot\), supposing that each of the central double stars has a solar mass. The expansion energy is thus obtained as \((1 - 3) \times 10^{49}\) erg. The turbulent velocity will be at most one third of the expansion velocity, and the turbulent energy is regarded as large as the magnetic energy; the corresponding magnetic field strength is on the average \(10^{-3}\) gauss. These forms of energy are supposed to be generated rather long, at least one year (of the order of \(t_\odot\) estimated in (IV·10)), after the explosion. The rates of energy consumption in the radiation and of energy supply due to radioactivity are integrated over from \(t_\odot \geq 3 \times 10^7\) sec to the present. For the consumption in the electrons we roughly assume that the rate is nearly constant over this period. Thus we obtain Table 3-2, in which various modes of energy contents are compared.

From the energy contents in Table 3-2, we can draw the following conclusions.

(i) The efficiency of acceleration of relativistic particles is surpris-
S. Hayakawa, K. Ito and Y. Terashima

Table 3-2. Energy contents (erg) in various modes

<table>
<thead>
<tr>
<th>Expansion</th>
<th>(1-3)×10^{49}</th>
<th>Electromagnetic radiation</th>
<th>(0.4-0.8)×10^{48}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbulence</td>
<td>(1-3)×10^{48}</td>
<td>Electrons</td>
<td>(0.3-3)×10^{48}</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>(1-3)×10^{48}</td>
<td>Radioactivity</td>
<td>(0.2-1.0)×10^{48}</td>
</tr>
</tbody>
</table>

ingly good; the turbulent and magnetic energies may be converted almost completely to relativistic particles for a rather short period. If substantial part of the energy available can be converted to the energy of relativistic particles, the active lifetime of the Crab may be as long as $10^{11}$ sec.

(ii) The Fermi mechanism is not a major cause of acceleration, because this would result in the cosmic ray energy some thousand times (proportional to the mass of particles accelerated) larger than the electron energy, much larger than any form of energy.*

We suggest the following sequences of energy conversion, and also that the betatron-like process due to hydromagnetic shock waves is possibly responsible for acceleration, of which detailed discussion will be given in §3.3.

(iii) The fast moving light ripples, which were suggested by Oort and Walraven (Oo 56) as responsible for the injection of relativistic particles, should not be regarded as the motion of a gas, because the kinetic energy of the gas, in which the density could be larger than that in the medium, would correspond to the rate of energy consumption as large as $10^{39}$ erg sec$^{-1}$, larger than any other rate of energy supply. The interpretation of these ripples will be given also in §3.3 with reference to the scheme of energy conversion suggested above.

(iv) The proton flux $\sim 5\times10^{50}$ erg, assumed by Burbidge (Bu 58), is too large to be supplied, as far as we accept the hypotheses (a) and (b). To overcome the difficulty, one might be forced to assume that the kinetic and magnetic energy of a supernova at the explosion would be far larger than the energy of the visible light. This assumption would violate the proportionality of the visible luminosity to the radioactive energy even in the initial stage of a supernova.

*) If the injection energy is high enough, the mass dependence does no longer hold. In this case, however, one has to ask an acceleration mechanism for the injection.
3.2. Electromagnetic radiation from the Crab nebula

The radio intensity from the Crab nebula is observed in the frequency range from 60 Mc/s to $9.5 \times 10^3$ Mc/s (Ro 56), as is summarized in Fig. 3-1. For example,

$$J(\nu) = 2.0 \times 10^{-22} \text{ watt m}^{-2}(\text{c/s})^{-1} \text{ at } \nu = 10^8 \text{c/s}.$$

The absolute intensity $I(\nu)$ is estimated (not corrected for absorption inside the nebula) as

$$I(\nu) = 4\pi L^2 J(\nu)$$

$$= 4 \times 10^{24} \text{ erg sec}^{-1}(\text{c/s})^{-1} \text{ at } \nu = 10^8 \text{c/s},$$

where $L$ is the distance of the Crab nebula, $4 \times 10^{21}$cm.

The optical intensity is estimated from its visual magnitude $m_v = 9.14$ as

$$I(\nu) = 3.4 \times 10^{31} \text{ erg sec}^{-1}(\text{c/s})^{-1}$$

at $\nu = 7.1 \times 10^{14} \text{c/s}$. (3.2)

The intensity in the optical region is given by

$$I_{\text{total}} = 1.3 \times 10^{38} \text{ erg sec}^{-1}.$$  

(3.3)

The spectrum of the radio waves is rather flat and may be given approximately by the solid line in Fig. 3-1 as

$$I_\nu \propto \nu^{-0.20}.$$  

(3.4)

The optical radiation is mainly of a continuous spectrum whose shape is not accurately determined yet.

The distribution contours of surface intensities of the radio wave and the light have recently been observed by Oort and Walraven (Oo 56), which are shown in Figs. 3-2 and 3-3. The radio distribution is flatter than the light distribution. The former can be approximately obtained from the latter by multiplying the distances from the center by about 1.7, or roughly:

for radio region, $R_r = 3 \times 10^{18}$ cm and $V_r = 10^{56}$ cm$^3$,

for optical region, $R_0 = 2 \times 10^{18}$ cm and $V_0 = 3 \times 10^{55}$ cm$^3$, (3.5)

where $R$ is the radius of the region of interest and $V$ the corresponding volume.
From the above data we can estimate the total intensity of the radiation in the range from radio to light; it amounts to about

\[ I_r \approx 6 \times 10^{38} \text{ erg sec}^{-1}. \quad (3.6) \]

This was cited in §3.1.

Recent observations have shown that the optical radiation is strongly polarized and the polarization may be nearly complete in some places (Oo 56, Ba 56, Hil 57). Polarization experiments have also shown that the magnetic field is comparatively regular and the scales of the irregularities actually found are of the order of 1/2 light year.

It was suggested by Shklovskij that the characteristics of the radiation from the Crab nebula can be explained in terms of the synchrotron radiation (Sh 53a). We estimate the number and the energy of fast electrons on the basis of this hypothesis.

The strength of the magnetic field is as large as

\[ H = 10^{-8} \text{ gauss}, \quad (3.7) \]

in the inner region of the nebula, as argued by Oort and Walraven (Oo 56), but it may be weaker by one order in the outer region. They show that the energy of the magnetic field is comparable to the turbulent energy, if we adopt this, and not smaller than the energy of fast electrons. On the other hand Burbidge (Bu 58) assigned a much larger value of \( H \) as

\[ H = 10^{-2} \text{ gauss}, \quad (3.7') \]
This gives too short life-times for high energy electrons responsible to radiate light.\(^*\)

As shown in App. VI, the spectrum of the electromagnetic waves radiated by a fast electron has a rather sharp peak at the frequency
\[
\nu_m = 1.40 \times 10^6 H_1 (E/m_e c^2)^2 \tag{VI \cdot 7'}
\]
where \(H_1\) means the average strength of the magnetic field perpendicular to the direction of motion of an electron, being of the same order of that given in \((3 \cdot 7')\) or \((3 \cdot 7)\). Thus we obtain the optimum energies of electrons radiating radio waves and light, which are given in Table 3-3.

Table 3-3.

<table>
<thead>
<tr>
<th>(\lambda) (cm)</th>
<th>(\nu (\text{c/s}))</th>
<th>(E(H_1=10^{-3}\text{gauss})) (eV)</th>
<th>(E(H_1=10^{-2}\text{gauss})) (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio 5x10^2</td>
<td>6x10^7</td>
<td>1.1x10^8</td>
<td>3.2x10^7</td>
</tr>
<tr>
<td>3x10^2</td>
<td>10^4</td>
<td>1.4x10^8</td>
<td>4.3x10^7</td>
</tr>
<tr>
<td>10</td>
<td>10^4</td>
<td>7.5x10^8</td>
<td>2.4x10^8</td>
</tr>
<tr>
<td>10000A</td>
<td>3.0x10^14</td>
<td>2.4x10^{11}</td>
<td>7.4x10^{10}</td>
</tr>
<tr>
<td>Light 1200Å</td>
<td>7.1x10^{14}</td>
<td>3.6x10^{11}</td>
<td>1.1x10^{11}</td>
</tr>
<tr>
<td>3000Å</td>
<td>10^{12}</td>
<td>4.3x10^{11}</td>
<td>1.4x10^{11}</td>
</tr>
</tbody>
</table>

The intensity of the synchrotron radiation at the source, \(I(\nu)\) radiated by the electrons which have the energy spectrum
\[
n(r,E)dE = r n_0(r)(E/E_0)^{7}dE/E \tag{3 \cdot 8}
\]
is given by Eq. (VI \cdot 13) in App. VI as
\[
I(\nu) = 1.63 \times 10^{-21} 2^{7/2} U(\gamma) (\nu_0/\nu_m)^{7/2} H_1 N_0 \text{erg sec}^{-1} (\text{c/s})^{-1}, \tag{VI \cdot 13}
\]
where
\[
N_0 = \int n_0(r)dr
\]
and \(\nu_m\) is the one defined by \((VI \cdot 7')\) for \(E=E_0\). For \(E=E_0\) and \(\nu=\nu_m\), we can deduce the number of electrons responsible for the synchrotron radiation by comparing the above equation with \((3 \cdot 1-4)\) and taking \(H=H_1\)
\[
N_0 = 2.4 \times 10^{45} (2^{7/2} U(\gamma))^{-1} H^{-1} \quad \text{for} \quad E_0 = 4.3 \times 10^6 \text{eV},
\]
\[
N_0 = 2.1 \times 10^{42} (2^{7/2} U(\gamma))^{-1} H^{-1} \quad \text{for} \quad E_0 = 1.1 \times 10^{10} \text{eV}. \tag{3 \cdot 9}
\]
\(^*\) If the magnetic field strength were much weaker than \(10^{-3}\) gauss, the magnetic energy would be too small compared with the electron energy, so that the magnetic field could not persist long enough.
These numerical values give us a rough idea on the number and the energy of the radiating electrons.

Prior to going into detailed discussions of the distributions of fast electrons, we examine various modes of interactions of a fast electron in the nebula.

**Synchrotron radiation.** This is the most important process for the energy loss of a fast electron, of which the rate is given by

\[-\frac{dE}{dt} = b_s E^2 , \quad b_s = \frac{2c}{3} \left( \frac{e^2}{m_e c^2} \right) \left( \frac{eH_e}{m_e c^2} \right)^2 \frac{1}{m_e c^2} = 3.90 \times 10^{-16} \, \text{eV}^{-1} \text{sec}^{-1},\]

(VI·1')

and

\[-\frac{dE}{dt} = 3.90 \times 10^{-21} \, \text{eV}^{-1} \text{sec}^{-1} \quad \text{for} \quad H_e = 10^{-3} \text{gauss} , \]

\[-= 3.90 \times 10^{-19} \, \text{eV}^{-1} \text{sec}^{-1} \quad \text{for} \quad H_e = 10^{-2} \text{gauss} .\]

**Ionization.** Since the gas in the Crab nebula is highly ionized and mostly consist of hydrogen, we refer to the theory worked out by Hayakawa and Kitao (Ha 56b).*1 The rate of energy loss is expressed by

\[-\frac{dE}{dt} = b_i (E) , \quad b_i (E) = \rho B_i (E) .\]

(3·10)

\[B_i (E)\] is a slowly varying function of energy. The density of gas, \(\rho\), is hardly known exactly, but presumed to be about

\[\rho = 10^{-23} - 10^{-22} \, \text{g} \, \text{cm}^{-3} .\]

(3·11)

The degree of ionization is inferred to be nearly complete and \(\rho = 10^{-22} \, \text{g} \, \text{cm}^{-3}\). Then \(b_i (E) = 3 \times 10^{-5} \, \text{eV} \, \text{sec}^{-1}\). If this is compared with (VI·1'), the energy region in which the ionization loss is greater than the synchrotron radiation loss is restricted as

\[E \ll E_c \approx 8 \times 10^7 \, \text{eV} .\]

(3·12)

From Table 3-3, this energy range lies below the region concerned.

**Bremsstrahlung.** The bremsstrahlung (He 54) in the Coulomb fields of atomic nuclei and electrons contributes to the energy loss by a small fraction of the ionization loss. The rate of energy loss in a completely ionized medium is expressed as

\[-\frac{dE}{dt} = b_r E , \quad b_r = \frac{32\pi}{3} \frac{c \rho}{M} \left( \frac{e^2}{\hbar c} \right) \left( \frac{e^2}{m_e c^2} \right)^2 Z^2 \ln \left( \frac{D_h / m_e c}{m_e c} \right) = 3.3 \times 10^9 \rho \, \text{sec}^{-1} ,\]

(3·13)

*1 A factor \(2m_e c^2 \Gamma\) in the logarithm in Eq. (2·9) of reference (Ha56b) should be replaced by \(M c^4\) for energies above \((M/2m_e) M c^4\), where \(M\) is the nucleon mass. But this correction is insignificant in any numerical discussion.
where $D$ is the Debye length, i.e. $D = 6.90(T/n_e)^{1/2}$ cm, and we take the electron density $n_e \approx 10^8$ cm$^{-3}$ and the electron temperature $T \approx 10^4$ °K. Comparing this with (VI·1'), we know that the bremsstrahlung loss may become greater than the synchrotron radiation loss in the energy region given by (3·12), in which the former amounts to only a fraction of the ionization loss.

**Inverse Compton effect.** The Crab nebula involves photons whose average energy density is approximately

$$W_\gamma \approx 20 \text{ eV cm}^{-3}. \quad (3.14)$$

The rate of energy loss of an electron due to collisions with these photons has the expressions (Fe 48) as

$$\frac{dE}{dt} = b_e E^2, \quad b_e = \frac{8\pi}{3} \left( \frac{e^2}{m_e c^2} \right)^2 \frac{3.6 c kT}{(m_e c^2)^2} n_\gamma$$

$$= 1.0 \times 10^{-25} W_\gamma \text{eV}^{-1} \text{sec}^{-1}, \quad (3.15a)$$

$$u_e \gg 1; \quad \frac{dE}{dt} = b_e E, \quad b_e = \frac{8\pi}{3} \left( \frac{e^2}{m_e c^2} \right)^2 \frac{W_\gamma}{kT}$$

$$= 7.4 \times 10^{-15} (W_\gamma/kT) \text{sec}^{-1}, \quad (3.15b)$$

where $n_\gamma$ is the photon number density, $T$ is the temperature of the radiation and $u_e$ is defined by

$$u_e = (m_e c^2)^2 / E \cdot kT. \quad (3.16)$$

For $E \approx 10^8 \text{eV} - 10^{11} \text{eV}$ and $kT \approx 1 \text{eV}$, $u_e \gg 1$ and we use the formula (3.15a) with (3.14). This is again a fraction of percent of the synchrotron radiation loss. The inverse Compton effect was very important at the early stage of the nebula when the luminosity was enormously large. The electrons injected at this stage must have been decelerated by this process.

Comparing the above processes, only the synchrotron radiation has to be taken into account for practical purposes. Only at low energies, however, the ionization loss plays a significant role.

**Fermi mechanism.** Electrons injected into the nebula may be accelerated by the Fermi mechanism. The rate of energy gain is expressed by Eq. (V·8') as

$$\frac{dE}{dt} = a_f E,$$

and

$$a_f = \frac{2c}{l} \left( \frac{V_0}{c} \right)^2,$$

where $l$ is the transport mean free path and $V_0$ the mean value of random
velocities of magnetic clouds. For \( V_0 = 3 \times 10^7 \text{ cm sec}^{-1} \) and \( l = 10^{17} \text{ cm} \), we have

\[
a_f \cong 6 \times 10^{-18} \text{ sec}^{-1}.
\] (3.17)

The energy gain due to the Fermi mechanism ceases at an energy, above which the energy loss due to the synchrotron radiation is more effective than the gain. An electron cannot gain a greater energy beyond the barrier energy \( E_b \), which is given by

\[
E_b = a_f/b_f \cong 10^8 \text{ eV}.
\] (3.18)

The barrier energy is so low that the electrons responsible for the optical radiation are not attainable due to the Fermi mechanism, as seen from the comparison of (3.18) with the figures given in Table 3-3. Although the smaller value of \( l \) gives the larger value of \( E_b \), \( l \) cannot be smaller than \( 10^{17} \text{ cm} \) because the scales of the irregularities of the magnetic field are of the order of 1/2 light year.

The consideration above leads to the same conclusion as argued in §3.1, that the Fermi mechanism is not efficient in the Crab nebula.

The existence of the barrier energy may be interpreted in such a term that high energy electrons must be continuously supplied in the Crab, since the lifetimes of electrons, \( T_{1/2} \), for the synchrotron radiation in the visible region are about \( 10^9 \text{ sec} \) in a magnetic field of \( 10^{-3} \text{ gauss} \) as known from Eq. (VI-8) in App. VI. The medium energy of these electrons is \( 2 \times 10^{11} \text{ eV} \) as shown in Table 3-3. Even if the Fermi mechanism had been efficient in the early stage of the Crab, high energy electrons could have not survived till now. It needs some mechanism, by which electrons are accelerated rapidly to \( 10^{11} - 10^{12} \text{ eV} \). Oort and Walraven estimated the intensity of the electrons in the nebula along this line.

The fact that the spatial distribution of the radio waves is wider than that of the light shows that the distribution of electrons is not uniform, and (or) so would not be of the magnetic field. If, however, the strength of the magnetic field varies little over the main part of the nebula, the difference between the radio and the optical region is ascribed to that the high energy electrons responsible for the optical emission are more concentrated towards the center than the comparatively low energy electrons responsible for the radio emission.

We propose the following model, the continuous generation of the electrons, and the non-uniformity of the electron distribution under the
Origin of Cosmic Rays

constant magnetic field.\(^{2}\) If the electrons are injected from the central region of the nebula, then high energy electrons lose their energy before they reach the outer part of the nebula, while the low energy electrons diffuse into the whole nebula. The average diffusion time for the latter is of the order of \(10^9\) sec as shown in (3.34), being shorter than the lifetime for the synchrotron radiation, \(T_{1/2} \approx 10^{11}\) sec. The time scale concerned here is \(10^9\) sec and it is much shorter than the age of the Crab itself. Then the injection rate of the electrons is assumed as constant for practical purpose.

Let the source of the electrons be

\[
Q(E, r, t) = q(E)S(r),
\]

where the injection spectrum \(q(E)\) is given by

\[
q(E) = r_1q_0(E_0/E)^{\gamma_1}/E \quad \text{for} \quad E < E_0,
\]

\[
q(E) = r_2q_0(E_0/E)^{\gamma_1}/E \quad \text{for} \quad E > E_0.
\]

For \(S(r)\) we assume a uniform source,

\[
S(r) = \text{const} \quad \text{for} \quad r < r_0,
\]

\(^{2}\) One might think that the difference between the contours of surface intensities of the radio waves and the light, as seen in Fig. 3-2, 3, could be explained by the non-uniform distribution of the magnetic field. But this difference cannot be attributed only to the non-uniformity of the magnetic field as shown in the following.

To simplify the argument, we assume the constant density of the fast electrons throughout the nebula. The fact that the radio distribution \(I_{\nu}\) is approximately obtained from the light distribution \(I_{\nu_0}\) by multiplying the distance from the center \(d\) by about 1.7 tells us

\[
I_{\nu_0} = f(d) \quad \text{and} \quad I_{\nu} = f(d/1.7).
\]

These equalities are satisfied when \(f(d)\) has the functional shape:

\[
I_{\nu_0} = A - B \ln d,
\]

\[
I_{\nu} = A - B \ln (d/1.7) = A' - B \ln d.
\]

As is known from (3.8), (VI.13) and (3.25),

\[
I_{\nu_0} \propto H(d)^{\gamma_1+1}/2 \quad \text{and} \quad I_{\nu} \propto H(d)^{\gamma_2+1}/2
\]

where \(\gamma_1\) and \(\gamma_2\) are defined by (3.26) and (3.27), respectively.

