Appendix materials for "The Growing Importance of Social Skills in the Labor Market"

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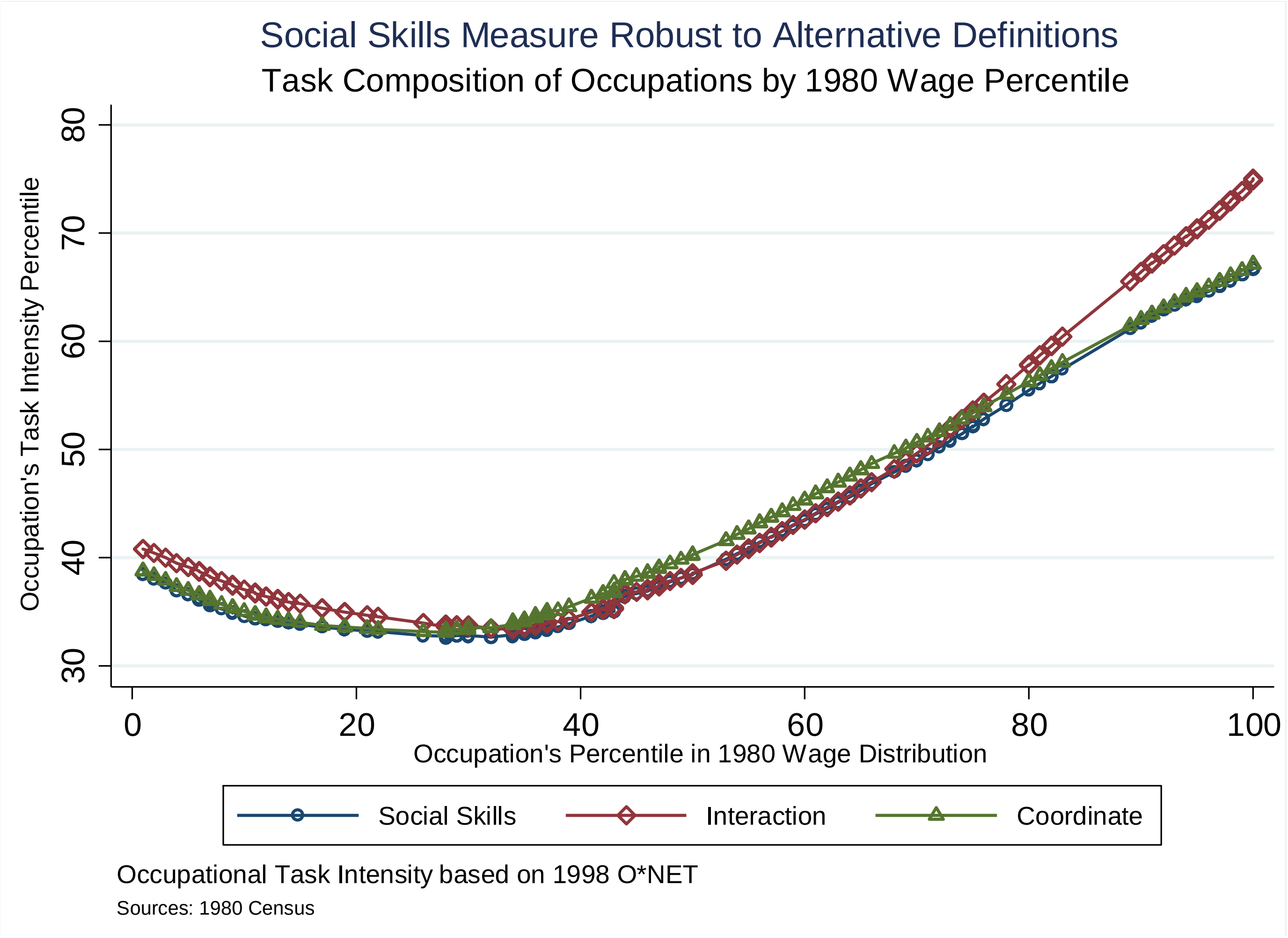
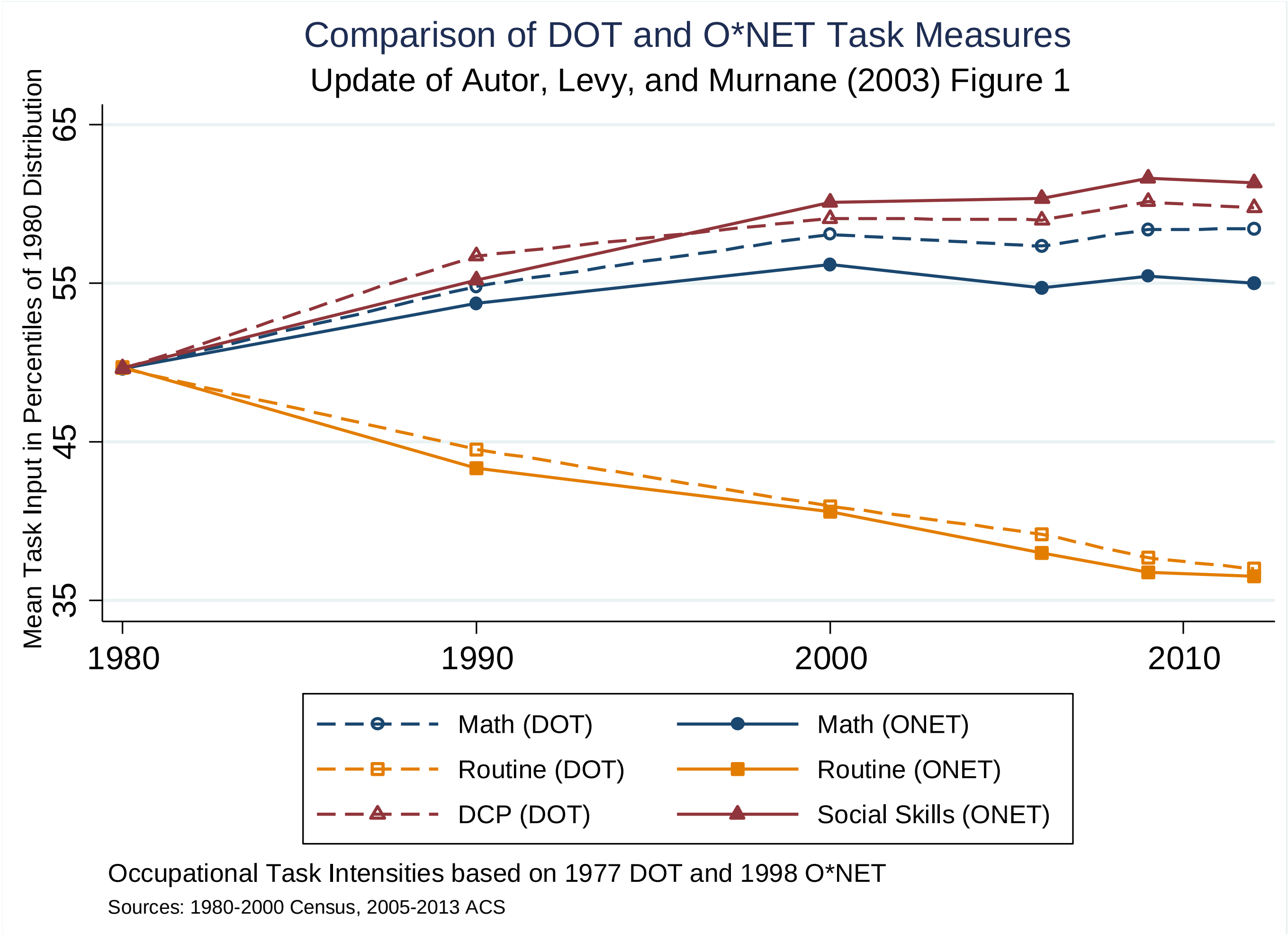
FIGURE A.1