If we put

\[
H(d)^{\gamma_1+1}/2 \propto A - B \ln d
\]

then

\[
I_{\nu_0} \propto H(d)^{\gamma_2+1}/2 \propto (A - B \ln d)^{\gamma_2+3/2}\gamma_1+2.
\]

Since \(\gamma_1 < \gamma_2\), the last relation cannot be written in the form required above. This indicates that the intensity contours argue for the necessity of the diffusion of electrons out of the nebula.
or a uniform shell source

\[ S(r) = \text{const} \quad \text{for} \quad r_0 - \epsilon < r < r_0. \]

We normalize \( S(r) \) as

\[ \int S(r) \, dr = 1. \]

The diffusion equation for the stationary state is of the form

\[ D \Delta N - \frac{\partial}{\partial E} (b_i E^2 N) + q(E) S(r) = 0, \quad (3 \cdot 21) \]

with

\[ D = \frac{1}{3} cl. \quad (3 \cdot 22) \]

The transport mean free path \( l \) is of the order of the scale of irregularities of the magnetic field. The ionization loss is neglected here, because of the reason stated above.

The solutions of the above equation are worked out in App. VII (iii). From Eqs. (VII•25, 26) we have \( N(E, r) \) for \( E < E_0 \):

\[ N(E, r) = \frac{2^{1/2} e^{3/2} r q_0 (E_0/E)^{\gamma_1} \frac{1}{E}}{4 \pi D r_0^2} f \left( \frac{r}{r_0} \right), \quad (3 \cdot 23) \]

where

\[ r < r_0; \quad f \left( \frac{r}{r_0} \right) = \frac{3}{2} - \frac{1}{2} \left( \frac{r}{r_0} \right)^2 \]

for the uniform source,

\[ r < r_0; \quad f \left( \frac{r}{r_0} \right) = 1 \]

for the shell source,

and

\[ r > r_0; \quad f \left( \frac{r}{r_0} \right) = \frac{r_0}{r} \]

for either case.

\( f(x) = 1 \) means that the electron density inside the cavity, \( r < r_0 \), is constant. For \( E > E_0 \), \( N(E, r) \) is

\[ N(E, r) = \frac{2^{1/2} e^{3/2} r q_0 (E_0/E)^{\gamma_2} \frac{1}{E}}{4 \pi D r_0^2} f \left( \frac{r}{r_0} \right), \quad (3 \cdot 24) \]

\( f(r/r_0) \) has the same expressions as before for \( r < r_0 \), but rather complex ones for respective cases in the region \( r > r_0 \):

\[ (6D/b_i E)^{3/2} r > r_0; \quad f \left( \frac{r}{r_0} \right) = \frac{r_0}{r}, \]
Origin of Cosmic Rays

\[(6D/bE)^{1/2} \sim r; \quad f\left(\frac{r}{r_0}\right) = \left(\frac{2\pi}{3}\right)^{\frac{1}{2}} e^{-\eta} \left(\frac{r_0}{r}\right) \exp\left(-\frac{bE r^2}{4D}\right) F\left(\frac{bE r^2}{4D} \right),\]

where \(F(K)\) is given by Eq. (VI.28) and \(\eta = r_2 - 2\).

Integrating \(N(E, r)\) over \(r\)

\[N(E) = \int 4\pi r^2 N(E, r) dr\]

we have

\[N(E) \propto E^{-(r_1+1)} \quad \text{for} \quad E \leq E_0,\]

\[N(E) \propto E^{-(r_2+2)} \quad \text{for} \quad E > E_0.\]  

(3.25)

Comparing these with (3.8) and (VI.13), the power index of the frequency spectrum of the radio waves in (3.4) gives

\(r_1 = 0.40.\)  

(3.26)

It seems natural to assume the continuity of the injection spectrum \(q(E)\). Further \(N(E, r)\) with suitable values of \(r_1, r_2\) and \(E_0\) must give the observed intensities of the radiation (3.1', 2, 3). Then we get the possible values of \(r_2\) and \(E_0\), assuming \(H \approx 10^{-3}\) gauss, with the aid of (3.9) and on account of the similarity to the cosmic ray spectrum for \(E > E_0\), as

\[r_2 = 1.5,\]

\[E_0 = 7.5 \times 10^{10} \text{ eV}.\]  

(3.27)

This results in the predicted value of the index of the frequency spectrum of the light as

\[I(\nu) \propto \nu^{-1.25}.\]  

(3.28)

The following assignment of other parameters gives the calculated distribution of the surface intensity of the light in good agreement with the observation as indicated in Fig. 3-2. (the shell source gives a more satisfactory result),

\[r_0 = 0.5 \times 10^{18} \text{ cm},\]

\[D \approx (1-4) \times 10^{27} \text{ cm}^2 \text{ sec}^{-1},\]

\[I \approx (1-4) \times 10^{17} \text{ cm}.\]  

(3.29)

The full spectrum of the electrons is thus determined and is plotted in Fig. 3-4.

The calculated distribution of the radio waves is a little steeper than the observed distribution. However, it is flattened, as shown in Fig. 3-3, if we take into account the absorption of the radio waves by the free-free tran-
sition (Al 55) as follows.

The absorption coefficient of the free-free transition is given by

$$\kappa_s = \frac{8}{3} \left( \frac{\pi}{6} \right)^{1/2} \frac{e^6}{c(m_e k T)^{3/2}} \frac{Z^2 g}{v^2} n_e n_i$$

$$= 0.0178 Z^2 n_e n_i g T^{-3/2} \nu^{-2} \text{cm}^{-1}, \quad (3.30)$$

where

$$g = \frac{\sqrt{3}}{\pi} \ln \left( \frac{4 k T}{e^2 n_e^{1/3}} \right) = 1.27$$

$$\times \left( 3.38 + \ln T - \frac{1}{3} \ln n_e \right).$$

$n_e$ and $n_i$ are the number densities of electrons and ions in cm$^{-3}$, respectively, $T$ is the kinetic temperature in °K and $\nu$ in c/s. For the present analysis we take $n_i = n_e = 10^9$, $Z = 1$ (corresponding to $\rho \approx 10^{-22}$ g cm$^{-3}$ in (3.11)) and $T = 10^4$ °K, then

$$\kappa_s = 1.52 \times 10^{-10} (10^9/\nu)^2 \text{cm}^{-1}. \quad (3.31)$$

The total number of the electrons $N_e$ (energy greater than $10^8$ eV) and their energy are now estimated as

$$N_e = \int_0^{\infty} dE d\tau N(E, \tau) \approx 2 \times 10^{49}, \quad (3.32a)$$

$$W_e = \int_0^{\infty} dE d\tau E N(E, \tau) \approx 10^{59} \text{eV}, \quad (3.32b)$$

where the integral over energy is extended to infinity with the spectrum (3.24), since the result is insensitive to the cut-off of energy.

The rate of generation of the electrons is

$$L_e = \int_{10^8}^{\infty} q(E) dE \approx 2 \times 10^{40} \text{sec}^{-1}, \quad (3.33a)$$

$$U_e = \int_{10^8}^{\infty} E q(E) dE \approx 1.5 \times 10^{50} \text{eV sec}^{-1}. \quad (3.33b)$$

It should be noted that the mean lifetime of the electrons with energy around $10^{10}$ eV is evaluated with use of the value of $D$ in (3.29) as

$$T_e \approx R^2/4D \approx 10^9 \text{sec}. \quad (3.34)$$

We summarize the results in Table 3-4. The assignment of many
parameters introduced is fairly consistent, though our model is nothing but one example. For comparison the values of the parameters in Burbidge’s model (Bu 58) are also shown. According to the latter model, protons are accelerated at the earliest stage and electrons are their secondary products coming from the decays of pions produced by nuclear collisions.

Table 3-4.

<table>
<thead>
<tr>
<th></th>
<th>Our model</th>
<th>Burbidge’s model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>H</strong></td>
<td>$10^{-3}$ gauss</td>
<td>$10^{-2}$ gauss</td>
</tr>
<tr>
<td>Total number of</td>
<td>$2 \times 10^{49}$</td>
<td>$10^{48}$</td>
</tr>
<tr>
<td>fast electrons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total energy of</td>
<td>$10^{59}$ eV = $1.6 \times 10^{47}$ erg</td>
<td>$3 \times 10^{48}$ eV = $5 \times 10^{46}$ erg</td>
</tr>
<tr>
<td>fast electrons</td>
<td>$\sim 10^8$ sec</td>
<td>$\sim 10^8$ sec</td>
</tr>
<tr>
<td>Average lifetime of</td>
<td></td>
<td></td>
</tr>
<tr>
<td>fast electrons</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total number of</td>
<td>$2 \times 10^{49}$</td>
<td>$4 \times 10^{49}$</td>
</tr>
<tr>
<td>cosmic ray particles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total energy of</td>
<td>$1.5 \times 10^{62}$ eV = $2.5 \times 10^{47}$ erg</td>
<td>$3 \times 10^{42}$ eV = $5 \times 10^{46}$ erg</td>
</tr>
<tr>
<td>cosmic ray particles</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.3. Acceleration mechanism and nuclear particles in the Crab nebula

In the preceding subsections we discussed that the total energy of relativistic particles in the Crab nebula would be comparable to that of magnetic fields and turbulent motions. This requires some mechanism which rapidly converts the energy of other modes into the energies of these particles. Also in the analysis of the synchrotron radiation we assumed that high energy electrons are injected from the central region of the nebula without reference to the details of the injection mechanism.

Oort and Walraven (Oo 56) suggested that the central stars in the Crab are still active and the moving light ripples, being emitted sporadically from them, might be a possible source of relativistic particles. As stated in §3.1 the moving light ripples could not be regarded as the motion of a gas, but as a shock motion. The magnetic field in the central region of the Crab is quite regular and may be of a ring shape as known from polarization experiments (Hil 57). Therefore, the ripples travel nearly perpendicular to the magnetic field and there may develop a hydromagnetic shock that will result in the rapid acceleration of relativistic particles.

If we apply the formula (V.21), the strength of the magnetic field induced by the ripple is of the order of

$$H_z \approx H_1 \frac{V_s}{V_\lambda} \approx 10^{-2} \text{ gauss,}$$

where $V_s$ is the velocity of the ripple, $\approx 3 \times 10^9 \text{ cm sec}^{-1}$, $H_1$ and $V_\lambda$ are
respectively the strength of the magnetic field and the Alfvén velocity in the medium. Their values are presumed to be

\[ H_1 \approx 10^{-3} \text{ gauss}, \]
\[ V_A = (4\pi \rho_1)^{-1} H_1 \approx 10^8 \text{ cm sec}^{-1}. \]

In the last expression \( \rho_1 \) is the density in the medium, \( \approx 10^{-23} \text{ g cm}^{-3} \). Thus the energy density of the ripple is estimated as

\[ H_1^2 / 8\pi \approx 10^{-5} \text{ erg cm}^{-3}. \]  \hspace{1cm} (3.36)

Since the ripple has the volume of \( 10^{49} - 10^{50} \text{ cm}^3 \), the total energy contained is

\[ W_r \approx 10^{44} - 10^{45} \text{ erg}. \]  \hspace{1cm} (3.37)

Taking into account that the frequency of occurrence of the ripples is about once in every three months, the rate of energy production by the ripples amounts to

\[ L_r \approx 10^{37} - 10^{38} \text{ erg sec}^{-1}, \]  \hspace{1cm} (3.38)

this being comparable to the energy consumption in the form of the electrons (see Table 3-1).

Although the energy generated is not completely converted into the kinetic energies of particles, we could ascribe the substantial part of the required injection mechanism to the ripple. At any rate, in the central region the acceleration by the hydromagnetic shocks seems probable, con-
sidering the situation that radial turbulent motions traverse the magnetic lines of force. As the magnetic field is subject to rapid variance, the average efficiency of the acceleration is given by Eq. (V.14) as

$$\frac{dp}{dt} = a_\theta \cdot p, \quad a_\theta = \frac{1}{2H} \frac{\partial H}{\partial t} \langle \sin \alpha \rangle = \frac{\pi}{8} \frac{1}{H} \frac{\partial H}{\partial t}. \quad (3.39)$$

The rate of increase of the magnetic flux in the central region will be as high as

$$\frac{1}{H} \frac{\partial H}{\partial t} \sim \frac{1}{H} \frac{H}{(l/V_e)} \sim 10^{-9} \text{sec}^{-1}, \quad (3.40)$$

where \( l \) is the scale of irregularities of the magnetic field \( \approx 10^{17} \) cm and \( V_e \) the expansion velocity \( \approx 10^8 \) cm sec\(^{-1}\). In the ripple a more rapid variation will be expected and the rate of change is at least

$$\frac{1}{H} \frac{\partial H}{\partial t} \sim \frac{1}{H} \frac{H}{(d/V_e)} \geq 10^{-7} \text{sec}^{-1}, \quad (3.41)$$

where \( d \) is the traveling distance of the ripple \( \approx 3 \times 10^{16} \) cm. Thus the efficiency of acceleration is much higher than the Fermi mechanism.

Further remarks of the moving light ripples are given by comparing the rate of energy gain estimated above with the rate of the synchrotron radiation loss for relativistic electrons in the ripple magnetic field of \( 10^{-2} \) gauss given by Eq. (VI.1'),

$$- \frac{dE}{dt} = b_e E^2, \quad b_e \approx 4 \times 10^{-19} \text{eV}^{-1} \text{sec}^{-1}.$$ 

Then an electron attains an energy of

$$a_\theta / b_e \approx 10^{11} \text{eV}.$$ 

The electrons having these energies are able to radiate visible light as shown in Table 3-3.

Assuming that the ripple supplies a fraction \( f \) of the electron generation evaluated in (3.32a, b), whose spectrum is given by Eq. (3.20, 26, 27), the optical intensity from the ripple is estimated from Eq. (VI.13) as

$$I_r = 0.7 \times 10^{30} f T_r \text{ erg sec}^{-1}, \quad (3.42)$$

where \( T_r \) means the average lifetime of the electrons in the ripple responsible for the optical emission. \( T_r \) may be shorter than the lifetime of the ripple, \( \approx 10^7 \) sec, since the electron diffusion is thought to be very fast. For example, with \( T_r \approx 10^5 \) sec and \( f = 0.1 \), we obtain

$$I_r \approx 7 \times 10^{33} \text{ erg sec}^{-1}. \quad (3.42')$$
This is luminous enough in contrast to the background luminosity, although $T_r$ is chosen too low.\footnote{Recently Shklovskij (Sh 57) has given a similar interpretation of the ripples, that the occurrence of the ripples is due to the fluctuations of the magnetic field. But the possibility of acceleration of fast particles in the ripple is abandoned, and the electron density there is assumed to be nearly equal to that in other places, irrespective of the fluctuations of the magnetic field.}

In the acceleration mechanism proposed, the energy attained is dependent on the initial energy. The acceleration of nuclear particles will start with energies of the order of MeV, or of the radioactive decay products. We rewrite (3.39) with (3.40) or (3.41) for non-relativistic case as

$$\frac{dE_k}{dt} = a_0 E_k, \quad a_0 = 10^{-9} \text{ or } 10^{-7} \text{ sec}^{-1},$$

(3.43)

where $E_k$ is the kinetic energy per nucleon of a nuclear particle.

On the other hand the energy loss is mainly due to the ionization process. The rate of energy loss of a nucleus with atomic number $Z$ and mass number $A$ is expressed by

$$-\frac{dE_k}{dt} = \rho(Z^2/A) B_i(E_k),$$

(3.44)

where $\rho$ is the density of matter, $v$ the velocity of the nucleus, and $B_i(E_k)$ the energy loss per gcm$^{-2}$ for proton with kinetic energy $E_k$, the relation (3.44) being plotted against the kinetic energy of the proton by Hayakawa and Kitao (Ha 56b). We take the complete ionization and $\rho \approx 10^{-22}$ gcm$^{-3}$. If this is compared with (3.43), the rate of energy gain is greater than the ionization loss if

$$E_k > 5 \times 10^{6} \text{ eV},$$

(3.45)

even for an iron nucleus. Thus the ionization loss is insignificant.

Since the particles having the same rigidity are subject to the same condition in the betatron-like acceleration, the total number and energy of cosmic rays are expected to be nearly equal to those of electrons, as the ionization loss can be discarded. Then for the cosmic ray particles having momenta per nucleon greater than $10^8$ eV/c, we obtain

$$N_{c.r.} \approx 2 \times 10^{49},$$

(3.46a)

$$W_{c.r.} \approx 1.5 \times 10^{59} \text{ eV}.$$  

(3.46b)

Here we take account of the only difference that high energy nuclear particles do not suffer the synchrotron radiation loss as the high energy electrons do. As cosmic ray particles escape from the nebula by diffusion,
Origin of Cosmic Rays

for which the lifetime is taken to be of the same value, \( \tau \cong 10^9 \) sec, as that given by (3.34) for electrons, the generation rates of the number and the energy of cosmic rays will be

\[
S_{\text{c.r.}} \cong 2 \times 10^{40} \text{ sec}^{-1}, \quad (3.47a)
\]

\[
U_{\text{c.r.}} \cong 1.5 \times 10^{50} \text{ eV sec}^{-1}. \quad (3.47b)
\]

The above consideration on the acceleration mechanism supports the energy conversion scheme proposed in §3.1. The ripples trigger the machine which converts the turbulent and magnetic energy into the energy of fast particles. The former modes of energy may come from expansion and radioactivity. The betatron action operates also outside the ripples and consumes the expansion energy directly. Thus the major part of energies of fast particles is supplied ultimately from the expansion energy. An answer is given to the question how long the continuous production of fast particles will continue. If all of the expansion energy, \( \cong 2 \times 10^{40} \) erg, is available for the fast particles, the present rate of generation of (3.33b) and (3.47b) will continue for

\[
t_{e} \cong 2 \times 10^{49}/(U_{e} + U_{\text{c.r.}}) \cong 10^{11} \text{ sec}. \quad (3.48)
\]

This is in good agreement with \( t_{e} \) in Eq. (IV\( \cdot 13' \)), another estimate of the active lifetime of the Crab nebula (see App. IV).

Thus the total production of cosmic ray particles from the Crab amounts to

\[
S_{\text{c.r.}} \cdot t_{e} = 2 \times 10^{51} \text{ particles and } U_{\text{c.r.}} \cdot t_{e} = 1.5 \times 10^{61} \text{ eV}. \quad (3.49)
\]

Our view that most of cosmic ray particles are produced continuously from the supernova remnant is contrary to Burbidge’s hypothesis, in which the enormous amount of protons must have been created in the initial stage of explosion.

Now we give a remark on a possible experiment to examine the yield of nuclear particles in the Crab (Ha 58). In Burbidge’s model, for example, the total energy in the proton flux amounts to \( \cong 5 \times 10^{50} \) erg = 3 \times 10^{62} eV and the mean energy per proton is 7 \times 10^{12} eV (see Table 3-4). Hence the Crab could be a strong source of high energy photons. The intensity of the photons at the top of the earth atmosphere is estimated to be about 3 \times 10^{-5} \text{ cm}^{-2} \text{ sec}^{-1}. Since the average energy of the photons is of the order

\[\text{After the preparation of this paper we have read a paper by Piddington (Pi 57). He reaches a similar conclusion independently. But his model of the Crab is different in some respects from the model we adopted. The ripple is interpreted as a hydromagnetic wave packet, whose energy content is evaluated as } 7 \times 10^{52} \text{ erg sec}^{-1}, \text{ in agreement with our estimate of (3.38). The acceleration is ascribed to the Fermi action in the ripple.} \]
of $10^{11}\text{eV}$, this intensity is compared with the intensity of nuclear particles of the same energy, about $10^{-4}\text{cm}^{-2}\text{sec}^{-1}$, or with that giving photons of the same energy, $10^{-4}\text{cm}^{-2}\text{sec}^{-1}$. Thus the high energy photons from the Crab will be possibly observed with nuclear emulsion; it may appear as a cascade shower without being accompanied with charged pions. Both in Oort and Walraven's model and ours, the intensity of such photons seems too low, $\sim 10^{-9}\text{cm}^{-2}\text{sec}^{-1}$, to be observable.

It is interesting to inspect the abundances of those elements ejected from supernovae which turn into cosmic rays. The relative abundances of the ejected materials are estimated by Burbidge et al. (Bu 57a) and they are found to be really rich in heavy elements. According to their estimate, the silicon and iron groups consist in 20% and 10% by mass. Since nuclei of the silicon group are supposed to be formed mainly by the helium capturing processes from nuclei of the medium group*, the abundances of these two are presumed to be of the same order of magnitude. It will not be far from reality, therefore, if we choose the relative abundances by number as

$$N_H : N_{Fe} : N_{Si} = 2 : 1 : 0.5.$$  \hfill (3.50)

This ratio is consistent with the composition of cosmic rays.

Among the medium group, the overabundance of carbon has been observed by Koshiba et al. (Ko 57), as was mentioned in §2, and this fact has been interpreted in terms of the generation of cosmic rays mostly at massive, young stars of high central temperatures, since the C/N ratio expected from the C/N cycle increases with temperature. We would, however, like to point out an alternative interpretation based on the supernova origin. In the early stage of the explosion when the temperature in the envelope is high enough, so that the rapid neutron capture processes take place, oxygen and nitrogen in the envelope are able to suffer the following reactions:

$$^{16}\text{O}(p, r)^{17}\text{F}(\beta^+\nu)^{17}\text{O}(p, \alpha)^{14}\text{N} \quad (T_{1/2}(^{17}\text{F} \rightarrow ^{17}\text{O}) \approx 66 \text{sec}),$$

$$^{14}\text{N}(p, r)^{15}\text{O}(\beta^+\nu)^{15}\text{N}(p, \alpha)^{12}\text{C} \quad (T_{1/2}(^{15}\text{O} \rightarrow ^{15}\text{N}) \approx 120 \text{sec}).$$

At the temperature as high as $10^9\,\text{°K}$ the proton capturing reactions occur quite rapidly in comparison with the period of the neutron capture processes, about 100 sec. Their reaction times are essentially determined by the lifetimes of the $\beta$-decays whose half-lifetimes are about 66 sec and 120 sec respectively. Therefore, a substantial part of oxygen nuclei are converted

* C. Hayashi, M. Nishida, N. Ohyama and H. Tsuda (Hay 58) have argued that the 4$n$ nuclei belonging to the silicon group are most likely to be produced at the moments of supernova explosions.
into nitrogen and so are nitrogen nuclei into carbon during the period concerned. This suggests that the supernova origin is likely to explain the overabundance of carbon, too.

The large rate of generation in (3.47a, b) might give rise to the anisotropy of cosmic rays on the earth, but this is not the case, provided that the diffusion in the interstellar space is effective.

The density of particles at a point of distance \( R \) from the origin and at time \( t \) is given, when particles of unit density are generated at the origin and at \( t=0 \), by

\[
\rho_0(R, t) = (3/4\pi Lct)^{3/2} \exp(-3R^2/4Lct),
\]

where \( L \) is the transport mean free path in the interstellar space. The density on the earth is obtained as

\[
\rho(R) = \int_0^t S_{\text{gen}}. \rho_0(R, t') \, dt'
\]

with

\[
R = 4 \times 10^{21} \text{ cm} \quad \text{and} \quad t = 3 \times 10^{10} \text{ sec}.
\]

Since \( L \) may not be larger than \( 10^{20} \text{ cm} \), \( 3R^2/4Lct \gg 1 \). This allows us to integrate (3.52) approximately, for the constant rate of generation, as

\[
\rho(R) \approx \left( \frac{3t}{4\pi^2 c L} \right)^{1/2} S_{\text{gen}} \frac{R}{R^2} \exp \left( -\frac{3R^2}{4Lct} \right) \approx 10^{-14} \ \text{e}^{-10^5} \text{cm}^{-3}.
\]

This is negligibly small compared with the density of general cosmic rays, \( 10^{-16} \text{cm}^{-3} \). At the time \( t_m = R^2/2cL \approx 3 \times 10^{13} \text{ sec} \), when the maximum intensity of \( \rho_0(R, t_m) \) is reached, \( \rho(R) \) will be only \( (2.7/4\pi) (S_{\text{gen}}/LcR) \approx 5 \times 10^{-14} \text{cm}^{-3} \), being still smaller by three order than the density of general cosmic rays.

If there occurred the very strong production of cosmic rays at the earliest stage, say as many as \( 10^{51} \) particles, we would take

\[
S_{\text{gen}} = S_0 \ \delta(t).
\]

Then, instead of (3.53), we have

\[
\rho(R) = S_0 \left( \frac{3}{4\pi Lct} \right)^{3/2} \exp \left( -\frac{3R^2}{4Lct} \right) \approx 10^{-11} \ \text{e}^{-10^5} \text{cm}^{-3}.
\]

At \( t_m = R^2/2cL \), \( \rho(R) \) will be only \( \approx 0.08 S_0 R^{-3} \approx 10^{-15} \text{cm}^{-3} \). This is again much smaller than our choice, the contribution from the Crab nebula is much less significant.

Thus a supernova which explodes at such a distance \( R \gg (Lct)^{1/2} \) as the Crab nebula can hardly disturb the isotropy of cosmic rays, provided that the scattering by magnetic clouds is effective.
§4. Cosmic rays associated with the matter ejection

During last two decades of the continuous observations of cosmic rays the large unusual increases in the intensity (Fo 46, Eh 48, El 52) have been observed at least five times, all of which can be identified with association with violent solar flares. This is the most direct evidence for the production of cosmic rays, although their contribution to the whole cosmic rays is regarded as small. The smallness of the contribution is considered as due to the quietness of the sun; more active stars may produce a greater amount of cosmic rays. In this section we investigate how important are such contributions. For this purpose the solar flare effect is discussed first of all as the most informative example.

4.1. Cosmic ray outbursts at solar flares

Here we consider the large flare effects alone, because the evidence for the small flare effects (Fi 54a) may be regarded as not definite yet. Some characteristic data obtained for the past flare effects are compiled in Table 4-1. The time sequence of important events for the latest flare is shown in Table 4-2.

In spite of meager data for the earlier ones and of the poor statistics based on five events, we may draw the following characteristic features of the solar flare effect.

1) All the cosmic ray increases were associated with intensive flares

a) This was found several minutes after the optical outburst, whenever the latter moment was identified definitely.

b) The cosmic ray data are taken from observations at Cheltenham using an ionization chamber which was located in the combined, first and second, impact zone for the first and the fourth flares.

Table 4-1. Data of cosmic ray flare effects

<table>
<thead>
<tr>
<th>Date</th>
<th>2/28/42</th>
<th>3/7/42</th>
<th>7/25/46</th>
<th>11/19/49</th>
<th>2/23/56</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heliographic latitude</td>
<td>14N</td>
<td>not observed</td>
<td>29N</td>
<td>2S</td>
<td>23N</td>
</tr>
<tr>
<td>and longitude of flare</td>
<td>-0</td>
<td>west edge (?)</td>
<td>15E</td>
<td>70W</td>
<td>74W</td>
</tr>
<tr>
<td>SID onset time(a)</td>
<td>1107</td>
<td>0407</td>
<td>1615</td>
<td>1030</td>
<td>0335</td>
</tr>
<tr>
<td>CR onset time(b)</td>
<td>≤1200</td>
<td>≤0500</td>
<td>1630</td>
<td>1045</td>
<td>0448</td>
</tr>
<tr>
<td>CR maximum time(b)</td>
<td>1243</td>
<td>0537</td>
<td>1853</td>
<td>1100</td>
<td>0400</td>
</tr>
<tr>
<td>% increase at maximum(b)</td>
<td>12</td>
<td>15</td>
<td>22</td>
<td>40</td>
<td>85</td>
</tr>
</tbody>
</table>

---

*a) The content of this subsection is based on the discussions with Professors T. Gold and P. Morrison while one of the authors (S.H.) stayed at Cornell University and Massachusetts Institute of Technology in 1956 and 1957. Essential results of our discussions are published in (Co 57).
Table 4-2. Time sequence of important events for the flare on February 23, 1956

<table>
<thead>
<tr>
<th>Time</th>
<th>Event Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0331±1</td>
<td>Onset of a strong optical outburst.</td>
</tr>
<tr>
<td>0332</td>
<td>Onset of the solar radio outburst of 3000 Mc/s.</td>
</tr>
<tr>
<td>0335</td>
<td>Onset of the solar radio outburst of 200 Mc/s—20 Mc/s. a) Onset of SID.</td>
</tr>
<tr>
<td>0342</td>
<td>Maximum of the optical intensity of the flare.</td>
</tr>
<tr>
<td>0342-45</td>
<td>Onset of CR increase in the combined impact zone.</td>
</tr>
<tr>
<td>0347-50</td>
<td>Onset of CR increase in the third impact zone.</td>
</tr>
<tr>
<td>0349-55</td>
<td>Maximum of CR intensity in the combined impact zone.</td>
</tr>
<tr>
<td>0353</td>
<td>Onset of CR increase in the polar region.</td>
</tr>
<tr>
<td>0355-0410</td>
<td>Maximum of CR intensity in the third impact zone.</td>
</tr>
<tr>
<td>0415</td>
<td>End of the optical flare.</td>
</tr>
<tr>
<td>0415-18</td>
<td>Maximum of CR intensity in the polar region.</td>
</tr>
<tr>
<td>0430</td>
<td>Isotropic distribution and gradual decrease of CR intensity.</td>
</tr>
</tbody>
</table>

a) The intensity at 19.6 Mc/s scaled out at 0335-37 and 0343.5-50.

having occurred on the near side of the sun, whilst no increases have been observed for such flares on the far side that would have produced cosmic rays. Moreover, there seems to be a tendency that the flares associated with the cosmic ray increases occurred in the western quadrant, whereas large flares of importance 3 and 3+ were distributed uniformly over heliographic longitudes. These flare effects were apt to take place during the Forbush decreases of cosmic rays.

(2) The onsets of the cosmic ray increases were behind the moments of the optical and radio outbursts and of the onset of the sudden ionospheric disturbance (SID).

(3) The solar cosmic rays in earlier stages come from the direction of the sun, thus forming the impact zones (Lü 57a, 57b). a) In later stages when the intensity decreases the directional distribution becomes nearly isotropic.

(4) The energy spectrum of the solar cosmic rays is so steep that only at the latest flare effect an increase was observed at equatorial stations.

Based on the above facts we shall construct a model of the flare effects. The fact that only the largest flares produce cosmic rays indicates the existence of a threshold; even if particles are accelerated to cosmic ray energies, they cannot break out a magnetic wall, unless the cosmic ray energy density becomes high enough. If a sufficient amount of energy is stored, this may open up the wall to let cosmic rays go out of the solar region. This moment may be identified with the radio outburst of low frequencies, between 0335 and 0337 for the flare effect on February 23, 1956.

a) Two impact zones around 3 hour and 9 hour of local time have been considered as important (Fl 54a), but for a steep energy spectrum these two have been found to form a combined zone and the third impact zone around 20 hour has been noticed as accessible for particles of the lowest possible energies.
These injected particles propagate towards the earth with rather good collimation, as is indicated by (1) together with (3) above. It seems reasonable to assume such a path for cosmic rays that connects the sun to the earth; the path is formed by magnetic fields which may be responsible for the Forbush decrease (Ka 57, Co 57). The magnetic fields should not consist of too many eddies on one hand, in order that the transmission of particles is fast enough to form a fairly sharp front; it should not be too regular on the other hand, in order that lower energy particles suffer more delay (Lü 57b). With such a field the isotropy and the slow decrease in the last phase might be hardly expected.

These contradictory requirements seem to be satisfied by a rather regular magnetic field of tongue shape, allowing some irregularity to scatter particles slightly (Co 57). Such a magnetic field guides particles towards the earth, the particles making helical motions. Since the magnetic lines of force diverge, as one goes far from the sun, most of particles propagate nearly parallel to the lines of force in the neighbourhood of the earth, resulting in the rapid transmission and the existence of the impact zones. A considerable fraction of the transmitted particles are reflected at the end of the tongue and go back towards the sun. In the neighbourhood of the sun the lines of force are squeezed to make another reflector, and consequently particles travel back and forth between these reflectors. This results in the essential isotropy at the earth, allowing the particles to diffuse away from the tongue field.

According to this model, the yield of cosmic rays at the flare may be estimated from the intensity integrated over the first half hour of the increase. For this purpose we refer to the neutron intensities observed on February 23, 1956, at Göttingen (MeB 56). The flux intensity thus estimated is at least $2 \times 10^8$ sec$^{-1}$ cm$^{-2}$ (see App. VIII), if averaged over the initial phase of half an hour. The time integrated intensity is, therefore, obtained as $4 \times 10^9$ cm$^{-2}$ or larger. On account of that the flare concerned was the most violent one in this century, the value for an average flare effect may be taken as

$$\text{Time integrated intensity at the earth}=10^6 - 10^7 \text{ cm}^{-2}.$$ (4.1)

The scale of the magnetic field guiding the cosmic rays can be inferred from the duration of the Forbush decrease during which the cosmic ray outburst occurs and also by taking the fact into account that the flare particles of the western quadrant of the solar surface are accessible. Thus the area through which the flare particles cover may be as large as $10^{26}$ cm$^2$. If this is multiplied by (4.1), we obtain

$$\text{The number of cosmic ray particles per flare}=10^{32} - 10^{33}.$$ (4.2)
Since five flare effects have been observed for last twenty years, and about a quarter of them would have been observable, we estimate

Average rate of production of the solar cosmic rays\(=3 \times (10^{24} - 10^{25})\) sec\(^{-1}\). \hspace{1cm} (4.3)

If each of about \(10^{11}\) stars in the Galaxy were to produce cosmic rays at the above rate, the total rate of cosmic ray production would be of the order of \(10^{48}\) sec\(^{-1}\) which is far smaller than the required rate, \(10^{42}\) sec\(^{-1}\), as was shown in §2. However, there may be those stars which produce cosmic rays more efficiently. Since there seems to be no direct way of estimating the production rate, we are obliged to take such an assumption that the yield of cosmic rays relative to that of ejected matter of rather low energies is the same for any star. The ratio of these two yields will be called the efficiency of cosmic ray production.

Now we estimate the efficiency for the sun. The corpuscular emission from the sun is argued on various bases, particularly on geomagnetic storms. It has been estimated (Mu 54) that

\[
\text{Rate of particle emission} = 10^{33} - 10^{34}\text{ sec}^{-1}. \quad (4.4)
\]

The ratio of (4.3) to (4.4), \(3 \times (10^{-10} - 10^{-8})\), gives us the efficiency which we may fix to be of the order of

\[
\text{Efficiency of cosmic ray production} = 3 \times 10^{-9} \quad (4.5)
\]

for definiteness. This is smaller than the efficiency for the Crab nebula, of the order of \(10^{-6}\).

### 4.2. Cosmic ray production at active stars

It has often been mentioned that such active stars as novae, magnetic variable stars, red giants and supergiants of late type may be important sources of cosmic rays. However, the shortage in information on the properties of these objects prevents one to draw any definite conclusion. Since we are also subject to the same difficulty, we shall make only crude considerations in what follows.

**Novae.** The energy out of a nova, roughly \(10^{45}\) erg, is by a factor of about ten thousands smaller than that of a supernova, while its frequency of occurrence, roughly \(10^{-6}\) sec\(^{-1}\), is by a factor of a thousand larger. Hence, even if the efficiency of cosmic ray production at novae were as large as that at supernovae, the contribution of novae to galactic cosmic rays could not be larger than that of supernovae, the latter alone being probably unable to supply the whole cosmic rays.
**Magnetic variable stars.** The very strong magnetic fields and the overabundances of some particular elements of peculiar A stars suggest the possibility of the acceleration of particles to cosmic ray energy (Fo 55). In order that a sufficient amount of nuclear reactions take place at the surface of such a star due to the energetic particles, their time integrated intensity should be as large as \(10^{25} \text{ cm}^{-2}\), because the reaction cross section is of the order of \(10^{-25} \text{ cm}^2\). If the spot area, in which the active acceleration is supposed to take place, consists in a substantial part of the total surface area, say \(10^{32} \text{ cm}^2\), the yield of the energetic particles per magnetic variable star may be as large as \(10^{47}\). Since there are about \(10^8\) such stars in the Galaxy and each of them has a lifetime not shorter than \(10^{16} \text{ sec}\), the rate of cosmic ray production may be at most \(10^{37} \text{ sec}^{-1}\), this being only slightly larger than that from the whole normal stars.

**Supergiants and red giants.** A supergiant with enormous atmosphere is observed to be ejecting matter with a considerable velocity of about \(10^6 \text{ cm sec}^{-1}\) (De 56). Burbidge *et al.* (Bu 57a) argued that most of the interstellar matter should come from such ejection at supergiants and red giants and estimated the mass of the ejected matter as \(2 \times 10^{10} \text{ M}_\odot\) or about \(3 \times 10^{57}\) nucleons. If we assume the same efficiency as that given by (4.5) for the solar ejection also for the ejection at the supergiants and red giants, although the velocity of the latter ejection is smaller than that of the solar corpuscular streams which cause geomagnetic storms, we obtain

\[
\text{Rate of cosmic ray production at supergiants and red giants} = 10^{42} \text{ sec}^{-1}.
\]

This is in qualitative agreement with the required production rate given in (1.7), if one takes into account that the acceleration efficiency may be greater at giant stars, because the linear scale of the accelerating region is much larger than that of the sun spot. In addition to this argument, a comparison between the relative abundances of interstellar matter and cosmic rays also favours the contribution from the giant stars. For supernovae would supply too few H nuclei in comparison with heavy nuclei to cosmic rays, while the matter ejection could be responsible for the supply of a sufficient amount of H.

Another argument for the above hypothesis will be provided by considering the production of the cosmic light elements, Li, Be and B, due to the spallation caused by energetic particles (Ha 55). If each of \(n\) stars, whose average surface area is \(S\), emits cosmic rays of time integrated intensity \(I\) for the age of the Galaxy, the total number of cosmic rays generated is \(nSI\), which should be \(10^{60}\), according to the rate of production \(10^{43} \text{ sec}^{-1}\). Perhaps more than the above amount of particles do not get out of the star but are absorbed in the stellar atmosphere by nuclear collisions. The pro-
ducts due to the nuclear collisions are then ejected to the interstellar space. Since the cross section for producing the light elements by the spallation processes at the nuclear collisions is about $10^{-25} \text{cm}^2$, as is shown in App. II, the abundance of the light elements thus produced is $10^{-25} I$ relative to that of heavier elements. The ratio of the cosmic abundances of the light to the heavier elements $3 \times 10^{-6}$, requires $I$ to be about $3 \times 10^{19} \text{cm}^{-2}$. For $nSI=10^{10}$ and $n=10^{10}$, taking the large mass of giant stars into account for the latter, we have

$$S=3 \times 10^{20} \text{cm}^{2}. \quad (4.7)$$

This leads to a very large radius, about $10^4 R_0$. Such a large radius is actually observed for the envelope of supergiants (De 56).