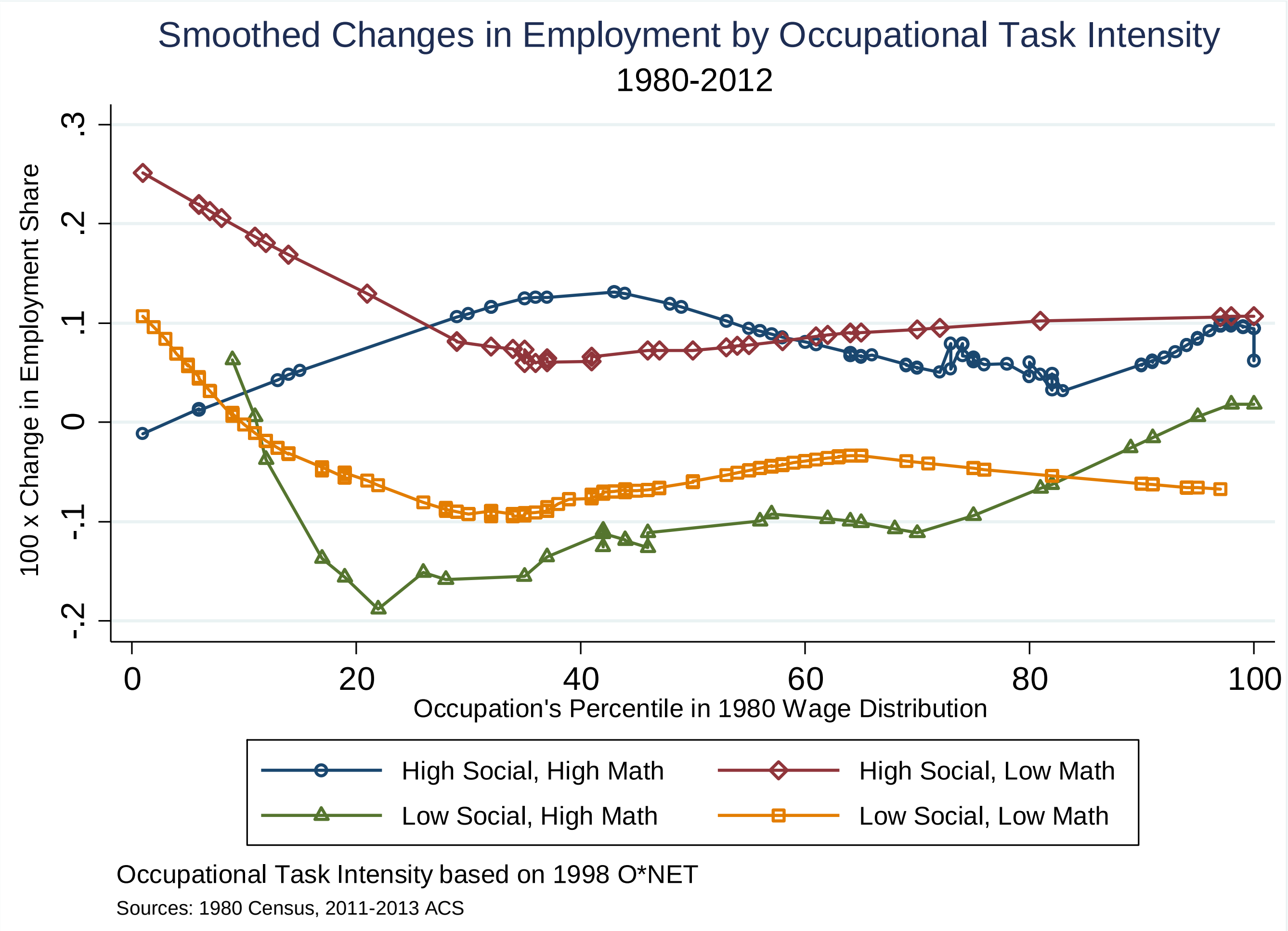
Each line plots the average task intensity of occupations by wage percentile, smoothed using a locally weighted regression with bandwidth 0.8. Task intensity is measured as an occupation's employment-weighted percentile rank in the Census IPUMS 1980 5 percent extract. All task intensities are taken from the 1998 O*NET. Mean log wages in each occupation are calculated using workers' hours of annual labor supply times the Census sampling weights. Consistent occupation codes for 1980-2012 are updated from Autor and Dorn (2013) and Autor and Price (2013).

FIGURE A.2



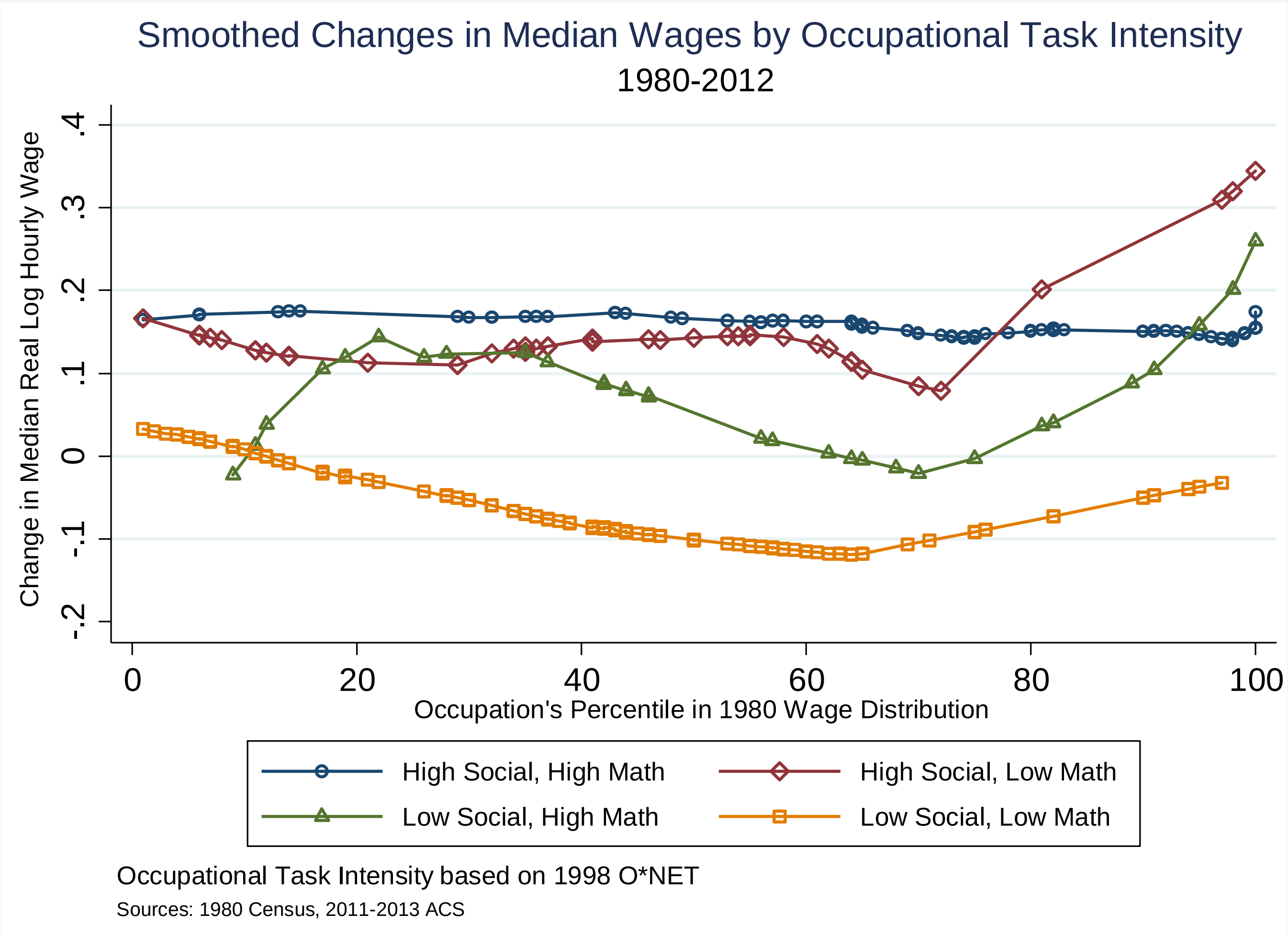
O*NET 1998 and DOT 1977 task measures by occupation are paired with data from the IPUMS 1980-2000 Censuses and the 2005-2013 American Community Survey samples. Consistent occupation codes for 1980-2012 are updated from Autor and Dorn (2013) and Autor and Price (2013). Data are aggregated to industry-education-sex cells by year, and each cell is assigned a value corresponding to its rank in the 1980 distribution of task input. Plotted values depict the employment-weighted mean of each assigned percentile in the indicated year. See the text and Appendix for details on the construction of O*NET task measures.

FIGURE A.3



Each line plots 100 times the change in employment share between 1980 and 2012 for occupations that are above and/or below the 50th percentile in nonroutine analytical and social skill task intensity as measured by the 1998 O*NET. Lines are smoothed using a locally weighted regression with bandwidth 1.0. Wage percentiles are measured as the employment-weighted percentile rank of an occupation's mean log wage in the Census IPUMS 1980 5 percent extract. Consistent occupation codes for 1980-2012 are updated from Autor and Dorn (2013) and Autor and Price (2013). See the text and Appendix for details on the construction of O*NET task measures.

FIGURE A.4



Each line plots 100 times the change in median log hourly real wages between 1980 and 2012 for occupations that are above and/or below the 50th percentile in nonroutine analytical and social skill task intensity as measured by the 1998 O*NET. Lines are smoothed using a locally weighted regression with bandwidth 1.0. Wage percentiles on the horizontal axis are measured as the employment-weighted percentile rank of an occupation's mean log wage in the Census IPUMS 1980 5 percent extract. Consistent occupation codes for 1980-2012 are updated from Autor and Dorn (2013) and Autor and Price (2013). See the text and Appendix for details on the construction of O*NET task measures.

TABLE A.1

CORRELATION BETWEEN ROUTINE AND SOCIAL SKILL TASK INTENSITY

Outcome is the Routine Task Intensity of an Occupation	(1)	(2)
Social Skill Intensity of Occupation	-0.679***	-0.560***
	[0.113]	[0.155]
Add Other O*NET and DOT tasks		X
Observations	337	337
R-squared	0.439	0.662

Notes: Data from the 1980-2000 Census, 2006-2013 ACS, 1991 DOT, and 1998-2013 O*NET. Observations are at the occupation level. Additional DOT task measures are nonroutine analytical, nonroutine interactive, routine cognitive, routine manual and nonroutine manual. Additional O*NET task measures are Number Facility, Inductive/Deductive Reasoning, Use/Analyze Information, the Service task composite and Require Social Interaction. See text and Appendix for details on all O*NET task measures. All models also control for median log hourly wage and are weighted by total labor supply in each cell. Standard errors are clustered at the occupation level. *** p<0.01, ** p<0.05, * p<0.10

TABLE A.2

LABOR MARKET RETURNS TO COGNITIVE SKILLS AND SOCIAL SKILLS IN THE NLSY79

Outcome is Hourly Wage (in 2012 dollars)	(1)	(2)	(3)	(4)	(5)
Cognitive Skills (AQT, standardized)		4.36***	4.35***	3.95***	2.32***
		[0.17]	[0.16]	[0.17]	[0.18]
Social Skills (standardized)	2.67***	1.59***	1.20***	1.07***	0.75***
		[0.15]	[0.12]	[0.12]	[0.12]
Cognitive * Social			1.04***	1.04***	0.74***
			[0.17]	[0.16]	[0.16]
Non-cognitive Skills (standardized)				1.12***	0.89***
				[0.15]	[0.15]
Demographics and Age / Year Fixed Effects		X	X	X	X
Years of completed education					X
R-squared	0.132	0.171	0.173	0.176	0.195
Observations	133,603	133,603	133,603	133,539	133,539

Notes: Each column reports results from an estimate of equation (18) in the paper, with real hourly wages as the outcome and person-year as the unit of observation. The data source is the National Longitudinal Survey of Youth 1979 cohort (NLSY79). Cognitive skills are measured by each NLSY79 respondent's score on the Armed Forces Qualifying Test (AFQT), and are normalized to have a mean of zero and a standard deviation of one. I use the AFQT score crosswalk developed by Altonji, Bharadwaj and Lange (2012). Social skills is a standardized composite of four variables - 1) sociability in childhood; 2) sociability in adulthood; 3) participation in high school clubs; and 4) participation in team sports - see the text for details on construction of the social skills measure. My measure of "non-cognitive" skills is the normalized average of the Rotter and Rosenberg scores in the NLSY. The regression also controls for race-by-gender indicator variables, age, year, census region, and urbanicity fixed effects - plus additional controls as indicated. Standard errors are in brackets and clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10

TABLE A.3
HETEROGENEITY IN RETURNS TO SKILLS

Outcome is Log Hourly Wage	Males	Females	Nonwhite	White	No College	Some College
(in 2012 dollars)	(1)	(2)	(3)	(4)	(5)	(6)
Cognitive Skills (AQT, standardized)	0.167***	0.213***	0.208***	0.185***	0.131***	0.169***
	[0.009]	[0.010]	[0.009]	[0.008]	[0.010]	[0.012]
Social Skills (standardized)	0.055***	0.036***	0.017*	0.044***	0.027***	0.032***
	[0.008]	[0.008]	[0.009]	[0.008]	[0.009]	[0.009]
Cognitive * Social	0.031***	0.004	-0.010	0.023***	-0.002	0.021**
	[0.008]	[0.008]	[0.009]	[0.008]	[0.008]	[0.010]
Non-cognitive Skills (standardized)	0.050***	0.047***	0.056***	0.046***	0.036***	0.051***
	[0.009]	[0.009]	[0.008]	[0.008]	[0.008]	[0.009]
Demographics and Age / Year Fixed Effects	X	X	X	X	X	X
R-squared	0.361	0.294	0.324	0.346	0.335	0.319
Observations	65,008	61,187	57,135	69,060	67,841	58,354

Notes: Each column reports results from an estimate of equation (18) in the paper, with the natural log of real hourly wages as the outcome and person-year as the unit of observation. The data source is the National Longitudinal Survey of Youth 1979 cohort (NLSY79). Cognitive skills are measured by each NLSY79 respondent's score on the Armed Forces Qualifying Test (AFQT), and are normalized to have a mean of zero and a standard deviation of one. I use the AFQT score crosswalk developed by Altonji, Bharadwaj and Lange (2012). Social skills is a standardized composite of four variables - 1) sociability in childhood; 2) sociability in adulthood; 3) participation in high school clubs; and 4) participation in team sports - see the text for details on construction of the social skills measure. My measure of "non-cognitive" skills is the normalized average of the Rotter and Rosenberg scores in the NLSY. The regression also controls for race-by-gender indicator variables, age, year, census region, and urbanicity fixed effects - plus additional controls as indicated. Standard errors are in brackets and clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10

TABLE A.4
HETEROGENEITY IN RETURNS TO SKILLS

Outcome is Hourly Wage (in 2012 dollars)	Males	Females	Nonwhite	White	No College	Some College
	(1)	(2)	(3)	(4)	(5)	(6)
Cognitive Skills (AQT, standardized)	3.93***	3.73***	3.89***	3.96***	1.95***	3.92***
	[0.25]	[0.21]	[0.19]	[0.20]	[0.18]	[0.31]
Social Skills (standardized)	1.48***	0.76***	0.66***	1.00***	0.75***	0.66***
	[0.22]	[0.13]	[0.21]	[0.16]	[0.16]	[0.19]
Cognitive * Social	1.46***	0.61***	0.31	1.19***	0.13	1.17***
	[0.26]	[0.18]	[0.21]	[0.22]	[0.16]	[0.27]
Non-cognitive Skills (standardized)	1.45***	0.85***	0.94***	1.17***	0.55***	1.42***
	[0.24]	[0.19]	[0.14]	[0.19]	[0.14]	[0.25]
Demographics and Age / Year Fixed Effects	X	X	X	X	X	X
R-squared	0.176	0.133	0.157	0.173	0.116	0.181
Observations	68,187	65,352	60,012	73,527	68,322	65,217