The above consideration suggests the following two facts. (i) Cosmic rays should be produced in a rarefied gas, this being in accordance with the argument in App. IV that the dissipation should be small for the acceleration to be efficient. (ii) The generation of the light elements, Li, Be and B, could be explained by the spallation caused by energetic particles, as was inferred by a number of authors (Fo 55, Ha 55) and has recently been termed as the x-process (Bu 57a).

§5. Extragalactic origin

By the extragalactic origin one usually means either of the following two alternatives: (i) most of the observed cosmic rays are originated at extragalactic sources or (ii) most of the observed cosmic rays are produced in our Galaxy, but only most energetic particles are chiefly of extragalactic origin. A number of arguments for and against the above hypotheses based on cosmic rays and astronomy may be summarized in the following.

For the first alternative there seems to have been rather weak arguments: (ia) The average thickness of traversed matter, about $3 \text{gcm}^{-2}$ as shown in §2, roughly corresponds to the straight passage of the universe. (ib) The rotation of nearby galaxies with magnetic fields, the large Magellanic cloud in the case of our Galaxy, can accelerate particles by electromotive forces due to the rotating magnetic fields (He 55).

The absence of the Compton-Getting effect has been regarded as a strong argument against (ia). Moreover, the presence of magnetic fields is hardly incorporated with the straight passage. The idea of (ib) may have some bearing on the origin of cosmic rays, but this origin alone seems difficult to explain the overabundance of the heavy primaries, because no reason can be found for the higher concentration of heavy elements in the intergalactic space than in the extragalactic space. Anyway the evidence on the Crab nebula and that discussed in §4 favour a significant contribution from the
galactic origin, although the extragalactic origin like (ib) could replace the origin associated with the matter ejection. This problem will be answered by a number of methods to examine the hypothesis (ib) suggested by Heidmann, specially the secular variation of the cosmic ray intensity in the period of the rotation (He 55).

The second alternative looks more attractive than the first (Co 58), because (iia) the existence of those high energy particles which are essentially isotropic and produce the largest size of air showers, the number of electrons at sea level being observed more than $10^9$ per shower (Cl 57), and (iib) there are such peculiar nebulae that emit strong radio waves, like M87 and Cygnus A, and presumably produce cosmic rays, too (Bu 56b). The arguments can be further strengthened by the possible statistical acceleration due to moving galaxies, particularly efficient in a dense cluster like the Coma (Bu 57b).

The argument (iia) is based on the fact that a particle of energy higher than $10^{18}$ eV is not sufficiently stirred in the Galaxy, so that we would expect neither isotropy nor a smooth energy spectrum up to this energy in the case of the galactic origin. The energy of such a high energy particle is evaluated from the number of electrons in an extensive air shower observed at sea level. In the latter procedure there arises a question on the effect of fluctuation; the number of electrons depends greatly upon the altitude at which the first nuclear collision occurs to initiate a shower. Conventionally this altitude has been assumed at the top of the atmosphere, but this assumption is correct only if the primary spectrum is steep enough and the attenuation of a shower near sea level is slow enough. Since these are not the cases, the primary energy currently adopted is found as an overestimate. Kraushaar (Kr 57) pointed out that the conventional estimate was too large by a factor of 1.4. According to a more refined analysis made independently by Miyake (Mi 58), an overestimate of factor ten may have come out at energies between $10^{16}$ eV and $10^{17}$ eV and the primary particles of energies above $10^{17}$ eV would be absent. Since the result depends rather sensitively on the values of some parameters, the final conclusion should be reserved after more detailed studies. Nevertheless, this point must be kept in mind with regard to the argument (iia).

Concerning (iib), Burbidge (Bu 56b) estimated the number of relativistic particles in M87 as about $10^{56} - 10^{58}$. Assuming that the mean lifetime of nuclear particles is determined mainly by nuclear collisions, that is, $10^{13} - 10^{15}$ sec$^{-1}$, the rate of cosmic ray production may be as large as $10^{41} - 10^{43}$ sec$^{-1}$. If there are $10^6$ such active galaxies, as can be inferred from radio astronomy, we have in the intergalactic space
Density of cosmic rays from active galaxies
\[ =10^{-18} - 10^{-14} \text{ cm}^{-3}. \]  
(5.1)

Hence the contribution to the general cosmic rays is negligibly small. Even if their average energy is as high as $10^{12}$ eV, their contribution to high energy cosmic rays seems to be of minor fraction. The intensity of directly coming particles from the nearest of those galaxies, M87, is also small, $10^{-11} - 10^{-7} \text{ cm}^{-2} \text{ sec}^{-1}$, provided that the scattering in the intergalactic space is negligible. Although these figures may increase by assuming a larger yield, the contribution could be appreciable only at the highest energy.

In addition to such peculiar ones ordinary galaxies like ours are emitting cosmic rays, provided that the escape out of the galaxy is so important as to be assumed in §2. If there are $10^{8}$ ordinary galaxies, each of which emits cosmic rays at a rate of $10^{43} \text{ sec}^{-1}$, they contribute to the extragalactic cosmic rays by

Density of cosmic rays from ordinary galaxies
\[ =10^{-15} \text{ cm}^{-3}. \]  
(5.2)

The smallness of the extragalactic density compared with the density inside a galaxy indicates that the galactic boundary is highly reflective against cosmic rays, thus being in accordance with the spherical model of the Galaxy adopted in §1. The high reflectivity suggests that only a small fraction of the extragalactic cosmic rays can enter a galaxy. Perhaps the effect of the extragalactic cosmic rays can be seen only for very high energy particles and gamma rays. The observability of such gamma rays has been discussed by Burbidge (Bu 56b) for M87. Other possibilities of observing line gamma rays of low energies, such as those arising from the annihilation of positons and negatons, will be discussed in other occasions.

In concluding this section, we have to add a remark, which may be shared with everybody else, that the information on the extragalactic origin is too meager to make any definite statement.

Acknowledgments

In the course of this work we have received much help from various people, to whom we should like to express many thanks taking this occasion, among them to Professor C. Hayashi for his continuous discussions and helpful comments, to Professor T. Hatanaka for his advice in astrophysical problems, to Dr. G. Burbidge and Professor P. Morrison for their frequent communications and critical comments. We are also grateful to the Mainichi Press by which this work has been partly supported.
S. Hayakawa, K. Ito and Y. Terashima

Appendices

I. Yields of nucleides due to spallation processes

On the bombardment of heavy nuclei with protons having energies in the GeV range, we devide the spallation process into two steps: the nucleon cascade and the evaporation.

We can calculate the nucleon cascade in a nucleus under some simplified assumptions. Then, the mean number of nucleons knocked-on by this process is obtained as a function with respect to the incident proton energy $E_0$ and the radius of nuclei $R$. The nucleons falling below $E_v$ in the nucleus may not contribute to the further cascade process, but are captured by the nucleus and then emitted from its surface as slow evaporated particles. It seems reasonable to assume that $E_v=45$ MeV. The excitation energy $U$, is also calculated as a function with respect to $E_0$ and $R$ by means of the nucleon cascade theory.

The relation between the excitation energy and emitted particles will be discussed according to Fujimoto and Yamaguchi (Fu 50). However, we make such a simplification that a nucleus consists of neutrons alone and retains a constant temperature $T$ through the evaporation process, on account of that the final result is not too sensitive to the value of temperature. The energy distribution of evaporated particles is now approximately given by

$$\frac{(E/T^2)}{\exp(-E/T)}dE,$$

and then the probability that $N_e$ particles are evaporated and the excitation energy fall in the energy region below neutron binding energy $E_b$ is approximated by the Gaussian distribution.

Thus, the probability for evaporating $n_e$ neutrons and $p_e$ protons ($n_e+p_e=N_e$) by the evaporation process will be proportional to

$$\frac{(n_e+p_e)!}{n_e!p_e!}\left(\frac{r}{1+r}\right)^{n_e}\left(\frac{1}{1+r}\right)^{p_e}\exp\left[-\frac{(n_e+p_e)-N_e^2}{a}\right],$$

provided that we assume the binary distribution for $n_e$ and $p_e$, where $r$ is the average ratio of the numbers of neutrons and protons. Here, $N_e$, $T$ and $a$ are given as

$$N_e=(U+3T)/(2T+E_0)\approx U/(2T+E_0),$$

$$T=(10U/A)^{1/2} \quad \text{and} \quad a=4(T/E_0)^2N_e,$$

where $A$ is the mass number of nuclei.

For the fluctuation of particles knocked-on by the cascade process, we shall assume the same distribution as above. Then, the probability for emitt-
ing \(n_e\) neutrons and \(p_e\) protons \((n_e + p_e = N_e)\) by the cascade process will be proportional to

\[
\frac{(n_e + p_e)!}{n_e! p_e!} \left( \frac{N}{A} \right)^{n_e} \left( \frac{Z}{A} \right)^{p_e} \exp \left[ -\frac{(n_e + p_e) - \overline{N}_e}{b} \right],
\]

where \(N\) and \(Z\) are respectively the neutron number and the proton number of a nucleus \((N+Z = A)\), \(b\) is the mean square deviation of \(\overline{N}_e\), and \(\overline{N}_e\) is the average value of \(N_e\).

Thus, it may be possible to conclude that the probability \(P\) for emitting \(n\) neutrons and \(p\) protons \((n + p = \triangle)\) from a nucleus is proportional to the product of \((\triangle \cdot 2)\) and \((\triangle \cdot 6)\). Now, assuming \(N \approx Z = A/2\) and using the Stirling formula, \(P\) is simplified as

\[
P \propto \exp \left\{ -\frac{(\triangle - \overline{\triangle})^2}{2(a + b)} \right\} \exp \left\{ -\frac{\ln r}{2N_e} |(n - p) - d| \right\},
\]

where \(\overline{\triangle}\) is the mean number of \(\triangle\) and \(d\) is a constant depending on \(r\) and \(N_e\). The coefficient of \(|n - p|\) in the exponent, \(-\ln r/2N_e\), which is deduced from the difference between the probabilities for evaporating \(n_e\) or \(p_e\), has a rather small absolute value for \(r = 1.5 - 2.0\) and \(N_e = 3 - 15\). However, since the correlation between the evaporating neutrons and protons seems to be very strong, its value will become larger than the above. Now, if we replace \(-\ln r/2N_e\) by unity and also use an approximate value \(\triangle / \overline{\triangle}\) for \(d\), taking account of the cascade process, the formula (I•7) is reduced to

\[
P \propto \exp \left\{ -\frac{(\triangle - \overline{\triangle})^2}{2(a + b)} \right\} \exp \left\{ -\frac{|(n - p) - \triangle|}{\triangle} \right\}.
\]

The cross sections for the production of nuclides by the bombardment of copper with protons in the GeV range (Fr. 54) are in good agreement with our results predicted by the formula (I•8).

II. Fragmentation probabilities

We give the values of the fragmentation probabilities which are defined in §2.1. \(P_{Fe}, P_{FeH}, P_{n}\) and \(P_{HL}\) are obtained by extrapolating the previous result of the spallation to lighter nuclei (App. I). For \(P_{n}, P_{HM}\) and \(P_{HL}\), we take their relative abundances into account since rich elements are \(^{24}\text{Mg}\), \(^{28}\text{Si}\) and \(^{32}\text{S}\) among heavy nuclei. This consideration leads us to fragmentation probabilities of a smaller value for \(P_{n}\), and larger ones for \(P_{HM}\) and \(P_{HL}\) than our previous estimates (Ha 56a). \(P_{y}\) and \(P_{yL}\) are deduced from the following experimental and theoretical information. The cross section for the \(^{12}\text{C}(p, pn and pd)^{11}\text{C}\) reactions is observed as about 20% of the geometrical one at energies of our interest (Ki 57). The cross sections
producing $^9$H from N and O are measured to be about 10% of the geometrical cross sections (Fi 55a). Theoretically, the cross sections for $(p, pn)$, $(p, d)$ and $(p, p\alpha)$ reactions of medium nuclei are estimated as about 30% of the geometrical cross sections except for Ne$+p$ reactions, for which one may expect a considerable yield of $p+\alpha$, about 80% of the geometrical one, because $^{20}$Ne easily turns into a stable nucleus $^{16}$O. The energy dependence of these reactions seems to be weak. On the other hand, the rate of complete break-up processes is estimated as large as 50% of the total reaction cross section for $^{12}$C$+p$ reactions, taking account of the reaction $^8$Be$\rightarrow 2\alpha$. For other medium nuclei 20% - 30% of the geometrical cross sections may be appropriate for the complete break-up processes. Other partial break-up processes would mainly contribute to $P_{M, L}$.

From the above arguments we thus get $P_M$ and $P_{M, L}$, by taking an average over the relative abundances of medium nuclei.

<table>
<thead>
<tr>
<th>secondary ($j$)</th>
<th>Fe</th>
<th>H</th>
<th>M</th>
<th>L</th>
<th>$\alpha$</th>
<th>$P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>primary ($i$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Fe$</td>
<td>0.10-0.20</td>
<td>0.80-0.90</td>
<td>$\sim$0</td>
<td>$\sim$0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$H$</td>
<td>0</td>
<td>0.10-0.30</td>
<td>0.45-0.55</td>
<td>0.30-0.40</td>
<td>1.0-2.0</td>
<td>3.0-5.0</td>
</tr>
<tr>
<td>$M$</td>
<td>0</td>
<td>0</td>
<td>0.10-0.20</td>
<td>0.30-0.40</td>
<td>1.0-2.0</td>
<td>3.0-4.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\sim$4.0</td>
</tr>
</tbody>
</table>

### III. Radioactive energy source in Type I supernovae

It was suggested by Burbidge et al. (Bu 56a) that the exponential form of the light curves with a half-life of 55 nights of Type I supernovae is due to the spontaneous decay of $^{254}$Cf. A recent investigation (Hu 57) has found the spontaneous fission half-life of $^{254}$Cf to be $56.2 \pm 0.7$ days, which might be regarded as in good agreement with the above interpretation. Burbidge et al. (Bu 57a) discussed the way in which $^{254}$Cf may be synthesized in supernova outbursts.

Transuranic elements are synthesized in the so-called rapid process, a rapid neutron capture process which takes place in the outbursts of Type I supernovae. In detailed discussions of the process, we consider two alternative cases. Case I; the cycling between $A=110$ and $A=260$ takes place in the course of the syntheses, because the fission fragments of the heaviest elements supply elements in the region $A=110$ to $A=150$, and in the extreme case of the complete cycling most of materials, being subject to the rapid process, are converted to heavier elements of $A \geq 110$. Case II; the
synthesizing process gives rise to the element abundances similar to these in the solar system.

When one percent of the solar mass is converted in this way the total radioactive energy is estimated, in respective cases, as

\[
\begin{align*}
1.43 \times 10^{48} \text{erg} & \quad \text{for Case I,} \\
4.9 \times 10^{47} \text{erg} & \quad \text{for Case II. (III·1)}
\end{align*}
\]

The rate of radioactive energy release is shown in Fig. III-1 for respective cases. At the explosion $^{254}\text{Cf}$ gives a main contribution, $7/8$ of the total energy release for case I and about a half of that for case II. Hence the curve of the energy release of case I is of the same form as the light curve of supernovae, and that of case II deviates a little from this. We prefer case I, which also gives the reasonable mass of a supernova, as will be shown in the following.

![Graph showing the radioactive energy release over time for Case I and Case II.](image-url)
Shortly after the explosion the fission energy of $^{254}$Cf is estimated as

$$0.693 T_{1/2} Q_g f \frac{M}{254 M} = 2.3 \times 10^{45} \text{ erg day}^{-1} \text{ for Case I},$$

$$= 4.17 \times 10^{44} \text{ erg day}^{-1} \text{ for Case II}, \quad (III \cdot 2)$$

where $Q$ is the energy release per $^{254}$Cf nucleus (220 MeV), $T_{1/2}$ its half-life (56 days), $f$ the fraction of the mass converted and $g$ the formation ratio of $^{254}$Cf, the values of $f$ and $g$ being also estimated by Burbidge et al. (Bu 57a) $f = 0.01$, and $g = 0.0113$ for case I and $g = 0.002$ for case II, respectively.

The rate of the total energy release is thus found to be

$$W_t = 2.36 \times (8/7) \times 10^{45} = 2.7 \times 10^{45} \text{ erg day}^{-1} \text{ for Case I},$$

$$= 4.17 \times 2.1 \times 10^{44} = 8.7 \times 10^{44} \text{ erg day}^{-1} \text{ for Case II}. \quad (III \cdot 3)$$

On the other hand the total energy emitted under the exponential decline of the light curve is given by

$$E_t = 3.8 \times 10^{33} \times 10^{0.4 (1.6-M_a-0.75) \times 8.64} \times 10^4 \text{ erg day}^{-1}, \quad (III \cdot 4)$$

where $M_a$ is the absolute magnitude at the maximum luminosity of a supernova and $\delta M_a$ is the difference between the maximum absolute magnitude at the onset of the exponential form.

Comparing $E_t$ with $W_t$, we can estimate the mass of a supernova by the relation

$$E_t = s W_t, \quad (III \cdot 5)$$

where $s$ is the mass of a supernova in units of the solar mass, $s = \frac{M}{M_\odot}$.

For the Crab, in which $M_a$ is possibly in the region from $-16$ to $-17$ and $\delta M_a$ is likely to be taken as 2.5, we find

$$s = 2.3 - 5.9 \quad \text{for Case I},$$

$$s = 6.5 - 16.5 \quad \text{for Case II}. \quad (III \cdot 6)$$

Thus Case I gives a reasonable value of the mass, but Case II too large a mass value.

The theory developed by Burbidge et al. is a tentative one and certain modifications may be necessary in some cases. As pointed out by Huizenga and Diamond (Hu 57), the fact that the light curve of the supernova IC 4182 keeps an exponential decline between the period 100 to 640 days after the maximum implies that the ratio $^{254}$Cf/$^{252}$Cf is greater than 10, while the theory gives the ratio $\sim 1.4$($^{252}$Cf is the main source of energy next to $^{254}$Cf).

In old remnants of Type I supernovae, e.g., the Crab nebula, the spon-
taneous decay of $^{250}$Cm and $^{251}$Cf give the largest contribution to the energy supply. In Case II $^{32}$S and $^{39}$A besides $^{250}$Cm and $^{251}$Cf are important sources of energy.

We show in the following tables the elements which give main contributions for energy supply at various stages of the supernova after explosion, taking account of the recent determination of half-lives of $^{254}$Cf and $^{250}$Cm (Hu 57).