Notes: Each column reports results from an estimate of equation (18) in the paper, with real hourly wages as the outcome and person-year as the unit of observation. The data source is the National Longitudinal Survey of Youth 1979 cohort (NLSY79). Cognitive skills are measured by each NLSY79 respondent's score on the Armed Forces Qualifying Test (AFQT), and are normalized to have a mean of zero and a standard deviation of one. I use the AFQT score crosswalk developed by Altonji, Bharadwaj and Lange (2012). Social skills is a standardized composite of four variables - 1) sociability in childhood; 2) sociability in adulthood; 3) participation in high school clubs; and 4) participation in team sports - see the text for details on construction of the social skills measure. My measure of "non-cognitive" skills is the normalized average of the Rotter and Rosenberg scores in the NLSY. The regression also controls for race-by-gender indicator variables, age, year, census region, and urbanicity fixed effects - plus additional controls as indicated. Standard errors are in brackets and clustered at the individual level. *** p<0.01, ** p<0.05, * p<0.10

TABLE A.5
CHANGES IN EMPLOYMENT BY OCCUPATION TASK INTENSITY IN THE CENSUS/ACS

	1980-2012			1980-1990	1990-2000	2000-2012	2000-2012
Outcome is Log Employment (LS Weighted)	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Math Task Intensity	-0.053**	-0.075*	-0.127	0.130**	-0.196**	-0.054	-0.117**
	[0.024]	[0.042]	[0.099]	[0.058]	[0.089]	[0.048]	[0.056]
Social Skill Task Intensity	0.054***	0.029	-0.025	0.003	0.052	-0.050	-0.069*
	[0.019]	[0.038]	[0.071]	[0.045]	[0.049]	[0.036]	[0.036]
Math * Social		0.006	0.018*	0.002	0.001	0.012*	0.019***
		[0.008]	[0.010]	[0.007]	[0.009]	[0.007]	[0.007]
Routine Task Intensity			-0.049	-0.012	-0.018	-0.024	-0.019
			[0.039]	[0.020]	[0.031]	[0.017]	[0.019]
Service Task Intensity			0.036	-0.019	0.023	0.034**	0.028
			[0.040]	[0.024]	[0.029]	[0.016]	[0.020]
Sex-Education-Industry Fixed Effects	X	X	X	X	X	X	X
Controls for other O*NET Task Measures			X	X	X	X	X
Exclude Mgmt, Health Care and Education							X
R-squared	0.516	0.516	0.521	0.714	0.663	0.716	0.698
Observations	74,212	74,212	74,212	74,212	74,212	74,212	60,739

Notes: Each column reports results from a regression of the natural log of employment in the indicated end year on log employment in the indicated base year, the O*NET task measures and sex-education-industry fixed effects. The data come from the 1980-2000 U.S. Censuses and the 2005-2013 American Community Surveys and are collapsed to year-occupation-industry-sex-education cells, with each cell weighted by labor supply. The O*NET task measures are percentiles that range from 0 to 10 and are weighted by labor supply to conform to the 1980 occupation distribution. The additional O*NET task measures are the O*NET variables Number Facility, Inductive and Deductive Reasoning, and Analyzing and Using Information, Require Social Interaction, Coordinating the Work and Activities of Others, and Communicating with Supervisors, Peers, or Subordinates.. See the text and Appendix for details on the construction of each O*NET task measure and for details on which occupations are classified as Managers, Health Care or Education (Column 7). Standard errors are in brackets and clustered at the occupation level. *** p<0.01, ** p<0.05, * p<0.10

TABLE A.6

CHANGES IN WAGES BY OCCUPATION TASK INTENSITY IN THE CENSUS/ACS

	1980-2012			1980-1990	1990-2000	2000-2012	2000-2012
Outcome is the Log Hourly Wage	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Math Task Intensity	0.0451***	0.0418***	0.0007	0.0044	0.0046	0.0003	-0.0060
	[0.0069]	[0.0101]	[0.0230]	[0.0165]	[0.0162]	[0.0207]	[0.0250]
Social Skill Task Intensity	0.0492***	0.0453***	0.0615***	0.0532***	0.0434***	0.0564***	0.0675***
	[0.0059]	[0.0124]	[0.0159]	[0.0117]	[0.0113]	[0.0143]	[0.0157]
Math * Social		0.0009	-0.0029	-0.0020	-0.0012	-0.0028	-0.0032
		[0.0024]	[0.0023]	[0.0018]	[0.0016]	[0.0021]	[0.0025]
Routine Task Intensity			0.0034	0.0038	0.0027	0.0028	0.0049
			[0.0057]	[0.0046]	[0.0042]	[0.0051]	[0.0052]
Service Task Intensity			-0.0126	-0.0216***	-0.0150***	-0.0118	-0.0255***
			[0.0085]	[0.0063]	[0.0058]	[0.0077]	[0.0097]
Sex-Education-Industry Fixed Effects	X	X	X	X	X	X	X
Controls for other O*NET Task Measures			X	X	X	X	X
Exclude Mgmt, Health Care and Education							X
R-squared	0.501	0.501	0.516	0.564	0.557	0.525	0.501
Observations	74,212	74,212	74,212	74,212	74,212	74,212	60,739

Notes: Each column reports results from a regression of the natural log of real (indexed to 2012) median hourly wages in the indicated end year on log hourly wages in the indicated base year, the O*NET task measures and sex-education-industry fixed effects. The data come from the 1980-2000 U.S. Censuses and the 2005-2013 American Community Surveys and are collapsed to year-occupation-industry-sex-education cells, with each cell weighted by labor supply. The O*NET task measures are percentiles that range from 0 to 10 and are weighted by labor supply to conform to the 1980 occupation distribution. The additional O*NET task measures are the O*NET variables Number Facility, Inductive and Deductive Reasoning, and Analyzing and Using Information, Require Social Interaction, Coordinating the Work and Activities of Others, and Communicating with Supervisors, Peers, or Subordinates. See the text and Appendix for details on the construction of each O*NET task measure and for details on which occupations are classified as Managers, Health Care or Education (Column 7). Standard errors are in brackets and clustered at the occupation level. *** p<0.01, ** p<0.05, * p<0.10

Data Appendix

January 2016

1 Changes to the *Occ1990dd* Occupation System

I made edits to the Autor and Dorn (2013) *Occ1990dd* Occupation System to:

1. extend the system to cover the 2010 Census/ACS occupation codes;

2. attempt to improve consistency of definitions of occupations over time; and

3. dissagregate codes in *occ1990dd* when possible.

This edited and updated version of the *occ1990dd* occupation system contains 341 occupation codes.

1.1 2010 Occupation Codes

To extend the system to cover the 2010 occupation codes, I examined the mapping between the 2005-2009 ACS OCC codes and the 2010-2013 ACS OCC codes.[1] For each 2010 OCC code with an equivalent 2005 OCC code, I assigned the *occ1990dd* code that was associated with the equivalent 2005 OCC code, as given by Autor and Dorn's crosswalk between *occ1990dd* codes and 2005 OCC codes. For example, the 2005 OCC code 12 (Financial Managers) is mapped to the in the *occ1990dd* code 7. Therefore, I map the 2010 OCC code 120 (Financial Managers) to the *occ1990dd* code 7 as well. For the few new 2010 OCC codes that did not have an obvious equivalent in the set of 2005 OCC codes, I used my best judgment. For example, I mapped the 2010 OCC code 0425 (Emergency Management Directors) to *occ1990dd* code 22 (Managers and administrators, n.e.c.). Using this procedure, I created a crosswalk between the 2010 Census/ACS occupation codes and the existing *occ1990dd* codes provided by Autor and Dorn.

1.2 Improving Consistency of Definitions Over Time

After creating the crosswalk between the 2010 occupation codes and the existing *occ1990dd* codes, I attempted to improve the consistency of definitions of occupations over time. To do so, I examined each *occ1990dd* code and the associated 1980, 1990, 2000, 2005, and now 2010 OCC codes. I checked for consistency in definitions across time, using the 1990-2000 OCC codes crosswalk in Table 2 of Scopp (2003) as a reference. When I found an inconsistency, I attempted to resolve it by remapping OCC codes to the appropriate *occ1990dd* code. For example, prior to editing, the *occ1990dd* code 308 (Computer and peripheral equipment operators) was linked to the 1980 and 1990 OCC codes: 304, 308, and 309 and the 2000 OCC code 580 as shown in Panel A of Table 1.