<table>
<thead>
<tr>
<th>$A$</th>
<th>Element</th>
<th>Decay Mode</th>
<th>$T_{1/2}(d)$</th>
<th>$Q$</th>
<th>$n_0$</th>
<th>$\xi_i$</th>
<th>$\xi$ (365 days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>33</td>
<td>P</td>
<td>$\beta$</td>
<td>25</td>
<td>0.12</td>
<td>50</td>
<td>0.165</td>
<td>---</td>
</tr>
<tr>
<td>35</td>
<td>S</td>
<td>$\beta$</td>
<td>87</td>
<td>0.08</td>
<td>50</td>
<td>2.99x10^{-2}</td>
<td>1.63x10^{4}</td>
</tr>
<tr>
<td>45</td>
<td>Ca</td>
<td>$\beta$</td>
<td>160</td>
<td>0.11</td>
<td>1</td>
<td>4.8x10^{-4}</td>
<td>9.89x10^{4}</td>
</tr>
<tr>
<td>47</td>
<td>Ca</td>
<td>$\beta$</td>
<td>4.7</td>
<td>1.83</td>
<td>100</td>
<td>26.9</td>
<td>---</td>
</tr>
<tr>
<td>59</td>
<td>Fe</td>
<td>$\beta$</td>
<td>45</td>
<td>1.36</td>
<td>30</td>
<td>0.628</td>
<td>2.27x10^{4}</td>
</tr>
<tr>
<td>89</td>
<td>Sr</td>
<td>$\beta$</td>
<td>54</td>
<td>0.66</td>
<td>3.4</td>
<td>2.87x10^{-2}</td>
<td>2.64x10^{4}</td>
</tr>
<tr>
<td>91</td>
<td>Y</td>
<td>$\beta$</td>
<td>58</td>
<td>0.70</td>
<td>2.2</td>
<td>1.83x10^{-3}</td>
<td>2.34x10^{4}</td>
</tr>
<tr>
<td>95</td>
<td>Zr</td>
<td>$\beta$</td>
<td>65</td>
<td>1.73</td>
<td>2.2</td>
<td>4.08x10^{-2}</td>
<td>8.32x10^{4}</td>
</tr>
<tr>
<td>131</td>
<td>I</td>
<td>$\beta$</td>
<td>8.05</td>
<td>0.50</td>
<td>1.73</td>
<td>7.32x10^{-2}</td>
<td>---</td>
</tr>
<tr>
<td>140</td>
<td>Ba</td>
<td>$\beta$</td>
<td>12.8</td>
<td>2.4</td>
<td>0.45</td>
<td>5.8x10^{-2}</td>
<td>---</td>
</tr>
<tr>
<td>144</td>
<td>Ce</td>
<td>$\beta$</td>
<td>285</td>
<td>1.5</td>
<td>0.45</td>
<td>1.6x10^{-3}</td>
<td>6.58x10^{4}</td>
</tr>
<tr>
<td>182</td>
<td>Ta</td>
<td>$\beta$</td>
<td>112</td>
<td>0.8</td>
<td>0.057</td>
<td>2.85x10^{-4}</td>
<td>2.96x10^{4}</td>
</tr>
<tr>
<td>188</td>
<td>W</td>
<td>$\beta$</td>
<td>65</td>
<td>1.0</td>
<td>0.09</td>
<td>9.6x10^{-4}</td>
<td>1.96x10^{4}</td>
</tr>
<tr>
<td>225</td>
<td>Ra</td>
<td>$\alpha+\beta$</td>
<td>14.8</td>
<td>27.9</td>
<td>0.045</td>
<td>5.9x10^{-2}</td>
<td>---</td>
</tr>
<tr>
<td>254</td>
<td>Cf</td>
<td>SF(a)</td>
<td>56.2</td>
<td>220</td>
<td>0.139</td>
<td>0.377</td>
<td>3.80x10^{4}</td>
</tr>
<tr>
<td>257</td>
<td>Fm</td>
<td>$\alpha$, SF(6%)</td>
<td>$\sim$10</td>
<td>18.5</td>
<td>0.091</td>
<td>0.116</td>
<td>---</td>
</tr>
<tr>
<td>(252)</td>
<td>Cf</td>
<td>$2\gamma$</td>
<td>2.2</td>
<td>12.9</td>
<td>0.097</td>
<td>1.1x10^{-2}</td>
<td>4.72x10^{5}</td>
</tr>
</tbody>
</table>

Table III-2. Radioactive decays at the age of 10 years

<table>
<thead>
<tr>
<th>$A$</th>
<th>Element</th>
<th>Decay Mode</th>
<th>$T_{1/2}(y)$</th>
<th>$Q$</th>
<th>$n_0$</th>
<th>$\xi_i$</th>
<th>$\xi$ (10 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>Ni</td>
<td>$\beta$</td>
<td>80</td>
<td>$\sim$0.03</td>
<td>10</td>
<td>7.0x10^{-6}</td>
<td>6.42x10^{4}</td>
</tr>
<tr>
<td>85</td>
<td>Kr</td>
<td>$\beta$</td>
<td>10.4</td>
<td>0.31</td>
<td>3.4</td>
<td>1.9x10^{-4}</td>
<td>9.75x10^{4}</td>
</tr>
<tr>
<td>90</td>
<td>Sr</td>
<td>$\beta$</td>
<td>28</td>
<td>1.26</td>
<td>2.2</td>
<td>1.9x10^{-4}</td>
<td>1.48x10^{4}</td>
</tr>
<tr>
<td>137</td>
<td>Cs</td>
<td>$\beta$</td>
<td>30</td>
<td>0.55</td>
<td>0.45</td>
<td>1.6x10^{-5}</td>
<td>1.27x10^{4}</td>
</tr>
<tr>
<td>194</td>
<td>Os</td>
<td>$\beta$</td>
<td>$\sim$2</td>
<td>1.01</td>
<td>0.88</td>
<td>8.4x10^{-4}</td>
<td>2.61x10^{4}</td>
</tr>
<tr>
<td>210</td>
<td>Pb</td>
<td>$\alpha+\beta$</td>
<td>20</td>
<td>5.8</td>
<td>0.17</td>
<td>9.36x10^{-5}</td>
<td>6.62x10^{4}</td>
</tr>
<tr>
<td>227</td>
<td>Ac</td>
<td>$\alpha+\beta$</td>
<td>22</td>
<td>34.8</td>
<td>0.032</td>
<td>9.57x10^{-5}</td>
<td>7.0x10^{4}</td>
</tr>
<tr>
<td>228</td>
<td>Ra</td>
<td>$\alpha+\beta$</td>
<td>6.7</td>
<td>35.2</td>
<td>0.032</td>
<td>3.2x10^{-4}</td>
<td>1.14x10^{4}</td>
</tr>
<tr>
<td>252</td>
<td>Cf</td>
<td>$\alpha$, SF(6%)</td>
<td>2.2</td>
<td>12.9</td>
<td>0.097</td>
<td>1.1x10^{-2}</td>
<td>4.72x10^{5}</td>
</tr>
</tbody>
</table>
Table III-3. Radioactive decays at the age of 900 years

<table>
<thead>
<tr>
<th>$A$</th>
<th>Element</th>
<th>Decay Mode</th>
<th>$T_{1/2}$ (y)</th>
<th>$Q$</th>
<th>$n_0$</th>
<th>$\varepsilon_i$</th>
<th>$\varepsilon$ (900 years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>Si</td>
<td>$\beta$</td>
<td>300</td>
<td>0.82</td>
<td>50</td>
<td>$2.6 \times 10^{-4}$</td>
<td>$3.25 \times 10^{-5}$</td>
</tr>
<tr>
<td>39</td>
<td>A</td>
<td>$\beta$</td>
<td>260</td>
<td>0.22</td>
<td>50</td>
<td>$9.3 \times 10^{-3}$</td>
<td>$8.44 \times 10^{-4}$</td>
</tr>
<tr>
<td>226</td>
<td>Ra</td>
<td>$\alpha+\beta$</td>
<td>$1.62 \times 10^3$</td>
<td>31.6</td>
<td>0.045</td>
<td>$1.66 \times 10^{-6}$</td>
<td>$1.13 \times 10^{-6}$</td>
</tr>
<tr>
<td>229</td>
<td>Th</td>
<td>$\alpha+\beta$</td>
<td>$7.3 \times 10^3$</td>
<td>33.8</td>
<td>0.032</td>
<td>$2.8 \times 10^{-7}$</td>
<td>$2.57 \times 10^{-7}$</td>
</tr>
<tr>
<td>230</td>
<td>Th</td>
<td>$\alpha+\beta$</td>
<td>$8 \times 10^4$</td>
<td>36.4</td>
<td>0.123</td>
<td>$1.06 \times 10^{-7}$</td>
<td>$1.06 \times 10^{-7}$</td>
</tr>
<tr>
<td>231</td>
<td>Pa</td>
<td>$\alpha+\beta$</td>
<td>$3.4 \times 10^4$</td>
<td>39.9</td>
<td>0.127</td>
<td>$2.8 \times 10^{-7}$</td>
<td>$2.7 \times 10^{-7}$</td>
</tr>
<tr>
<td>239</td>
<td>Pu</td>
<td>$\alpha$</td>
<td>$2.43 \times 10^4$</td>
<td>5.2</td>
<td>0.267</td>
<td>$1.08 \times 10^{-7}$</td>
<td>$1.05 \times 10^{-7}$</td>
</tr>
<tr>
<td>240</td>
<td>Pu</td>
<td>$\alpha$</td>
<td>$6.6 \times 10^4$</td>
<td>5.3</td>
<td>0.110</td>
<td>$1.68 \times 10^{-7}$</td>
<td>$1.53 \times 10^{-7}$</td>
</tr>
<tr>
<td>241</td>
<td>Am</td>
<td>$\alpha$</td>
<td>470</td>
<td>5.6</td>
<td>0.221</td>
<td>$5.01 \times 10^{-6}$</td>
<td>$1.32 \times 10^{-6}$</td>
</tr>
<tr>
<td>243</td>
<td>Am</td>
<td>$\alpha+\beta$</td>
<td>$7.6 \times 10^3$</td>
<td>5.4</td>
<td>0.157</td>
<td>$2.1 \times 10^{-7}$</td>
<td>$1.93 \times 10^{-7}$</td>
</tr>
<tr>
<td>245</td>
<td>Cm</td>
<td>$\alpha+\beta$</td>
<td>$1.1 \times 10^4$</td>
<td>11.4</td>
<td>0.430</td>
<td>$8.43 \times 10^{-7}$</td>
<td>$7.95 \times 10^{-7}$</td>
</tr>
<tr>
<td>246</td>
<td>Cm</td>
<td>$\alpha$</td>
<td>$4 \times 10^4$</td>
<td>5.5</td>
<td>0.103</td>
<td>$2.69 \times 10^{-7}$</td>
<td>$2.30 \times 10^{-7}$</td>
</tr>
<tr>
<td>249</td>
<td>Cf</td>
<td>$\alpha$</td>
<td>470</td>
<td>6.3</td>
<td>0.139</td>
<td>$3.54 \times 10^{-6}$</td>
<td>$9.35 \times 10^{-7}$</td>
</tr>
<tr>
<td>250</td>
<td>Cm</td>
<td>$\alpha$, SF(75%)</td>
<td>$2.8 \times 10^4$</td>
<td>170</td>
<td>0.139</td>
<td>$1.60 \times 10^{-6}$</td>
<td>$1.57 \times 10^{-6}$</td>
</tr>
<tr>
<td>251</td>
<td>Cf</td>
<td>$\alpha$</td>
<td>700</td>
<td>6.2</td>
<td>0.236</td>
<td>$3.96 \times 10^{-6}$</td>
<td>$1.62 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

The initial abundance $n_0$ of each element is a relative ratio to $\text{Si}=10^6$, $Q$ decay energy in MeV, $T_{1/2}$ half-life in days (d) or years (y), and the initial rate of energy release $\varepsilon_i=0.693n_0QT_{1/2}^{-1}$ and the rate of energy release at time $t$, $\varepsilon(t)$, are calculated in MeV day$^{-1}$.

The total mass units and the total energy release of the elements converted in the rapid process are in this scale.

for Case I, \[ \sum n_0 A = 3.15 \times 10^3 \text{ mass units, } \]
\[ \sum n_0 Q = 234 \text{ MeV, } \]

for Case II, \[ \sum n_0 A = 2.1 \times 10^4 \text{ mass units, } \]
\[ \sum n_0 Q = 534 \text{ MeV. } \]

IV. Dynamics of supernova explosion

The course of evolution of stars that leads to supernova outbursts has been discussed by Burbidge et al. (Bu 57a). We consider a star with mass greater than a certain limit. The massive star will shrink due to too weak mechanical support and its internal temperature rises so high that the so-called $e$-process takes place, by which nuclei around the iron peak may be formed. According to Burbidge et al. the values of the density $\rho$ and the temperature $T$ for the $e$-process are estimated respectively as $\rho \approx 10^5$ gcm$^{-3}$ and $T \approx 4 \times 10^9$ °K. On further shrinking the temperature in the central
region will exceed $7 \times 10^9$ °K, and then the dissociation of Fe to He brings about the implosion of the central region. By the central region we mean the inner part of the star containing some fraction of mass greater than the Schönberg-Chandrasekhar limit. (Sc 42). At the implosion the gravitational energy is released and the envelope will be subject to sudden heating. Since all nuclear fuel has not been consumed in the envelope, some elements can generate a large amount of energy. A large flux of neutrons is also produced and gives rise to the rapid process, which is explained briefly in App. III.

In this stage the fast conversion of the thermonuclear energy into the kinetic energy results in heating of the inner part of the star to about $10^9$ °K, and the explosion will start. The rapid process will proceed to $T \approx 10^9$ °K and $\rho \approx 10^3$ gcm$^{-3}$.

For a schematic model of a supernova at the beginning of the explosive expansion we tentatively take such a gas sphere with temperature $T_1 \approx 10^9$ °K and matter density $\rho_{m1} \approx 10^9$ gcm$^{-3}$ that is surrounded by the outer layer with lower temperature $T_0 \approx 10^4$ °K and density $\rho_0 \approx (10 - 10^2)$ gcm$^{-3}$. We assign the mass of the core as $M \approx 1.5 M_\odot$, the corresponding radius being $r \approx 10^{10}$ cm, and that of the envelope as $M \approx (0.5 - 1.0) M_\odot$.

The equation of state for the adiabatic expansion of the gas sphere described above is

$$\frac{d}{dr^3} (\rho r^2) = -P_t,$$  \hspace{1cm} (IV.1)

$\rho_t$ is the sum of the matter density $\rho_m$ and the radiational mass density $\rho_r$. $P_t$ is the sum of the matter pressure $P_m$ and the radiation pressure $P_r$, which are defined by

$$P_r = \frac{1}{3} \rho r c^2, \quad \rho r = \frac{a T^4}{c^2} \quad \text{and} \quad P_m = (\rho_m/M_m)RT,$$ \hspace{1cm} (IV.2)

where $a$ is the radiation density constant, $k$ the Boltzmann constant, and $M_m$ the mean mass of gas particles.

Since the present model with $T \approx 10^9$ °K and $\rho_m \approx 10^9$ gcm$^{-3}$ satisfies the conditions

$$\rho_m \gg \rho_r \quad \text{and} \quad P_m \ll P_r \quad \text{or} \quad 10^{-93} T^3 \gg \rho_m \gg 10^{-35} T^4,$$ \hspace{1cm} (IV.3)

we confine ourselves to this limiting case, for which the equation (IV.1) is easily solved (Hay 56). Taking account of $\rho r^3 \approx \text{const.}$ and $P_t \approx P_r$, the equation (IV.1) with (IV.2) is reduced to

$$\frac{d}{dr^3} (\rho r^2) = \frac{d}{dr^3} (\rho r^2) = -\frac{1}{3} \rho r.$$
Hence we have
\[ \rho \propto r^{-4} \quad \text{and} \quad T \propto r^{-1}. \] (IV.4a)

As the kinetic energy exceeds the potential energy at the surface of the central region,

\[ K.E. \approx 10^{17} \text{ erg g}^{-1} \gg P.E. \approx 2 \times 10^{16} \text{ erg g}^{-1}, \]

and then the linear expansion of the system is permissible, or

\[ r \propto t \quad \text{and} \quad \rho_m \propto t^{-3}. \] (IV.4b)

We can apply these formulae, so far as the system is in radiative equilibrium. This is no longer valid when the absorption mean free path of radiation becomes greater than the dimension of the system. The former is defined by

\[ l_r = 1/n_e \sigma_{th}, \quad \sigma_{th} = 6.65 \times 10^{-25} \text{ cm}^2, \] (IV.5)

where \( \sigma_{th} \) is the cross section for the Thomson scattering and \( n_e \) the number density of electrons.

As the model proposed has the total mass \( M \approx 2.0 M_\odot \), that is, \( 2.5 \times 10^{57} \) nucleons, \( n_e \) is

\[ n_e = \alpha_e (2.5 \times 10^{57}) / \left( \frac{4\pi}{3} r^3 \right), \]

where \( \alpha_e \) is the fraction of free electrons to nucleons, being assumed as \( \alpha_e = 1 \). Therefore the critical dimension \( r_c = l_r \) is evaluated as

\[ r_c = 2 \times 10^{10} \text{ cm}. \] (IV.6)

The corresponding mass density is

\[ \rho_c = 10^{-15} \text{ g cm}^{-3}. \] (IV.7)

The time necessary for reaching \( r_c \) can be obtained simply by dividing \( r_c \) by the expanding velocity. The latter may be identified with the shock velocity associated with the explosion.

The shock wave may develop at the front of the expanding gas due to pressure gradient. Its velocity is deduced from the Rankine-Hugoniot relation as

\[ V_s = V_0 \sqrt{\gamma}, \] (IV.8)

where \( V_0 \) is the sound velocity in the medium into which the shock wave proceeds, and \( \gamma \) is the shock strength, the ratio of the pressure behind the shock front to the pressure in the medium,
\[ y = P_i / P_0. \]

\( V_0 \) and \( P_0 \) are related to the kinetic temperature and the number density in the medium, \( T_0 \) and \( n_0 \), respectively as

\[ P_0 = n_0 k T_0 \quad \text{and} \quad V_0 = \sqrt{\gammaRT_0}, \]

where \( R \) is the gas constant and \( \gamma \) is the ratio of specific heats.

In the envelope of our model it is plausible that \( T_0 \approx 10^4 \, ^\circ\text{K} \) and \( \rho_0 \approx (10 - 10^3) \, \text{g cm}^{-3} \) or \( n_0 = 10^{25} - 10^{26} \), hence

\[ V_0 \approx 10^8 \, \text{cm sec}^{-1}. \]

Since \( P_i \) is due almost exclusively to the radiation pressure given in (IV·2), we have

\[ y \approx 2 \times 10^6 - 2 \times 10^7. \]

Finally we find

\[ V_i \approx \sqrt{\gamma} V_0 \approx (1 - 3) \times 10^9 \, \text{cm sec}^{-1}. \quad \text{(IV·9)} \]

As the expansion goes on, the temperature of the inner part decreases as known from relation (IV·4a), and the velocity of the shock wave decreases. But there is a reason for that the shock velocity is maintained for a rather long period. Firstly, the nuclear energy supply inside the inner region is still possible and this prevents the temperature from falling.

Secondly, a decrease in the density of the outer region tends to maintain the shock velocity because the shock velocity is proportional to the inverse square-root of the density of the outer region. Therefore, the expansion velocity of \( 10^8 - 10^9 \, \text{cm sec}^{-1} \) would be easily attained. Assuming that \( V_i \) in (IV·9) is only slightly decreased, we have the expansion time to

\[ t_e \approx 10^{16} / 10^8 - 10^9 = 10^7 - 10^8 \, \text{sec}. \quad \text{(IV·10)} \]

Hence we may expect that in one year or so the supernova arrives at the critical point, at which it begins to deviate from the radiative equilibrium.

Once the expansion velocity of about \( 10^8 \, \text{cm sec}^{-1} \) is attained at \( t \approx t_e \), then almost all the internal energy of the system is converted into the energy of mass motion. The expansion velocity will maintain this value, so far as the retardation by the resistance of the interstellar medium is insignificant. Let us estimate the time when the latter effect becomes important (Am 52).