[1] Retrieved from https://usa.ipums.org/usa/volii/c2ssoccup.shtml on July 17, 2015

Table 1: Improving Consistency of Definitions (Example)

Panel A: *Occ1990dd* codes 303 and 308, prior to editing

***Occ1990dd* code**	**1980 Census Codes**	**1990 Census Codes**	**2000 Census (5% Sample) Codes**	**2005 ACS Codes**	**2010 ACS Codes**
303-Office supervisors	**303**-Supervisors, general office **305**-Supervisors, financial records processing	**303**-Supervisors, general office **305**-Supervisors, financial records processing	**500**-First-line supervisors/ managers of office and administrative support workers	**500**-First-line supervisors/ managers of office and administrative support workers	**500**0-First-line supervisors/ managers of office and administrative support workers
308-Computer and peripheral equipment operators	**304**-Supervisors, computer equipment operators **308**-Computer operators **309**-Peripheral equipment operators	**304**-Supervisors, computer equipment operators **308**-Computer operators **309**-Peripheral equipment operators	**580**-Computer operators	**580**-Computer operators	**5800**-Computer operators

Panel B: *Occ1990dd* codes 303 and 308, after editing

***Occ1990dd* code**	**1980 Census Codes**	**1990 Census Codes**	**2000 Census (5% Sample) Codes**	**2005 ACS Codes**	**2010 ACS Codes**
303-Office supervisors	**303**-Supervisors, general office **304**-Supervisors, computer equipment operators **305**-Supervisors, financial records processing	**303**- Supervisors, general office **304**-Supervisors, computer equipment operators **305**-Supervisors, financial records processing	**500**-First-line supervisors/ managers of office and administrative support workers	**500**-First-line supervisors/ managers of office and administrative support workers	**500**0-First-line supervisors/ managers of office and administrative support workers
308-Computer and peripheral equipment operators	**308**-Computer operators **309**-Peripheral equipment operators	**308**-Computer operators **309**-Peripheral equipment operators	**580**-Computer operators	**580**-Computer operators	**5800**-Computer operators

According to Scopp's (2003) 1990-2000 crosswalk, the 1990 OCC code 304 (Supervisors, computer equipment operators) gets entirely redistributed into the 2000 OCC code 500 (First-line supervisors/managers of office and administrative support workers), which is linked to *occ1990dd* code 303 (Office supervisors). Therefore, I remap the 1980 and 1990 OCC code 304 to the *occ1990dd* code 303 (Office supervisors) so that supervisors of computer equipment operators are consistently contained over time in the *occ1990dd* code 303. In contrast, the 1990 OCC code 309 (Peripheral equipment operators) largely redistributes into the 2000 OCC code 580 (Computer operators), so the 1980 and 1990 OCC code 309 remains mapped to *occ1990dd* code 308 as shown in Panel B of Table 1.

1.3 Disaggregating Codes

To disaggregate *occ1990dd* codes when possible, I also examined each *occ1990dd* code and the associated 1980, 1990, 2000, 2005, and 2010 OCC codes. Among the codes associated with each *occ1990dd* code, I searched for a set of 1980, 1990, 2000, 2005, and 2010 OCC codes that provided a consistent definition of an occupation that could stand alone as a separate occupation group. For example, prior to editing, the *occ1990dd* code 59 (Engineers and other professionals, n.e.c.) was mapped to OCC codes as shown in Panel A of Table 2. Among this group of codes, the occupation Marine Engineers and Naval Architects can be separated into its own group. Therefore, I created an additional *occ1990dd* code 58 (Marine engineers and naval architects) consisting of 1980 OCC code 58, 1990 OCC code 58, 2000 OCC code 144, 2005 OCC code 144, and 2010 OCC code 1440 as shown in Panel B of Table 2.

In contrast, the occupation Nuclear Engineers cannot stand alone as its own *occ1990dd* code. The occupation Nuclear Engineers has its own OCC code in 1980 (49), 1990 (49), and 2000 (151), but the occupation is joined with the miscellaneous engineers OCC code 153 in 2005 and 2010. This occupation cannot be separated from the other codes associated with *occ1990dd* code 59 (Engineers and other professionals, n.e.c.).

Table 2: Disaggregating Codes (Example)

Panel A: *Occ1990dd* code 59, prior to editing

***Occ1990dd* code**	**1980 Census Codes**	**1990 Census Codes**	**2000 Census (5% Sample) Codes**	**2005 ACS Codes**	**2010 ACS Codes**
59-Engineers and other professionals, n.e.c.	**49**-Nuclear engineers **54**-Agricultural engineers **58**-Marine engineers and naval architects **59**-Engineers, n.e.c	**49**-Nuclear engineers **54**-Agricultural engineers **58**-Marine engineers and naval architects **59**-Engineers, n.e.c	**142**-Environmental engineers **144**-Marine engineers **151**-Nuclear engineers **153**-Miscellaneous engineers, including agricultural and biomedical	**134**-Biomedical and agricultural engineers **142**-Environmental engineers **144**-Marine engineers and Naval architects **153**-Miscellaneous engineers including nuclear engineers	**1340**-Biomedical and agricultural engineers **1420**-Environmental engineers **1440**-Marine engineers and naval architects **1530**-Miscellaneous engineers including nuclear engineers

Panel B: *Occ1990dd* codes 58 and 59, after editing

***Occ1990dd* code**	**1980 Census Codes**	**1990 Census Codes**	**2000 Census (5% Sample) Codes**	**2005 ACS Codes**	**2010 ACS Codes**
59-Engineers and other professionals, n.e.c.	**49**-Nuclear engineers **54**-Agricultural engineers **59**-Engineers, n.e.c	**49**-Nuclear engineers **54**-Agricultural engineers **59**-Engineers, n.e.c	**142**-Environmental engineers **151**-Nuclear engineers **153**-Miscellaneous engineers, including agricultural and biomedical	**134**-Biomedical and agricultural engineers **142**-Environmental engineers **153**-Miscellaneous engineers including nuclear engineers	**1340**-Biomedical and agricultural engineers **1420**-Environmental engineers **1530**-Miscellaneous engineers including nuclear engineers
58-Marine engineers and naval Architects	**58**-Marine engineers and naval architects	**58**-Marine engineers and naval architects	**144**-Marine engineers	**144**-Marine engineers and naval architects	**1440**-Marine engineers and naval architects

The updated and edited *occ1990dd* codes, descriptions and associated OCC codes are displayed in Table 3.

Table 3: Updated and edited *occ1990dd* occupation system, based on earlier work by Autor and Dorn (2013)

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
4	Chief executives, public administrators, and legislators	3	3	1	1	10
		4	4	3		
7	Financial managers	7	7	12	12	120
8	Human resources and labor relations managers	8	8	13	13	135
						136
						137
9	Purchasing managers	9	9	15	15	150
13	Managers in marketing, advert., PR	13	13	4	4	40
				5	5	50
				6	6	60
14	Managers in education and related fields	14	14	23	23	230
15	Managers of medicine and health occupations	15	15	35	35	350
18	Managers of properties and real estate	16	18	41	41	410
19	Funeral directors	18	19	32	32	4465
22	Managers and administrators, n.e.c.	5	5	2	2	20
		17	16	10	10	100
		19	17	11	11	110
			21	14	14	140
			22	16	16	160
				22	22	220
				30	30	300
				31	31	310
				33	33	330
				34	34	340
				36	36	360
				40	42	420
				42	43	425
				43	60	430
				60	72	600
				72	430	725
				430		4300
23	Accountants and auditors	23	23	80	80	800
				93	93	930
24	Insurance underwriters	24	24	86	86	860
25	Other financial specialists	25	25	82	82	820
				83	83	830
				84	84	840
				85	85	850
				91	91	910
				94	94	940
				95	95	950
26	Management analysts	26	26	71	71	710
27	Personnel, HR, training, and labor rel. specialists	27	27	62	62	630
						640
						650
28	Purchasing agents and buyers of farm products	28	28	51	51	510
29	Buyers, wholesale and retail trade	29	29	52	52	520
33	Purchasing agents and buyers, n.e.c.	33	33	53	53	530
34	Business and promotion agents	34	34	50	50	500
35	Construction inspectors	35	35	666	666	6660
36	Inspectors and compliance officers, outside	36	36	56	56	565
				90	90	900
37	Management support occupations	37	37	73	73	726
						740
43	Architects	43	43	130	130	1300
44	Aerospace engineers	44	44	132	132	1320
45	Metallurgical and materials engineers	45	45	145	145	1450
47	Petroleum, mining, and geological engineers	46	46	152	152	1520
		47	47			

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
47	Petroleum, mining, and geological engineers	46	46	152	152	1520
		47	47			
48	Chemical engineers	48	48	135	135	1350
53	Civil engineers	53	53	136	136	1360
55	Electrical engineers	55	55	140	140	1400
				141	141	1410
56	Industrial engineers	56	56	143	143	1430
57	Mechanical engineers	57	57	146	146	1460
58	Marine engineers and naval architects	58	58	144	144	1440
59	Engineers and other professionals, n.e.c.	49	49	142	134	1340
		54	54	151	142	1420
		59	59	153	153	1530
64	Computer systems analysts and computer scientists	64	64	100	100	1005
				102	102	1006
				104	104	1007
				106	106	1020
				110	110	1030
				111	111	1050
						1060
						1105
						1106
						1107
65	Operations and systems researchers and analysts	65	65	70	70	700
				122	122	1220
66	Actuaries	66	66	120	120	1200
68	Mathematicians and statisticians	67	67	124	124	1240
		68	68			
69	Physicists and astronomists	69	69	170	170	1700
73	Chemists	73	73	172	172	1720
74	Atmospheric and space scientists	74	74	171	171	1710
75	Geologists	75	75	174	174	1740
76	Physical scientists, n.e.c.	76	76	176	176	1760
77	Agricultural and food scientists	77	77	160	160	1600
78	Biological scientists	78	78	161	161	1610
79	Foresters and conservation scientists	79	79	164	164	1640
83	Medical scientists	83	83	165	165	1650
84	Physicians	84	84	306	306	3060
85	Dentists	85	85	301	301	3010
86	Veterinarians	86	86	325	325	3250
87	Optometrists	87	87	304	304	3040
88	Podiatrists	88	88	312	312	3120
89	Other health and therapy occupations	89	89	300	300	3000
				326	326	3260
95	Registered nurses	95	95	313	313	3255
						3256
						3258
96	Pharmacists	96	96	305	305	3050
97	Dieticians and nutritionists	97	97	303	303	3030
98	Respiratory therapists	98	98	322	322	3220
99	Occupational therapists	99	99	315	315	3150
103	Physical therapists	103	103	316	316	3160
104	Speech therapists	104	104	314	314	3140
				323	323	3230
105	Therapists, n.e.c.	105	105	320	320	3200
				321	321	3210
				324	324	3245
106	Physicians' assistants	106	106	311	311	3110

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
154	Subject instructors, college	113	113	220	220	2200
		114	114			
		115	115			
		116	116			
		117	117			
		118	118			
		119	119			
		123	123			
		124	124			
		125	125			
		126	126			
		127	127			
		128	128			
		129	129			
		133	133			
		134	134			
		135	135			
		136	136			
		137	137			
		138	138			
		139	139			
		143	143			
		144	144			
		145	145			
		146	146			
		147	147			
		148	148			
		149	149			
		153	153			
		154	154			
155	Kindergarten and earlier school teachers	155	155	230	230	2300
156	Primary school teachers	156	156	231	231	2310
157	Secondary school teachers	157	157	232	232	2320
158	Special education teachers	158	158	233	233	2330
159	Teachers, n.e.c.	159	159	234	234	2340
				255	255	2550
163	Vocational and educational counselors	163	163	200	200	2000
164	Librarians	164	164	243	243	2430
165	Archivists and curators	165	165	240	240	2400
166	Economists, market and survey researchers	166	166	180	180	735
				181	181	1800
167	Psychologists	167	167	182	182	1820
169	Social scientists and sociologists, n.e.c.	168	168	186	186	1860
		169	169			
173	Urban and regional planners	173	173	184	184	1840
174	Social workers	174	174	201	201	2010
175	Religious workers, n.e.c.	177	177	205	205	2050
				206	206	2060
176	Clergy	176	176	204	204	2040
177	Welfare service workers	467	465	202	202	2015
						2016
						2025
178	Lawyers and judges	178	178	210	210	2100
		179	179	211		2105
183	Writersandauthors	183	183	285	285	2850
184	Technical writers	184	184	284	284	2840
185	Designers	185	185	263	263	2630
186	Musicians and composers	186	186	275	275	2750
187	Actors, directors, and producers	187	187	270	270	2700
				271	271	2710
188	Painters, sculptors, craft-artists, and print-makers	188	188	260	260	2600
189	Photographers	189	189	291	291	2910
				292	292	2920
193	Dancers	193	193	274	274	2740
194	Art/entertainment performers and related occs	194	194	276	276	2760
				286	286	2860

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
195	Editors and reporters	195	195	281	281	2810
				283	283	2830
197	Specialists in marketing, advert., PR	197	197	282	282	2825
198	Announcers	198	198	280	280	2800
199	Athletes, sports instructors, and officials	199	199	272	272	2720
203	Clinical laboratory technologies and technicians	203	203	330	330	3300
204	Dental hygienists	204	204	331	331	3310
205	Health record technologists and technicians	205	205	351	351	3510
206	Radiologic technologists and technicians	206	206	332	332	3320
207	Licensed practical nurses	207	207	350	350	3500
208	Health technologists and technicians, n.e.c.	208	208	340	340	3400
				341	341	3420
				353	353	3535
				354	354	3540
214	Engineering and science technicians	213	213	155	155	1550
		214	214	193	193	1930
		215	215			1940
		216	216			
		225	225			
217	Drafters	217	217	154	154	1540
218	Surveryors, cartographers, mapping scientists/techs	63	63	131	131	1310
		218	218	156	156	1560
		867	867			
223	Biological technicians	223	223	190	190	1900
				191	191	1910
224	Chemical technicians	224	224	192	192	1920
226	Airplane pilots and navigators	226	226	903	903	9030
227	Air traffic controllers	227	227	904	904	9040
228	Broadcast equipment operators	228	228	290	290	2900
229	Computer programmers	229	229	101	101	1010
233	Programmers of numerically controlled machine tools	233	233	790	790	7900
		714	714			
234	Legal assistants and paralegals	234	234	214	214	2145
		314	314	215	215	2160
235	Technicians, n.e.c.	235	235	196	196	1950
						1965
243	Sales supervisors and proprietors	243	243	470	470	4700
				471	471	4710
253	Insurance sales occupations	253	253	481	481	4810
254	Real estate sales occupations	254	254	81	81	810
				492	492	4920
255	Financial service sales occupations	255	255	482	482	4820
256	Advertising and related sales jobs	256	256	480	480	4800
258	Sales engineers	258	258	493	493	4930
269	Parts salesperson	269	269	475	475	4750
270	Sales workers	263	263	476	476	4760
		264	264			
		265	265			
		266	266			
		267	267			
		268	268			
		274	274			
274	Sales occupations and sales representatives	257	257	484	484	4840
		259	259	485	485	4850
				494	494	4940
275	Sales counter clerks	275	275	474	474	4740
276	Cashiers	276	276	472	472	4720
				513	513	5130
277	Door-to-door sales, street sales, and news vendors	277	277	495	495	4950
		278	278			
283	Sales demonstrators, promoters, and models	283	283	490	490	4900
285	Auctioneers and sales support occupations, n.e.c.	284	284	496	496	4965
		285	285			