As the system expands into the interstellar medium with mean density \( \rho' \), the mass of the expanding nebula will increase by sweeping up the gas particles of the medium. When the radius of the nebula is equal to \( r \), its
mass will increase from the initial value \( M \) to
\[
\frac{4}{3} \pi r^3 \rho' + M.
\]

The degraded velocity \( V \) is simply obtained by the law of conservation of momentum as
\[
\left( \frac{4}{3} \pi r^3 \rho' + M \right) V = M V_0, \tag{IV.11}
\]
where \( V_0 \) is the expansion velocity of the nebula at the moment of the explosion, say, at \( t = t_\alpha \). Substituting \( dr/dt \) in place of \( V \) and integrating (IV.11), we find
\[
\frac{1}{3} \pi r^3 \rho' + M r = M V_0 t. \tag{IV.12}
\]
This relation defines the radius \( r \) as a function of time \( t \).

We seek for example the time when the expansion velocity is reduced by a factor two. As is known from (IV.11) \( V \) will be equal to \( V_0/2 \) when
\[
\frac{4}{3} \pi r^3 \rho' = M.
\]
Substituting the last relation in (IV.12), we get the time sought as
\[
t_f = \frac{5}{4} \frac{V_0}{\sqrt{\frac{3M}{4\pi \rho'}}}. \tag{IV.13}
\]
For \( M \approx 2M_\odot \), \( \rho' \approx 10^{-24} \text{g/cm}^3 \), and \( V_0 = 10^8 \text{ cm/sec} \), we have
\[
t_f \approx 3 \times 10^3 \text{ year.} \tag{IV.13'}
\]
It should be noted that the density of the nebula at \( t = t_f \) is only twice that of the interstellar medium.

Again we consider the supernova within the time limit \( t_\alpha \) defined in (IV.10). Since the radiative equilibrium is maintained, thermal radiation is expected. The source of the radiation is regarded as the radioactive energy carried by \( \beta \)-rays, \( \alpha \)-rays, fission fragments, neutrons and \( r \)-rays. The former three are quickly absorbed by ionization, transferring their energy to the kinetic energy of particles. Neutrons are slowed down also quickly by collisions with protons and are absorbed eventually by producing \( r \)-rays. These and radioactive \( r \)-rays lose their energies by the Compton scattering. Since their mean free path is given approximately by (IV.5), substantial part of their energy contributes to the thermal one.
Since we have seen that most of the radioactive energy is converted to the thermal energy, we shall examine how much part of this energy will escape to outer space. The conductive flow of heat caused by the temperature gradient $\nabla T$ is given in the presence of the magnetic field of strength $H$ by

$$ Q = (1 + \omega_n^2 t_i^2)^{-1} \kappa \nabla T, $$

where $\omega_n = eH/m_e c$ is the cyclotron frequency of an electron and $t_i$ is the collision mean free time of electrons (Sp. 56). For the medium of number density $n$ this is given by

$$ t_i = 1.5 T^{n/2} n^{-1} \text{sec}. $$

$\kappa$ is the heat conductivity given by

$$ \kappa \simeq 3 \times 10^{-5} T^{5/2} \text{erg deg}^{-1} \text{cm}^{-1} \text{sec}^{-1}. $$

The temperature gradient may be estimated as

$$ \nabla T \simeq T/\langle v \rangle t_i = (T/t_i) \sqrt{m_e/3kT} \simeq 1.0 \times 10^{-8} n T^{-1} \text{deg cm}^{-1}. $$

Since $\omega_n t_i \gg 1$ in our case, we have

$$ Q \simeq (\omega_n t_i)^{-2} \kappa \nabla T \simeq 4 \times 10^{-28} n^3 H^{-2} T^{-3/2} \text{erg cm}^{-2} \text{sec}^{-1}. $$

This is compared with the energy transport due to the shock

$$ Q' = n k T V \simeq 1.4 \times 10^{-18} n V T \text{erg cm}^{-2} \text{sec}^{-1}. $$

For $T = 10^4 \degree \text{K}$, $n = 10 \text{cm}^{-3}$, $H = 10^{-3}$ gauss and $V = 10^8 \text{cm sec}^{-1}$, their ratio is

$$ Q/Q' \simeq 10^{-4}. $$

As this is an underestimate of $Q/Q'$, the heat conductivity is also negligible. The above considerations guarantee the validity of our treatment of the expansion up to the critical time $t_c$ given in (IV·10).

This situation is favourable to the Californium hypothesis, because the visible luminosity should be roughly proportional to the total radioactive energy for $t < t_c$. For $t > t_c$, however, the luminosity begins to deviate from the radioactive energy, because considerable part of energy is emitted in the ultraviolet region or in the X-ray region. In fact, the light curve of a supernova is found to deviate from the decay curve of whole radioactive substances after a year from its explosion, when the contribution from $^{252}\text{Cf}$ becomes as important as that from $^{254}\text{Cf}$. In order to examine this interpretation, the observation of neutron capture X-rays, mainly from $\text{p(n,r)\alpha}$, would be suggestive for a future supernova. Such $\gamma$-rays could be observed, if a supernova exploded in our vicinity, as close as the Crab.
5. Mechanisms of acceleration

Various mechanisms of the acceleration of cosmic rays have thus far been proposed, and most of them are related to the acceleration principles of accelerators, such as the betatron and the linear accelerator. Such proposals have always met an objection that the conditions required for acceleration are hardly realized under conceivable celestial circumstances; indeed, every one meets difficulties in constructing accelerators even in an artificially controlled way. However, there is one advantage in the celestial case in such a respect that the time scale for the acceleration is so long that a cumulative effect may turn out to be significant, as was emphasized by Fermi (Fe 49).

Suppose that a particle suffers a small amount of energy change $\Delta E$ in a time interval $\Delta t$ and the magnitude of $\Delta E$ is uncorrelated with that in the preceding time interval. Then the variation of the energy spectrum $N(E, t)$ against time $t$ can be described by the Fokker-Planck equation as

$$
\frac{\partial N}{\partial t} \Delta t = -\frac{\partial}{\partial E} (\langle \Delta E \rangle N) + \frac{1}{2} \frac{\partial^2}{\partial E^2} (\langle \Delta E^2 \rangle N) - \frac{N}{T} \Delta t + Q(E, t) \Delta t. 
$$

(V·1)

The first two terms in the right-hand side represent the main terms in the Fokker-Planck equation, in which $\langle \rangle$ means the average over many independent occasions of the energy changes. The second term was discarded in Fermi’s original work, but its importance was noticed by Kaplan (Ka 55) and Davis (Da 56). The third and the fourth terms in the right-hand side are added to take into account the loss of particles with a mean lifetime $T$ and the injection of particles at a rate of $Q(E, t)$, respectively. Since the average values of $\Delta E$ and $\Delta E^2$ appear in (V·1), this is called the statistical acceleration. The average values can be evaluated with the aid of the probability distribution $w(\tau) d\tau$, where $\tau$ represents variables on which $\Delta E$ depends, as

$$
\langle \Delta E^n \rangle = \int \Delta E^n w(\tau) d\tau, 
$$

(V·2)

where $d\tau$ is an infinitesimal interval suitable for the probability distribution. In what follows we shall calculate $\langle \Delta E \rangle$ and $\langle \Delta E^2 \rangle$ in a number of those cases which are supposed to be effective in the acceleration of cosmic rays.

(i) The Fermi I mechanism. Fermi (Fe 49) has pointed out that the scattering of cosmic ray particles by wandering magnetic clouds in the Galaxy results in the statistical energy gain, because the rate of head-on
collisions responsible for the energy gain is larger than that of overtaking collisions responsible for the energy loss. Here we give the energy change caused by such a collision, assuming the uniform spatial distribution of particles and the isotropic angular distribution of particles in the coordinate system in which the scattering cloud is at rest.

Let the velocity of the cloud be \( V \). Suppose that a particle of total energy \( E \) and velocity \( v \) impinges on the cloud and comes out with energy \( E' \). If the angle between \( v \) and \( V \) in the cloud system is \( \theta_c \), we have

\[
E' = r^2 E[(1 - v \cdot V) + (v - V)^2 - |v \times V|^2]^{1/2} V \cos \theta_c, \quad (V\cdot3)
\]

where \( r = (1 - V^2)^{-1/2} \) and the velocity is measured in the unit of the light velocity in this Appendix. \( E' \) is a function of \( \theta \), the angle between \( v \) and \( V \) in the space system, as well as of \( \theta_c \).

The probability distribution for the energy change,

\[
\Delta E = E' - E = r^2 E[V^2 - v \cdot V + (v - V)^2 - |v \times V|^2]^{1/2} V \cos \theta_c, \quad (V\cdot4)
\]

is proportional to the product of the relative velocity,

\[
v_r = (v - V)^2 - |v \times V|^2)^{1/2}/(1 - v \cdot V), \quad (V\cdot5)
\]

and the velocity distribution of the clouds, \( f(V) \, dV \). Hence the probability distribution is given by

\[
wdv \propto v_r f(V) \, dV \frac{1}{2} d(\cos \theta_c) \frac{1}{2} d(\cos \theta) \int v_r f(V) \, dV. \quad (V\cdot6)
\]

If the velocity distribution of the clouds is isotropic, it may be expressed as

\[
f(V) \, dV \propto (1/V_0) \exp(-V/V_0) \, dV, \quad (V\cdot7)
\]

in which the exponential distribution is chosen so as to be agreed with the observed one (Oo 55).

Substituting (V\cdot4) and (V\cdot6) in (V\cdot2), and taking into account the smallness of \( V^2 \) compared with unity, we obtain

\[
\langle \Delta E \rangle \approx (E/V_0) \int_0^V V^2 \exp(-V/V_0) \, dV = 2V_0^2 E, \quad (V\cdot8)
\]

\[
\langle \Delta E^2 \rangle \approx (v^2 E^2/V_0) \int_0^{(2/3) V_0^2} (2/3) V^2 \exp(-V/V_0) \, dV = (4/3) V_0^2 v^2 E^2. \quad (V\cdot9)
\]

Since \( \Delta t \) is equal to \( l/v \), \( l \) being the average distance between clouds, we finally find
If there is a long tail in the velocity distribution, as actually observed, the above quantities may become considerably large. Assuming the velocity distribution, for example, as

\[ f(V) dV = \{ (1-a) V_0^{-\nu} \exp(-V/V_0) + a V_{10}^{-\nu} \exp(-V/V_{10}) \} dV, \]

the average of \( V^2 \) yields \( 2(1-a) V_0^2 + 2a V_{10}^2 \), instead of \( 2V_0^2 \). For \( V_{10}/V_0 = 10 \), which seems to be the actual case, the long tail gives a major contribution for \( a > 0.01 \). Thus the role of the tail is found quite important.

Next we discuss the energy spectrum resulting from the Fokker-Planck equation (V·1) for the stationary state, \( t \gg T \), while the time dependent case will be solved in App. VI. Putting

\[ \frac{\langle \Delta E \rangle}{\Delta t} = a_1 E, \quad \frac{\langle \Delta E^2 \rangle}{\Delta t} = 2a_2 E^2, \]

we obtain a power spectrum

\[ n(E) \propto (E/E_0)^{-\nu - 1}, \tag{V·10} \]

assuming that particles are injected with energy \( E_0 \). The power index \( \nu \) is expressed in terms of \( a_1 \) and \( a_2 \) for \( t \approx a_1^{-1} \ln(E/E_0) \) and \( T \) as

\[ \nu = \left[ -(a_1 - a_2) + \sqrt{(a_1 - a_2)^2 + 4a_2 T} \right] / 2a_2. \tag{V·11} \]

If either \( a_1 \) or \( a_2 \) vanishes, \( \nu \) is given by

\[ \nu = 1/a_1 T \quad (a_2 = 0), \tag{V·12a} \]

or

\[ \nu = \frac{1}{2} + \frac{1}{2} \sqrt{1 + 4/a_2 T} \quad (a_1 = 0). \tag{V·12b} \]

If \( a_1 \) and \( a_2 \) are given respectively by (V·8') and (V·9'), we have

\[ \nu \approx 1/(a_1 - a_2) T \approx 3 \Delta t / 4 V_0^2 T, \tag{V·13} \]

on account of \( 4a_2 T^{-1} \leq (a_1 - a_2)^2 \) (here we take \( v = 1 \)). This result essentially is the same as that obtained from (V·12a), the simple Fermi theory.

(ii) The Fermi II mechanism. It has been known that magnetic fields in the Galaxy are rather regular in various places, such as the field along the spiral arm and that in the Crab nebula. Cosmic ray particles trapped in such a uniform field make a spiral motion along the line of force. If the strength of the magnetic field varies gradually in space, the pitch of the spiral motion changes gradually by conserving the energy of a particle.
The spiral of such a particle becomes flatter, as the field strength gets stronger, and the particle can be reflected when the spiral becomes a circle; this is a place where the lines of force form a jaw. If the jaw moves, the reflection gives rise to an energy change similar to that in the case of scattering by magnetic clouds. In a favourable case particles are trapped between such two jaws that move towards each other. Fermi (Fe 54) emphasized the importance of this mechanism for particles going along the galactic spiral arm, and the energy gain due to a head-on collision is assumed to be proportional to the velocity of the jaw.

There might, however, arise a criticism against the possibility of the acceleration, because no energy change should be expected by the adiabatic variation of the magnetic field. Indeed, no energy can be transferred to particles, as mentioned above, if the magnetic field varies slowly enough. This point will be clarified by taking the betatron acceleration into account.

(iii) The betatron acceleration. If the magnetic field strength increases in time, a particle gets an energy proportional to the rate of change in the field strength, as in the case of the betatron (Sw 35, Fa 56). For a particle of charge $Ze$ and velocity $v$ the rate of momentum change is given by

$$\frac{dp}{dt} = \frac{ZeR}{2} \frac{\partial H}{\partial t} = \frac{p \sin \alpha}{2H} \frac{\partial H}{\partial t},$$  \hspace{1cm} (V \cdot 14)$$

where $\alpha$ is the angle between the velocity of the particle and the direction of uniform magnetic field of strength $H$. $R$ is the radius of the spiral motion and it is related to the momentum of the particle, $p$, by

$$R = \frac{p \sin \alpha}{ZeH}. \hspace{1cm} (V \cdot 15)$$

In the adiabatic change of magnetic fields, the angular momentum of the spiraling particle is conserved:

$$pR \sin \alpha = \frac{p \sin^2 \alpha}{ZeH} = \text{const.}, \hspace{1cm} (V \cdot 16)$$

while the magnetic moment formed by the spiral current, $p v \sin^2 \alpha/2H$, is also conserved in the non-relativistic case. Hence the change of $H$ gives rise to the change of $\alpha$, conserving $p$. The adiabatic approximation holds in most of practical cases, since a period of the rotation of the particle,

$$T = \frac{2\pi R}{v \sin \alpha} = \frac{2\pi p}{Ze v H}, \hspace{1cm} (V \cdot 17)$$

is very short in cosmic scale.

One must, however, notice that the conservation of momentum results at the expense of changing $\alpha$; if the pitch of the spiral motion is not allowed to change, the change of $H$ requires the change of $p$, according to
This actually occurs when a particle is reflected at the place where \( \alpha = \pi/2 \). In this case the momentum of a particle varies, according to (V·14) or (V·16), as
\[
\frac{p^2}{H} = \text{const. or } \frac{(p/p_0)^2}{H} = \frac{H}{H_0},
\]
if the momentum is \( p_0 \) when the field strength is \( H_0 \). In a static field, however, without accelerating a particle the reflection takes place within an infinitesimal time interval, because the field component perpendicular to the direction of the guiding field pushes the particle away. The acceleration is effective, therefore, if the jaw of the magnetic field moves to overtake the particle motion. The condition for this may be given approximately by \( 2\pi v H_1/H < V \), where \( H_1 \) is the perpendicular component of the field and \( V \) the velocity of the jaw. As far as this condition holds, the particle can be accelerated, according to (V·18). If the jaw has taken over the particle, it begins to feel the decrease of the field strength. In this case the pitch angle is allowed to decrease, so that the momentum of the particle does not change, according to (V·16). The situation is the same, if the particle is reflected by the jaw. Thus the collision with a magnetic jaw is found to be an effective way of acceleration.

(iv) Acceleration by plasma waves. Bohm (Bo 48) has suggested that plasma waves can accelerate particles, if they stay in a favourable phase. Usually, however, the particles go out of phase, as they gain energy. The energy gain is possible, only if the plasma waves go towards a region of higher electron densities. Since the acceleration requires stringent matching of phase, this mechanism does not seem to be responsible for the most part of acceleration. Here we point out another mechanism which works when plasma waves go across a magnetic field.

As an example, we consider the case where the field strength of a standing plasma wave \( E \cos(\omega_0 t) \) is perpendicular to the magnetic field, around which a particle of velocity \( v \) makes a spiral motion of angular frequency \( \omega = 2\pi/T \), \( T \) being given by (V·17). If the plasma oscillation and the spiral motion are in resonance, namely \( \omega = \omega_0 \), the energy of the particle changes at a rate
\[
dE/dt = (Ze/2)vE|\sin \alpha \cos \delta|, \tag{V·19}
\]
where \( \delta \) is the phase angle between the plasma oscillation and the spiral motion. Hence the particles with favourable phases are accelerated, whereas those with unfavourable ones are decelerated. Since the distribution of the phase angles is supposed to be uniform, the energy spectrum of a group of particles after time \( t \) becomes
\[
dE\sqrt{(Ze/2)^2v^2 |E|^2 \sin^2 \alpha t^2 - (E - E_0)^2}, \tag{V·20}
\]
where the initial spectrum is assumed to be $\delta(E-E_0)\,dE$. Consequently a part of particles can be accelerated at the expense of a corresponding part of particles to be decelerated. However, this mechanism is also subject to a difficulty similar to Bohm's one, because the resonance becomes off as particles gain energy.

In practical cases, the plasma frequency is larger than the rotation frequency, so that the spiral motion is modulated nonadiabatically. This could make the statistical acceleration possible, due to the random modulation. Here we shall not enter detailed discussions on this problem.

(v) Acceleration by hydromagnetic shocks. The motion of the magnetic jaw discussed above may be due to hydromagnetic shocks. In general the hydromagnetic shock can strengthen a magnetic field, unless the shock propagates parallel to the field.

Suppose that a shock of velocity $V_s$ travels perpendicular to magnetic fields. If the field strengths in front and back of the shock front, $H_f$ and $H_b$ respectively, are so strong that the gas pressure in either side can be neglected in comparison with the magnetic pressure, the shock velocity is given, by reference to the Rankine-Hugoniot relation, as

$$V_s = V_A \sqrt{(1+3(H_b/H_f)^2)/2}, \quad (V \cdot 21)$$

$V_A$ is the velocity of the Alfvén wave in front of the shock,

$$V_A = H_f / \sqrt{4\pi \rho_f}, \quad (V \cdot 22)$$

where $\rho_f$ is the matter density there. Relation $(V \cdot 21)$ indicates the amount of increase in the magnetic field strength as the shock travels. Then the motion of spiraling particles under magnetic field $H_f$ is modified according to $(V \cdot 16)$. When the magnetic field increases beyond the upper limit corresponding to the case of $\alpha = \pi/2$, the momenta of the particles have to increase according to $(V \cdot 18)$. After all the field strength decreases as the shock becomes weak, but in this decrease the motion of the particles changes adiabatically, conserving their momenta. The situation is the same as that we discussed in subsection (iii).

The above mechanism seems to be the most effective, if magnetic fields lie nearly perpendicular to an expanding motion, such as in the Crab nebula and the solar flare associated with bipolar sun spots. The mechanism discussed in subsection (iv) may be considered as due to a shock, since the shock will excite the plasma wave. One may also call this the betatron acceleration.

As the magnetic field strength changes within a shock front of thickness $l$, the rate of change in the field strength may be as large as

$$\partial H / \partial t \approx H / (l/V_s).$$
Substituting this in \((V \cdot 14)\), we obtain the betatron acceleration,

\[
\frac{dp}{dt} = (\langle \sin \alpha \rangle V_s/2l) p,
\]

(provided that the drift away from the region of increasing field strength is neglected. Since the drift velocity across the shock front is considered to be of the same order of magnitude as \(V_s\), the acceleration may last as long as \(l/V_s\). Consequently the momentum gain of approximately \(\langle \sin \alpha \rangle p/2\) is expected, within the limit imposed by \((V \cdot 18)\) and \((V \cdot 21)\). If such acceleration is repeated once in a time interval \(\tau\) on the average, the resultant acceleration may be expressed as

\[
\frac{dp}{dt} = \langle \sin \alpha \rangle p/2\tau.
\]

This will result in an exponential increase of momentum in a way similar to the Fermi I mechanism, but this concerns the momentum, while the latter does the total energy. If the acceleration starts from the thermal equilibrium, therefore, the gained energy in a relativistic region is proportional to the square root of the mass of a particle concerned, whereas that is directly proportional to the mass in the Fermi I mechanism, provided that the energy loss in the course of the acceleration is neglected, although it is important for nuclear particles of low energies. Actually, however, the electron temperature can be raised higher than the proton temperature due possibly to the fluctuation in electric fields which may occur rather frequently in a plasma, and the energy exchange between electrons and protons takes considerable time in a rarefied gas. Such possibilities lead us to infer that the energies gained for electrons and protons may be not so different.

(vi) The motion of a magnetic field induces an electric field which may be responsible for the acceleration (Al 49, 50a, b). Usually an electric field is difficult to sustain, because of the large conductivity in cosmic plasmas, and merely electric polarization is associated with the moving magnetic field. The passage of a charged particle through such an electric field does not result in a gain or loss of energy. If, however, the particle is scattered in the electric field and goes out of it without passing through the whole field, the particle may gain or loss energy.\(^*)\)

Such scattering is regarded as due mainly to the multiple Coulomb scattering, for which the mean free path of a particle of charge \(Z\), momentum \(p\) and velocity \(v\) is approximately given by

\[
X_c = (vp/ZE_t) X_0,
\]

where \(E_t = 21\) MeV and \(X_0 = 10^2\) g cm\(^{-2}\) in the interstellar gas and the stellar atmosphere. If the linear radius of the cyclotron motion corresponds to

\(^*)\) This has been suggested by Professor C.Hayashi.
x g cm\(^{-2}\), a fraction of \(x/X\) could be accelerated to a considerable energy. Since the magnitude of \(x\) in most cases concerned, such as in the galactic cloud, in the Crab nebula and in the lower solar corona, are as large as \(10^{-12} - 10^{-10}\) g cm\(^{-2}\), only a fraction of about \(10^{-14} - 10^{-12}\) of electrons and protons could be accelerated to an energy of about \(10^3 - 10^5\) eV on the average.

A favourable condition is possibly provided by a helical magnetic field which is caused by twisting a rather uniform magnetic field. This may produce an electric field of rather large scale along the axis of the helics in the course of its shrinkage. This could be the case in the field of a pair of bipolar sun spots which would rotate in the same direction due to the cyclone motion on the solar surface; consequently the magnetic field is twisted gradually, storing an enormous amount of energy available for the solar eruption.

As has been seen from the above discussions, the acceleration of particles consists of two parts, the statistical one and the one associated with the change of electromagnetic fields for rather short periods in celestial scale. In the former the accumulation of the later processes is essential, while in the latter a lack of other modes of energy dissipation seems important, as will be argued in what follows.

It is well known that the relaxation of a magnetic field is very slow under celestial conditions. In App. IV we have shown that the heat conduction is also slow. For individual particles, however, energy received from magnetic fields dissipates through the collisions with other particles. For electron-electron collisions the mean life time for dissipation is of the order of \(t\) given in Eq. (IV·15). Since \(t\) increases with the energy of a particle, the acceleration becomes more efficient as the energy increases. If \(t\) is longer than the time for the increase of magnetic field strength, a considerable portion of particles gain energy. This seems to be the reason why substantial part of the magnetic energy is converted to the energy of relativistic particles, as has been shown in §3.1.

### VI. Synchrotron radiation

A high energy charged particle spiraling in a magnetic field \(\mathbf{H}\) loses its energy by continuously radiating electromagnetic waves. According to Schwinger (Sc 49) the total intensity of this synchrotron radiation is given by

\[
-d\frac{E}{dt} = P(E) = \frac{2}{3c} \left( \frac{e^2}{mc^2} \right) \left( \frac{eH}{mc} \right)^2 \left( \frac{E^3}{mc^2} \right) \left( \frac{v}{c} \right)^2,
\]

where

\[ (VI\cdot1) \]

\*\* Such a mechanism is pointed out by Professor T. Gold.\*
\[ H_{\perp} = \mathbf{H} \cdot \frac{\mathbf{p}}{p}, \]  

(VI•2)

\( m \) is the mass of the particle, \( E \) its total energy and \( p \) its momentum.

The frequency spectrum of the radiation can be expressed as

\[ P(E) = \int d\nu P(E, \nu), \]

with

\[ P(E, \nu) = \frac{3^{3/2}e^3H_1}{mc^2} F\left( \frac{\nu}{\nu_c} \right) \]

(VI•3)

\[ F\left( \frac{\nu}{\nu_c} \right) = \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta) \, d\eta, \]

(VI•4)

where \( K_{5/3}(x) \) is the modified Hankel function of the order 5/3, and the critical frequency \( \nu_c \) is defined by

\[ \nu_c = \frac{3}{4\pi} \left( \frac{eH_1}{p} \right) \left( \frac{E}{mc^2} \right)^{3/2} \approx \frac{3}{4\pi} \left( \frac{eH_1}{mc} \right) \left( \frac{E}{mc^2} \right)^{3/2}. \]

(VI•5)

The last expression is valid in the extremely relativistic cases, in which \( pc \ll E \). The spectrum \( P(E, \nu) \) has a maximum at the frequency \( \nu_m \), where

\[ P(E, \nu_m) = 1.60 \frac{e^3H_1}{mc^2}, \]

(VI•6)

\[ \nu_m = \frac{1}{3} \nu_c = \frac{1}{4\pi} \left( \frac{eH_1}{mc} \right) \left( \frac{E}{mc^2} \right)^{3/2}. \]

(VI•7)

The electromagnetic waves are radiated mostly along the direction of motion of the charged particle and the root mean square value of angle between the electromagnetic waves and the velocity vector of the particle is

\[ \langle \theta^2 \rangle^{1/2} \approx mc^2/E. \]

Another characteristic feature of this radiation is that it is strongly polarized. The degree of polarization is shown by Sokolov et al. (So 56) that 7/8 of the radiation has the electric field vector perpendicular to the external magnetic field and 1/8 of the radiation has the vector almost parallel to that field.

Synchrotron radiation is significant only for electrons because \( P(E) \) in the equation (VI•1) decreases strongly with increasing particle mass. Hence we give numerical values of the coefficients in the above equations only for an electron.
\[-\frac{dE}{dt} = P(E) = b_r E^2 \, \text{eV sec}^{-1}, \quad b_r = 3.90 \times 10^{-15} H^5 \, \text{eV}^{-1} \text{sec}^{-1}, \quad (\text{VI}\cdot1')\]

and

\[\nu_m = 1.40 \times 10^6 H_1 E (E/m_e c^2)^2 \, \text{c/s}. \quad (\text{VI}\cdot7')\]

Hereafter we measure the strength of the magnetic field in gauss.

The lifetime in which the initial energy \(E\) of an electron decreases to \(E/2\) due to the synchrotron radiation is given by

\[T_{1/2} = E / P(E) = 0.513 \times 10^9 (EH_f^2 / m_e c^2)^{−1} \, \text{sec}. \quad (\text{VI}\cdot8)\]

We consider the total intensity of the synchrotron radiation, \(I(\nu)\), emitted by electrons at position \(r\), whose energy spectrum at position \(r\) is

\[n_\nu(E, r) dE = \gamma n_\nu(r) (E_0/E)^\gamma dE/E. \quad (\text{VI}\cdot9)\]

\(I(\nu)\) is calculated by Ginzburg (Gi 56), assuming \(r\) to be independent of energy and position, as

\[I(\nu) d\nu = d\nu \int \int dEdr P(E, \nu) n_\nu(E, r) \]

\[= \frac{12 e^3}{m_e c^2} \gamma^2 U(\gamma) \nu^{-\frac{3}{2}} \int d\nu n_\nu(r) H_1(r) \nu_m^5 \nu_m^\frac{3}{2}(r), \quad (\text{VI}\cdot10)\]

where the integration goes over the region of interest and \(\nu_m(r)\) is related to \(E_0\) and the strength of the magnetic field \(H(r)\) at position \(r\) as shown in (VI\cdot7') as

\[\nu_m(r) = 1.40 \times 10^6 H_1(r) (E_0/m_e c^2)^2 \, \text{c/s}. \quad (\text{VI}\cdot7'')\]

The function \(U(\gamma)\) is given by

\[U(\gamma) = \int_0^\infty Y(u) u^{(3\gamma - 1)/4} du, \quad (\text{VI}\cdot11)\]

its numerical values being in Table VI-1, and \(Y(u)\) is in the relation to \(F(\nu/\nu_m)\) as

\[F\left(\frac{\nu}{\nu_m}\right) = \frac{2^4}{3^{1/4}} \left(\frac{3\nu}{4\nu_m}\right)^{1/3} Y\left(\left(\frac{3\nu}{4\nu_m}\right)^{3/4}\right). \quad (\text{VI}\cdot12)\]

Strictly speaking, \(\gamma\) is a function of energies and positions, but may be regarded as independent of energy since the spectrum of the radiation \(P(E, \nu)\) has a rather sharp peak at \(\nu_m\), in other word, the range of energies of electrons emitting the radiation of the frequency \(\nu_m\) is narrow. If both the magnetic field and the energy spectrum of the electrons vary little over the region of interest, the equation (VI\cdot10) is reduced to
\[ I(\nu) \approx (12e^2/m_e c^2)\gamma^2 U(\gamma) (\nu_m/\nu)^{7/2} H_1 N_0 \]
\[ = 1.63 \times 10^{-21} \gamma^2 U(\gamma) (\nu_m/\nu)^{7/2} H_1 N_0 \text{ erg sec}^{-1} (c/s)^{-1}, \]  
\( \text{VI} \cdot 13 \) and

\[ N_0 = \int dr n_0(r), \]

\( \text{VI} \cdot 7'' \)

\( H_1 \) is the average strength of the magnetic field perpendicular to the direction of motion of electrons \((H_1(r) \text{ in } \text{VI} \cdot 7'')\) giving \( \nu_m \) is also replaced by \( H_1 \).

Further a convenient formula of \( I(\nu) \) for the order of magnitude estimation is obtained under the optimum condition as

\[ I(\nu) \approx P(E, \nu_m) n_e V = 2.16 \times 10^{-22} H_1 n_e V \text{ erg sec}^{-1} (c/s)^{-1}, \]  
\( \text{VI} \cdot 14 \)

where \( V \) is the volume of the source and \( n_e \) is the density of electrons having an energy corresponding to the frequency \( \nu = \nu_m \) as defined in \( \text{VI} \cdot 7' \).

When the radiation is isotropic due to the random distribution of the magnetic field, the observed flux at the earth (energy per unit time and unit area) is simply related to \( I(\nu) \) estimated above as

\[ j_\nu = (4 \pi L^2)^{-1} I(\nu), \]  
\( \text{VI} \cdot 15 \)

where \( L \) is the distance of the source from the earth.

For such a case that the distribution of electrons spreads over the Galaxy, it is convenient to use the following formula, instead of \( \text{VI} \cdot 15 \) for local sources. Assuming again the isotropic distribution of radiation, the intensity observed at the earth (energy per unit time, unit area and unit solid angle) is given by

\[ j_\nu = \frac{1}{4\pi} \int P(\nu, E) n_e(E, s) dE \]  
\( \text{VI} \cdot 16 \)

where \( n_e(E, s) \) is the differential electron spectrum at the point \( s \) and the integration is carried out along the line of sight.

If \( n_e(E, s) \) and \( H_1 \) are independent of \( s \) and the former has the form along a path \( R \)

\[ n_e(E, s) dE = \tau n_0(E_0/E)^{7/2} dE/E, \]

we have from \( \text{VI} \cdot 10 \) and \( \text{VI} \cdot 7'' \)

\[ j_\nu = 1.30 \times 10^{-22} \gamma^2 U(\gamma) (\nu_m/\nu)^{7/2} H_1 R n_0 \text{ erg cm}^{-2} \text{sec}^{-1} (s/c)^{-1} \text{ster}^{-1}, \]  
\( \text{VI} \cdot 17 \)

where \( R \) and \( n_0 \) are measured in cm and cm\(^{-3}\) respectively.
Table VI-1

<table>
<thead>
<tr>
<th>( \gamma )</th>
<th>( U(\gamma) )</th>
<th>( \gamma^2 U(\gamma) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.37</td>
<td></td>
</tr>
<tr>
<td>2/3</td>
<td>0.163</td>
<td>0.172</td>
</tr>
<tr>
<td>1</td>
<td>0.128</td>
<td>0.256</td>
</tr>
<tr>
<td>1.5</td>
<td>0.101</td>
<td>0.430</td>
</tr>
<tr>
<td>2</td>
<td>0.087</td>
<td>0.696</td>
</tr>
<tr>
<td>2.5</td>
<td>0.0805</td>
<td>1.14</td>
</tr>
<tr>
<td>3</td>
<td>0.0785</td>
<td>1.18</td>
</tr>
<tr>
<td>3.5</td>
<td>0.0805</td>
<td>3.18</td>
</tr>
<tr>
<td>4</td>
<td>0.085</td>
<td>5.50</td>
</tr>
<tr>
<td>4.5</td>
<td>0.095</td>
<td>9.70</td>
</tr>
<tr>
<td>5</td>
<td>0.109</td>
<td>17.4</td>
</tr>
<tr>
<td>5.5</td>
<td>0.129</td>
<td>32.2</td>
</tr>
<tr>
<td>6</td>
<td>0.152</td>
<td>58.3</td>
</tr>
</tbody>
</table>

Functions \( U(\gamma) \) and \( Y(\gamma) \) are tabulated by Ginzburg (Gi 56) and \( F(a) \) by Oort and Walraven (Oo 56).

VII. Diffusion problems

The diffusion theory of cosmic rays in the stellar atmosphere and in interstellar space has been developed by many authors (Mo 54, Ka 55, Po 56, Te 56). We can apply the same methods to our problem as used in the cascade shower theory and the neutron optics (Mo 53, Ro 41).

The diffusion equation of fast particles in the nebulae or the interstellar medium is of the form:

\[
\frac{\partial N}{\partial t} + V(VN) - D\Delta N + \frac{\partial}{\partial E} \left( \frac{\langle \Delta E^2 \rangle}{\Delta t} N \right) - \frac{1}{2} \frac{\partial^2}{\partial E^2} \left( \frac{\langle \Delta E^2 \rangle}{\Delta t} N \right) - \frac{\partial}{\partial E} (b(E)N) + \frac{1}{T} N = Q. \tag{VII.1}
\]

\( N(E, r, t) \) is the density of particles at \( r \) and \( t \) with energies between \( E \) and \( E + dE \). \( V \) is the drift velocity of magnetic clouds. \( D \) is the diffusion constant which is related to the mean free path \( l \) of collisions of fast particles with magnetic clouds as

\[
D = \frac{1}{3} l v(E) \approx \frac{1}{3} l c, \tag{VII.2}
\]

where \( v(E) \) is the velocity of particles with energy \( E \), being equal to \( c \) in most cases. The first three terms in the above equation show the diffusion in space.

The fourth and fifth term denote the statistical acceleration, for which the detailed discussion has been given in App. V. \( \langle \Delta E \rangle \) and \( \langle \Delta E^2 \rangle \)
are the average and the mean square of the energy increase in time $\Delta t$. We can put

$$\frac{\langle \Delta E \rangle}{\Delta t} = a_1 E, \quad \frac{\langle \Delta E^2 \rangle}{\Delta t} = 2a_2 E,$$

and

$$\Delta t = \frac{l}{v(E)} \approx \frac{l}{c}.$$  \hspace{1cm} (VII.3)

For example, the coefficients $a_1$ and $a_2$ for the Fermi mechanism are calculated in Eqs. (V.8') and (V.9').

The sixth term means the deceleration of particles, $b(E)$ is the rate of energy loss which is due to the synchrotron radiation, the bremsstrahlung, the ionization loss and the inverse Compton effect. Synchrotron radiation is most important for electrons

$$-\frac{dE}{dt} = b_s E^2,$$  \hspace{1cm} (VII.4)

where $b_s$ is given in Eq. (VI.1').

The last term in the left-hand side shows the nuclear absorption of particles and $T$ is its lifetime. $Q$ in the right-hand side indicates the injection of particles, being a function with respect to $E$, $r$ and $t$.

To solve the equation (VII.1), involved mathematics is needed, but we can obtain simple solutions in some cases allowing to drop one or two terms in the equation. We give examples in the following.

(i) Suppose that the acceleration is efficient enough compared with the energy loss and the injection energy of particle is not high. We start with,

$$\frac{\partial n}{\partial t} + v(Vn) - D \Delta n + \frac{\partial}{\partial E} (a_1 En) - \frac{\partial^2}{\partial E^2} (a_2 E^2 n) + \frac{1}{T} n = Q,$$  \hspace{1cm} (VII.1A)

where

$$Q = q(E) \delta (r - r_0) \delta (t - t_0).$$

As is easily found, the steady-state solution of this equation has the simple form, the power spectrum of energy $E$, which is given in Eq. (V.10).