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
303	Office supervisors	303	303	500	500	5000
		304	304			
		305	305			
		306	306			
		307	307			
308	Computer and peripheral equipment operators	308	308	580	580	5800
		309	309			
313	Secretaries and administrative assistants	313	313	570	570	5700
315	Typists	315	315	582	582	5820
316	Interviewers, enumerators, and surveyors	316	316	523	523	5230
				531	531	5310
				534	534	5340
317	Hotel clerks	317	317	530	530	5300
318	Transportation ticket and reservation agents	318	318	483	483	4830
				541	541	5410
319	Receptionists and other information clerks	319	319	540	540	5400
		323	323			
326	Correspondence and order clerks	325	325	535	535	5350
		326	326			
		327	327			
328	Human resources clerks, excl payroll and timekeeping	328	328	536	536	5360
329	Library assistants	329	329	244	244	2440
				532	532	5320
335	File clerks	335	335	526	526	5260
336	Records clerks	336	336	520	520	5200
				542	542	5420
337	Bookkeepers and accounting and auditing clerks	337	337	512	512	5120
338	Payroll and timekeeping clerks	338	338	514	514	5140
344	Billing clerks and related financial records processing	339	339	511	511	5110
		343	343			
		344	344			
347	Office machine operators, n.e.c.	345	345	590	590	5900
		347	347			
348	Telephone operators	348	348	501	501	5010
				502	502	5020
349	Other telecom operators	349	353	503	503	5030
		353				
354	Postal clerks, exluding mail carriers	354	354	554	554	5540
				556	556	5560
355	Mail carriers for postal service	355	355	555	555	5550
356	Mail clerks, outside of post office	346	346	585	585	5850
		356	356			
357	Messengers	357	357	551	551	5510
359	Dispatchers	359	359	552	552	5520
364	Shipping and receiving clerks	364	364	550	550	5500
				561	561	5610
365	Stock and inventory clerks	365	365	515	515	5150
		374	374	562	562	5620
366	Meter readers	366	366	553	553	5530
368	Weighers, measurers, and checkers	368	368	563	563	5630
		369				
373	Material recording, sched., prod., plan., expediting cl.	363	363	560	560	5600
		373	373			
375	Insurance adjusters, examiners, and investigators	375	375	54	54	540
				584	584	5840
376	Customer service reps, invest., adjusters, excl. insur.	376	376	524	524	5240
				533	533	5330
377	Eligibility clerks for government prog., social welfare	377	377	525	525	5250
378	Bill and account collectors	378	378	510	510	5100
379	General office clerks	379	379	586	586	5860
383	Bank tellers	383	383	516	516	5160
384	Proofreaders	384	384	591	591	5910

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
385	Data entry keyers	385	385	581	581	5810
386	Statistical clerks	386	386	592	592	5920
387	Teacher's aides	387	387	254	254	2540
389	Administrative support jobs, n.e.c.	389	389	522	522	5165
				583	593	5220
				593		5940
405	Housekeepers, maids, butlers, and cleaners	405	405	423	423	4230
		407	407			
		449	449			
408	Laundry and dry cleaning workers	748	748	830	830	8300
413	Supervisors, firefighting and fire prevention occupations	413	413	372	372	3720
414	Supervisors, police and detectives	6	6	371	371	3710
		414	414			
415	Supervisors of guards	415	415	373	373	3730
417	Fire fighting, inspection, and prevention occupations	416	416	374	374	3740
		417	417	375	375	3750
418	Police and detectives, public service	418	418	370	370	3700
				382	382	3820
				385	385	3850
423	Sheriffs, bailiffs, correctional institution officers	423	423	380	380	3800
		424	424	384	384	3840
425	Crossing guards	425	425	394	394	3940
426	Guards and police, except public service	426	426	391	391	3910
				392	392	3930
427	Protective service, n.e.c.	427	427	390	390	3900
				395	395	3955
433	Supervisors of food preparation and service	433	433	401	401	4010
434	Bartenders	434	434	404	404	4040
435	Waiters and waitresses	435	435	411	411	4110
436	Cooks	404	404	400	400	4000
		436	436	402	402	4020
		437				
439	Food preparation workers	439	439	403	403	4030
444	Miscellanious food preparation and service workers	438	438	405	405	4050
		443	443	406	406	4060
		444	444	412	412	4120
				413	413	4130
				414	414	4140
				415	415	4150
445	Dental assistants	445	445	364	364	3640
447	Health and nursing aides	446	446	360	360	3600
		447	447	361	361	3610
				362	362	3620
				365	365	3645
				461	461	3646
						3647
						3648
						3649
						3655
						4610
448	Supervisors of cleaning and building service	448	448	420	420	4200
450	Superv. of landscaping, lawn service, groundskeeping	485	485	421	421	4210
451	Gardeners and groundskeepers	474	474	425	425	4250
		486	486			
453	Janitors	453	453	422	422	4220
455	Pest control occupations	455	455	424	424	4240
457	Barbers	457	457	450	450	4500
458	Hairdressers and cosmetologists	458	458	451	451	4510
				452	452	4520
459	Recreation facility attendants	459	459	440	440	4400
				443	443	4430

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
461	Guides	463	461	454	454	4540
462	Ushers	464	462	442	442	4420
464	Baggage porters, bellhops and concierges	466	464	453	453	4530
466	Recreation and fitness workers	175	175	462	462	4620
467	Motion picture projectionists	773	773	441	441	4410
468	Childcareworkers	406	406	460	460	4600
		468	466	464	464	4640
			467			
			468			
469	Personal service occupations, n.e.c	469	469	363	363	3630
				446	446	4460
				465	465	4650
470	Supervisors of personal service jobs, n.e.c	456	456	432	432	4320
471	Public transportation attendants	465	463	455	455	9050
						9415
472	Animal caretakers, except farm	487	487	435	435	4350
473	Farmers, ranchers, and other agricultural managers	473	473	20	20	205
		475	475	21	21	
		476	476			
479	Farm workers, incl. nursery farming, and marine life cultivation workers	479	479	434	434	4340
		483	483	605	605	6050
		484	484			
488	Graders and sorters of agricultural products	488	488	604	604	6040
489	Inspectors of agricultural products	489	489	601	601	6010
494	Supervisors, forestry and logging workers	477	477	600	600	6005
		494	494			
496	Timber, logging, and forestry workers	495	495	612	612	6120
		496	496	613	613	6130
498	Fishing and hunting workers	497	497	610	610	6100
		498	498			
		499	499			
503	Supervisors of mechanics and repairers	503	503	700	700	7000
505	Automobile mechanics and repairers	505	505	720	720	7200
		506	506			
507	Bus, truck, and stationary engine mechanics	507	507	721	721	7210
508	Aircraft mechanics	508	508	714	714	7140
		515	515			
509	Small engine repairers	509	509	724	724	7240
514	Auto body repairers	514	514	715	715	7150
				716	716	7160
516	Heavy equipement and farm equipment mechanics	516	516	722	722	7220
		517	517			
518	Industrial machinery repairers	518	518	733	733	7330
519	Machinery maintenance occupations	519	519	735	735	7350
523	Repairers of industrial electrical equipment	523	523	710	710	7100
				712	712	7120
525	Repairers of data processing equipment	525	525	701	701	7010
		538	538			
526	Repairers of household appliances and power tools	526	526	732	732	7320
527	Telecom and line installers and repairers	527	527	702	702	7020
		529	529	742	742	7420
533	Repairers of electrical equipment, n.e.c.	533	533	703	703	7030
				704	704	7040
				711	711	7110
534	Heating, air conditioning, and refrigeration mechanics	534	534	731	731	7315
535	Precision instrument and equipment repairers	535	535	743	743	7430
536	Locksmiths and safe repairers	536	536	754	754	7540
539	Repairers of mechanical controls and valves	539	539	730	730	7300
543	Elevator installers and repairers	543	543	670	670	6700
544	Millwrights	544	544	736	736	7360
549	Mechanics and repairers, n.e.c.	547	547	734	734	7340
		549	549	751	751	7510
		864		755	755	7550
				756	756	7560
				762	762	7630