We apply Méléllin transformation with respect to $E$,

$$f(s, r, t) = \int_0^\infty dE \cdot E^n(E, r, t),$$

and

$$u(s) = \int_0^\infty dE \cdot E^n q(E).$$  \hspace{1cm} (VII.5)

The transformed equation is
We separate this into two equations, letting

$$f(s, r, t) = \phi(r, s, t) K(s, t),$$  \hspace{1cm} (VII.7)

where $\phi$ and $K$ satisfy respectively

$$\frac{\partial K}{\partial t} - (sa_1 + s(s-1)a_2) K + \frac{1}{T} K = 0$$

and

$$\frac{\partial \phi}{\partial t} + F(V \phi) - D \Delta \phi = K^{-1} u \delta(r - r_0) \delta(t - t_0).$$

The solution of the first equation is straightforward and, if $V$ is assumed to be independent of $r$, the second is also of a typical type, being easily solved by the Fourier transformation. $K$ and $\phi$ have the expressions,

$$K(s, t) = \exp\left\{\left[sa_1 + s(s-1)a_2\right] t - \frac{t}{T}\right\},$$

and

$$\phi(r, s, t) = \frac{u}{8(\pi D \tau)^{3/2}} \exp\left\{\left[sa_1 + s(s-1)a_2 - \frac{1}{T}\right] \tau\right\} \times \exp\left\{-\frac{(R-V \tau)^2}{4D \tau}\right\},$$

where $R$ and $\tau$ are given by

$$R = r - r_0 \quad \text{and} \quad \tau = t - t_0.$$  \hspace{1cm} (VII.8)

Substituting these in (VII.7) and using the inverse theorem for the Méllin transformation, we find

$$n(E, r, t) = \frac{1}{8(\pi D \tau)^{3/2}} \exp\left\{-\frac{(R-V \tau)^2}{4D \tau} - \frac{\tau}{T}\right\} \times \frac{1}{2\pi} \int_{-\infty}^{+\infty} ds e^{is},$$

where

$$\lambda(s) = -(s+1) \ln E + [sa_1 + s(s-1)a_2] \tau + \ln u(s).$$

For given $q(E)$ we can integrate the above equation by the steepest descent method.

a) \hspace{1cm} $$q(E) dE = q_0 \delta(E - E_0) dE.$$  \hspace{1cm} (VII.10a)

By the transformation formula (VII.5), we have
\[ \nu(s) = q_0 E_0^{s+1}, \quad \lambda(s) = -(s+1) \ln\left(\frac{E}{E_0}\right) + [sa_1 + s(s-1)a_2]r + \ln q_0. \]

The integration \( I \) in the equation (VII.9) is evaluated as

\[ I = \frac{1}{2\pi i} \int_{\sigma-i\infty}^{\sigma+i\infty} ds e^{\lambda(s)} \frac{q_0}{(4\pi a_2^2)^{1/2}} \times \exp\{-r \ln\left(\frac{E}{E_0}\right) + (r-1)a_1r + (r-1)(r-2)a_2r\} \]

with

\[ r = 1 + s_0 = \frac{1}{2a_2^2} \ln\left(\frac{E}{E_0}\right) + \frac{3a_2 - a_1}{2a_2}, \quad (\text{VII.11}) \]

where \( s_0 \) is the root of the equation \( \lambda'(s_0) = 0 \).

The above relation shows that the time necessary for accelerating a particle from the injected energy \( E_0 \) to \( E \) is equal to \( a_1 r^{-1} \ln\left(\frac{E}{E_0}\right) \).

b) \( q(E) = \nu q_0 \left(\frac{E_0}{E}\right)^{r} dE/E \) for \( E_0 < E < E_m \), \( (E_m \text{ being the maximum value of the injection energy}) \)

\[ = 0 \quad \text{for } E < E_0, \quad E_m < E. \quad (\text{VII.10b}) \]

By the same procedure as above, we obtain

\[ I = \frac{1}{(4\pi a_2^2)^{1/2}} \frac{\nu q_0}{(\nu + 1 - r) E_0} \times \exp\{-r \ln\left(\frac{E}{E_0}\right) + (r-1)a_1r + (r-1)(r-2)a_2r\}, \]

where \( r \) is the same as that defined by (VII.11). It should be noticed that the acceleration is inefficient when \( r > \nu + 1 \).

Substituting these in (VII.9), we ultimately find that, for \( q(E) dE = q_0 \delta(E - E_0) dE \):

\[ n(E, r, t) = \frac{1}{2^4 \pi^2} \frac{q_0 E_0^{-1}}{(4\pi a_2^2)^{1/2}} \frac{(E_0/E)^{\gamma}}{\left(\frac{E}{E_0}\right)^{\gamma}} \times \exp\left\{ -\frac{(R - V\tau)^2}{4Dr} - \frac{r}{T} + (r-1)a_1r + (r-1)(r-2)a_2r \right\}, \quad (\text{VII.12a}) \]

and for \( q(E) dE = \nu q_0 \left(\frac{E_0}{E}\right)^{r} dE/E \):

\[ n(E, r, t) = \frac{1}{2^4 \pi^2} \frac{1}{(4\pi a_2^2)^{1/2}} \frac{\nu q_0 E_0^{-1}}{(\nu + 1 - r)} \frac{(E_0/E)^{\gamma}}{\left(\frac{E}{E_0}\right)^{\gamma}} \times \exp\left\{ -\frac{(R - V\tau)^2}{4Dr} - \frac{r}{T} + (r-1)a_1r + (r-1)(r-2)a_2r \right\}. \quad (\text{VII.12b}) \]

(ii) If the injection energies of particles are high and the synchrotron radiation is the main process, then we are concerned with
\[
\frac{\partial n}{\partial t} + V(Vn) - D \Delta n + \frac{1}{T} n - \frac{\partial}{\partial E} (b_r E^2 n) = Q,
\]
(VII•1B)

and

\[ Q = q(E) \delta (r - r_0) \delta (t - t_0). \]

We apply the Laplace transformation and the Fourier transformation at the same time:

\[
f(E, k, \lambda) = \frac{1}{(2\pi)^{3/2}} \int_0^\infty dr \int_0^\infty dte^{ikr}e^{-\lambda t}n(E, r, t),
\]

and

\[
u(E, k, \lambda) = \frac{1}{(2\pi)^{3/2}} \int_0^\infty dr \int_0^\infty dte^{ikr}e^{-\lambda t}Q(E, r, t).
\]

(VII•13)

Transforming the equation (VII•1B), we have

\[
\left( \lambda - iVk + Dk^2 + \frac{1}{T} \right) f - \frac{d}{dE} (b_r E^2 f) = u(E, k, \lambda)
\]

\[ = (2\pi)^{-\frac{3}{2}} q(E)e^{-ikr}e^{-\lambda t_0}.
\]

(VII•14)

We assume that \( q(E) = 0 \) for \( E > E_m \) and \( V \) is independent of \( r \) as before. At any \( t, n(E, r, t) = 0 \) for \( E > E_m \) and the same is for \( f(E, r, t) \), the solution of (VII•14) is therefore

\[
f(E, k, \lambda) = \int_0^{E_m} dE' \frac{1}{(2\pi)^{3/2}} q(E')e^{ikr}e^{-\lambda t_0}e^{-2\pi} \exp \left[ - \left( \int_{E_r}^{E_m} dE'' \left( \frac{2}{E''} - \frac{\lambda + \kappa}{b_r E''^2} \right) \right) \right],
\]

where \( \kappa = -iVk + Dk^2 + \frac{1}{T} \).

(VII•15)

By the inverse theorems for the Laplace and the Fourier transformations we have \( n(E, r, t) \) by

\[
n(E, r, t) = \frac{1}{(2\pi)^{3/2}} \int \int dE' dE'' \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} df(E, k, \lambda) e^{-ikr}e^{\lambda t}.
\]

(VII•17)

Substituting (VII•15) in (VII•17) and changing the order of integrations, we obtain

\[
n(E, r, t) = \frac{1}{(2\pi)^{3/2}} \int dke^{-ikR} \frac{E_{t_0}}{b_r E^2} \int_0^{E_m} dE' q(E') e^{-ikr} \cdot \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} d\lambda \left[ \tau^* \left( \frac{1}{\beta_0} \left( \frac{1}{\beta R} \right)^\frac{1}{R} \right) \right],
\]

(VII•18)

where \( R \) and \( \tau \) are \( r - r_0 \) and \( t - t_0 \), respectively.
Taking into account

\[ \frac{1}{2\pi i} \int_{\gamma - \infty}^{\gamma + \infty} d\lambda e^{\lambda \omega} = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\rho e^{\rho \omega} = \delta(C), \]

(VII·18) is reduced to

\[ n(E, r, t) = \frac{1}{(2\pi)^{3/2}} \int dk e^{-ik\omega} \frac{e^{-\frac{k}{b_1 E^2}}}{b_1 E^2} \int_{-\infty}^{\infty} dE' q(E') e^{\frac{E'}{b_2 E'}} \delta \left[ \tau - \frac{1}{b_1} \left( \frac{1}{E} - \frac{1}{E'} \right) \right]. \]

By the use of

\[ \int_0^\infty dx f(x) \delta \left( \frac{1}{bx} - a \right) = \int_0^\infty dy \frac{1}{by^2} f \left( \frac{1}{by} \right) \delta \left( y - a \right) = -\frac{1}{ba^2} f \left( \frac{1}{ba} \right), \]

the further integration can be easily performed, and we finally get

\[ n(E, r, t) = \frac{1}{(4\pi D\tau)^{3/2}} \int_{1-(1-\tau b_2 E)} q \left( \frac{E}{1-\tau b_2 E} \right) \exp \left[ -\frac{(R-V\tau)^2}{4D\tau} - \frac{\tau}{T} \right]. \]

(VII·19)

(iii) When the injection is of the form

\[ Q = q(E) S(r_0) T(t_0), \]

the corresponding solutions in (VII·1A) and (VII·1B) are given with (VII·9) and (VII·19) by

\[ N(E, r, t) = \int d\tau d\sigma n(E, r, t) S(r_0) T(t_0). \]

(VII·21)

If the distribution of fast particles is stationary in the case (ii), the density of particles \( N(E, r) \) is given, setting \( T(t_0) = \text{const.} \) and integrating over \( \tau = t - t_0 \), by

\[ N(E, r) = \frac{1}{(4\pi D)^{3/2}} \int dr_0 S(r_0) \int_0^{T_0} d\tau \frac{\tau - 3/2}{(1-\tau b_1 E)^2} q \left( \frac{E}{1-\tau b_1 E} \right) \exp \left[ -\frac{(R-V\tau)^2}{4D\tau} \right]. \]

(VII·22)

Change of variables

\[ \tau \to E' = E/(1-\tau b_1 E) \]

yields

\[ N(E, r) = \frac{1}{(4\pi D)^{3/2}} \int dr_0 S(r_0) \frac{1}{b_1 E^2} \int_{-\infty}^{+\infty} dE' q(E') \left( \frac{1}{b_1 E'} - \frac{1}{b_2 E'} \right)^{-\frac{1}{2}} \]

\[ \times \exp \left[ -\frac{(R-V\tau)^2}{4D} \left( \frac{1}{b_1 E} - \frac{1}{b_2 E'} \right)^{-1} - \frac{1}{T} \left( \frac{1}{b_1 E} - \frac{1}{b_2 E'} \right) \right]. \]

(VII·23)
Origin of Cosmic Rays

It is evident that \( N(E, r) \) satisfies the diffusion equation:

\[
V(N) - D \Delta N - \frac{\partial}{\partial E} (\beta E^2 N) - \frac{N}{\tau} = Q, \quad Q = q(E) S(r_0).
\] (VII·1C)

For example, we work out the case of a uniform shell source. We assume \( V = 0 \) and \( q(E) dE = \nu q_0 (E_0/E)^{\nu} dE/E \) as given by (VII·10b). Let \( S(r_0) \) be of the form

\[
S(r_0) = \frac{1}{V} \quad \text{for} \quad r_0 - \epsilon \leq r \leq r_0,
\]

\[
= 0 \quad \text{for} \quad r < r_0 - \epsilon \quad \text{or} \quad r > r_0,
\] (VII·24)

and

\[
V = 4\pi \epsilon r_0^2,
\]

where \( \epsilon \) is the thickness of the source and \( r_0 \) its radius. Hence the distribution of particles inside the source \( r < r_0 \) is uniform. When the nuclear absorption is negligible, or \( T \to \infty \), we find by the method of steepest descent the following solutions.

For \( \nu = 1 \):

\[
N(E, r) = N_0(E); \quad (6D/b_4E)^{1/2} > r > r_0, \quad N(E, r) = N_0 \left( \frac{r_0}{r} \right); \quad (6D/b_4E)^{1/2} < r \quad N(E, r) = kN_0 \left( \frac{r_0}{r} \right) \exp \left\{ \frac{-b_4E^2r^3}{4D} \right\} \cdot F \left( \frac{b_4Er^3}{4D} \right) \] (VII·25)

where \( N_0(E) \) represent the density of particles inside the cavity \( r < r_0 \) which has the expression:

\[
N_0 = \frac{2^{4\nu} \nu^{-\nu}}{4\pi Dr_0} \nu q_0 \left( \frac{E_0}{E} \right)^{\nu} \frac{1}{E}. \] (VII·26)

Constants \( k \) and \( \eta \) are defined by

\[
k = \left( \frac{2\pi}{3} \right)^{\nu} e^{\frac{\nu}{2}} \quad \eta = \nu - 1, \] (VII·27)

and \( F(K) \) is given by

\[
F(K) = \left( 1 + K \right)^{\frac{1}{2} - \eta} \left( 1 + \frac{4\eta}{3} K + \frac{2\eta}{3} K^2 \right)^{-1/2}. \] (VII·28)

For \( \nu = 1 \), we have simply

\[
r < r_0, \quad N(E, r) = \frac{1}{4\pi Dr_0} q_0 \frac{E_0}{E^2} \approx N_0.
\]
where
\[ G(K) = \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\infty} d\alpha e^{-\alpha}. \] (VII•29)

VIII. Intensity of solar cosmic rays

The absolute intensity of the solar cosmic rays observed on February 23, 1956 is estimated in the following way.* For this purpose we refer to the data obtained at Göttingen (Me B 56) and Chicago (Me P 56), because similar apparatus were employed at the same geomagnetic latitude (52°). At the initial phase of the cosmic ray increase, Göttingen was located in the 3 hour zone where particles with rigidity of about 4 GV are accessible, while Chicago was in the 20 hour zone where those of about 2.5 GV are accessible (Lü 57b). The intensities of neutrons detected at the respective stations were approximately twenty times larger than the normal intensities. The relative intensities at the top of the atmosphere can be evaluated with the aid of the neutron yield function (Si 53, Fo 53) as roughly 2:1, which is in qualitative agreement with the result calculated with a steep rigidity spectrum by Lüst and Simpson (Lü 57b), provided that the directions of arrival of the particles are a little more divergent than ±10°.

Now we compare the solar cosmic ray intensity with the normal one, referring to the Göttingen data. Assuming for simplicity that the incident particles are exclusively protons with energy spectra (Pf 57)
\[ j_p(E) dE = nE_0^n E^{-n-1} dE, \quad n=3 \] (VIII•1a)
\[ j_n(E) dE = N E_0 E^{-2} dE \] (VIII•1b)
respectively, where \( E \) is the total energy in the unit of the proton rest energy. The neutron yield function is given by (Si 53)
\[ Y(E) = Y_0 \ln(E/E_0), \quad E \geq E_0 = 2. \] (VIII•2)
At Göttingen the solar particles of energy \( E \) are assumed to be accessible with relative intensity
\[ A(E) = A_0 (E/E_0); \quad 3 \leq E \leq 5, \quad A_0 = 0.1, \] (VIII•3)
taking account of Lüst's calculation (Lü 57a). The limitation in the solid angles gives disadvantage of factor \( \Omega \) to the solar particles; \( \Omega \) may be chosen.

*) Our method of evaluation is somewhat different from that by Firor et al. (Fi 54b).
as about 1/2, taking the spread in the arrival directions into account.

Thus we obtain the counting rate of the solar particles proportional to

$$ C_s \propto \int j_s(E) Y(E) A(E) dE = \frac{n}{n-1} s Y_0 A_0 \theta \left( \frac{E_0}{E} \right)^{n-1} \left[ \ln \left( \frac{3}{E_0} \right) + \frac{1}{n-1} \right] ; \quad (VIII\cdot4a) $$

This is compared with the normal counting rate

$$ C_n \propto \int j_n(E) Y(E) dE = N Y_0 \frac{E_0}{E} \left( \ln \frac{E}{E_0} + 1 \right) \approx N Y_0. \quad (VIII\cdot4b) $$

where $E_c$ is the cut-off energy, approximately equal to 1.3 $E_0$ in our case. The ratio of the counting rates of about 20 gives us

$$ S/N \approx 10^3 C_s/N \approx 2 \times 10^3. \quad (VIII\cdot5) $$

If we take a larger value of $n$, say $n=5$ (Lü 57b), the ratio becomes slightly larger.

The ratio $S/N$ gives the intensity ratio for the energy region above $E \geq 2$. Since the energy of the solar particles extends to lower values, we shall tentatively extrapolate the spectra (VIII•1) to $E=1$, so that they give the total intensities. Their ratio at the top of the atmosphere is found to be

$$ I_s/I_n = 10^3, \quad (VIII\cdot6) $$

taking the difference in the angular distributions into account. Hence the absolute intensity of the solar particles is estimated as about $2 \times 10^3 \text{cm}^{-2} \text{sec}^{-1}$, which may be an underestimate, mainly because of the too crude extrapolation of the energy spectrum.

Such a strong intensity of charged particles will produce a considerable amount of ionization in the upper atmosphere. The specific ionization of cosmic rays at the top of the atmosphere is inferred as high as 200 ion pairs cm$^{-2}$ atm$^{-1}$, as is extrapolated from the data at lower altitudes (Ne 52). In the D-layer where pressure is about $10^{-3}$ atm, we expect the ion pair production of $4 \times 10^3 \text{cm}^{-2} \text{sec}^{-1}$. If this is as high as $10^3 \text{cm}^{-2} \text{sec}^{-1}$, one can observe the appearance of the D-layer. In fact, at the moment of the cosmic ray incidence the anomalous propagation of low frequency electromagnetic waves was observed over the Atlantic ocean (in the night side). It seems very reasonable to interpret this phenomenon in terms of the excess ionization produced by the solar cosmic rays. If this is taken as granted, our estimate of the intensity may be correct within the order of magnitude.
References

Al 49  H. Alfven, Phys. Rev. 75 (1949), 1732.
Bu 57a E. M. Burbidge, G. R. Burbidge, W. A. Fowler and F. Hoyle, Rev. Mod. Phys. 29 (1957), 547.
Eh 48 A. Ehmer, ZS. Naturforsch. 3a (1948), 264.
Fo 46 S. E. Forbush, Phys. Rev. 70 (1946), 771.
Fe 48 E. Feenberg and H. Primakoff, Phys. Rev. 73 (1948), 449.
Fe 49 E. Fermi, Phys. Rev. 75 (1949), 1169.
Fo 53 W. H. Fonger, Phys. Rev. 91 (1953), 351.
Fi 54a J. Firor, Phys. Rev. 94 (1954), 1017.
Fo 54 S. E. Forbush, J. Geophys. Res. 59 (1954), 523.
Ha 50 D. ter Haar, Rev. Mod. Phys. 22 (1950), 119.
References

Ha 56b S. Hayakawa and K. Kitao, Prog. Theor. Phys. 16 (1956), 139.
Ha 57b S. Hayakawa, Nuov. Cim. 5 (1957) 608.
Hi 57 S. Higashi, T. Oshio, H. Shibata, K. Watanabe and Y. Watase, Nuov. Cim. 5 (1957), 597.
Ka 58 S. Kaneko and M. Okazaki, Nuov. Cim. to be published.
MeB 56 B. Meyer, ZS. Naturforsch. 11a (1956), 326.
Mo 57 P. Morrison, Rev. Mod. Phys. 29 (1957), 235.
Mi 58 S. Miyake, private communication.
Ne 56 H. V. Neher, Phys. Rev. 103 (1956), 228.
Na 58 S. Nakagawa, E. Tamai and S. Nomoto, private communication.
References

Ro 41 B. Rossi and K. Greisen, Rev. Mod. Phys. 13 (1941), 240.
Ru 54 S. G. Rudstan, Phil. Mag. 46 (1955), 344.
Ros 56 A. H. Rosenfeld, R. A. Swanson and S. D. Warshaw, Phys. Rev. 103 (1956), 413.
Sw 33 W. F. G. Swann, Phys. Rev. 43 (1933), 217.
Sc 49 J. Schwinger, Phys. Rev. 75 (1949), 1912.
So 56 A. Sokolov, Nuov. Cim. Suppl. 3 (1956), 743.
Su 56 H. E. Suess and H. C. Urey, Rev. Mod. Phys. 28 (1956), 53.
Un 51 A. Unsöld, Phys. Rev. 82 (1951), 837.