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
558	Supervisors of construction work	553	553	620	620	6200
		554	554			
		555	555			
		556	556			
		557	557			
		558	558			
		613	613			
563	Masons, tilers, and carpet installers	563	563	622	622	6220
		564	564	624	624	6240
		565	565			
		566	566			
567	Carpenters	567	567	623	623	6230
		569	569			
573	Drywall installers	573	573	633	633	6330
575	Electricians	575	575	635	635	6355
		576	576	713	713	7130
577	Electric power installers and repairers	577	577	741	741	7410
579	Painters, construction and maintenance	579	579	642	642	6420
583	Paperhangers	583	583	643	643	6430
584	Plasterers	584	584	646	646	6460
585	Plumbers, pipe fitters, and steamfitters	585	585	644	644	6440
		587	587			
588	Concrete and cement workers	588	588	625	625	6250
589	Glaziers	589	589	636	636	6360
593	Insulation workers	593	593	640	640	6400
				672	672	6720
594	Paving, surfacing, and tamping equipment operators	594	594	630	630	6300
595	Roofers	595	595	651	651	6515
597	Structural metal workers	597	597	653	650	6500
				774	653	6530
					774	7740
598	Drillers of earth	598	598	682	682	6820
599	Misc. construction and related occupations	599	599	671	671	6710
				675	676	6765
				676		
614	Drillers of oil wells	614	614	680	680	6800
615	Explosives workers	615	615	683	683	6830
616	Miners	616	616	684	684	6840
617	Other mining occupations	617	617	694	694	6940
			868			
628	Production supervisors or foremen	633	628	770	770	7700
		863				
634	Tool and die makers and die setters	634	634	813	813	8130
		635	635			
637	Machinists	637	637	803	803	8030
		639	639			
643	Boilermakers	643	643	621	621	6210
644	Precision grinders and fitters	644	644	821	821	8210
645	Patternmakers and model makers, metal and plastic	645	645	806	806	8060
		676	676			
647	Jewelers and precious stone and metal workers	647	647	875	875	8750
649	Engravers	649	649	891	891	8910
		793	793			
653	Sheet metal workers	596	596	652	652	6520
		653	653			
		654	654			
657	Cabinetmakers and bench carpeters	657	657	850	850	8500
658	Furniture and wood finishers	658	658	851	851	8510
666	Tailors, dressmakers, and sewers	666	666	835	835	8350
		667	667			
668	Upholsterers	668	668	845	845	8450
669	Shoe and leather workers and repairers	669	669	833	833	8330
675	Hand molders, shapers, and casters, except jewelers	675	675	892	892	8920
		787	787			

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
677	Optical goods workers	677	677	352	352	3520
678	Dental laboratory and medical applicance technicians	678	678	876	876	8760
684	Miscellaneous precision workers, n.e.c.	646	646	816	822	8220
		684	684	822		
		705	705			
		715	715			
		717	717			
686	Butchers and meat cutters	686	686	781	781	7810
687	Bakers	687	687	780	780	7800
688	Batch food makers	688	688	784	784	7840
694	Water and sewage treatment plant operators	694	694	862	862	8620
695	Power plant operators	695	695	860	860	8600
696	Plant and system operators, stationary engineers	696	696	861	861	8610
699	Other plant and system operators	699	699	863	863	8630
703	Lathe and turning machine operatives	703	703	801	801	8010
		704	704			
706	Punching and stamping press operatives	706	706	795	795	7950
707	Rollers, roll hands, and finishers of metal	707	707	794	794	7940
708	Drilling and boring machine operators	708	708	796	796	7960
709	Grinding, abrading, buffing, and polishing workers	655	655	800	800	8000
		709	709			
713	Forge and hammer operators	713	713	793	793	7930
719	Molders and casting machine operators	719	719	810	810	8100
723	Metal platers	723	723	820	820	8200
724	Heat treating equipment operators	724	724	815	815	8150
727	Sawing machine operators and sawyers	727	727	853	853	8530
729	Nail, tacking, shaping and joining mach ops (wood)	726	726	854	854	8540
		728	728			
		729	729			
733	Misc. woodworking machine operators	656	656	855	855	8550
		659	659			
		733	733			
734	Bookbinders and printing machine operators, n.e.c.	679	679	823	823	8255
		734	734	824	824	8256
		737	737	826	826	
736	Typesetters and compositors	735	735	825	825	8250
		736	736			
738	Winding and twisting textile and apparel operatives	738	738	842	842	8420
739	Knitters, loopers, and toppers textile operatives	739	739	841	841	8410
743	Textile cutting and dyeing machine operators	743	743	836	840	8400
				840		
744	Textile sewing machine operators	744	744	832	832	8320
745	Shoemaking machine operators	745	745	834	834	8340
747	Clothing pressing machine operators	403	403	831	831	8310
		747	747			
749	Miscellanious textile machine operators	673	674	846	846	8460
		674	749			
		749				
753	Cementing and gluing machne operators	753	753	885	885	8850
754	Packers, fillers, and wrappers	754	754	880	880	8800
755	Extruding and forming machine operators	755	755	792	792	7920
		758	758	872	872	8720
756	Mixing and blending machine operators	725	725	865	865	8650
		756	756			
		768	768			
757	Separating, filtering, and clarifying machine operators	757	757	864	864	8640
763	Food roasting and baking machine operators	763	763	783	783	7830
764	Washing, cleaning, and pickling machine operators	764	764	886	886	8860

Occ1990dd code	Occ1990dd code description	Census 1980 Codes	Census 1990 Codes	Census 2000 (5% Sample) Codes	ACS 2005 Codes	ACS 2010 Codes
765	Paper folding machine operators	765	765	893	893	8930
766	Furnance, kiln, and oven operators, apart from food	766	766	804	804	8040
				873	873	8730
769	Slicing and cutting machine operators	769	769	871	871	8710
		786	786			
774	Photographic process machine operators	774	774	883	883	8830
779	Machine operators, n.e.c.	777	777	785	785	7850
		779	779	894	894	7855
		794	795	896	896	8940
		795				8965
783	Welders, solderers, and metal cutters	783	783	814	814	8140
		784	784			
785	Assemblers of electrical equipment	636	636	771	771	7710
		683	683	772	772	7720
		693	693	773	773	7730
		785	785	775	775	7750
789	Painting and decoration occupations	759	759	881	881	8810
		789	789			
799	Production checkers, graders, and sorters in manufacturing	689	689	874	874	3945
		796	796	941	941	8740
		797	797			9410
		798	798			
		799	799			
803	Supervisors of motor vehicle transportation	803	803	900	900	9000
		843	843			
			864			
804	Driver/sales workers and truck Drivers	804	804	913	913	9130
		805	806			
		806				
808	Bus drivers	808	808	912	912	9120
809	Taxi drivers and chauffeurs	809	809	914	914	9140
813	Parking lot attendants	813	813	935	935	9350
814	Motor transportation occupations, n.e.c.	814	814	915	911	9110
					915	9150
823	Railroad conductors and yardmasters	823	823	924	924	9240
824	Locomotive operators: engineers and firemen	824	824	920	920	9200
		826	826	926	926	9260
825	Railroad brake, coupler, and switch operators	825	825	923	923	9230
828	Ship and boat captains and operators	828	828	931	931	9310
829	Sailors and deckhands, ship/marine engineers	829	829	930	930	9300
		833	833	933		
834	Miscellanious transportation occupations	834	834	942	942	9420
844	Operating engineers of construction equipment	844	844	632	632	6320
		855	855			
848	Hoist and winch operators	848	848	956	956	9560
849	Crane and tower operators	845	845	951	951	9510
		849	849			
853	Excavating and loading machine operators	853	853	952	952	9520
856	Industrial truck and tractor operators	856	856	960	960	9600
859	Misc. material moving equipment operators	454	454	965	965	9650
		859	859	975	975	9750
865	Helpers, constructions	865	865	761	761	7610
866	Helpers, surveyors	866	866	660	660	6600
869	Construction laborers	869	869	626	626	6260
				673	673	6730
873	Production helpers	873	874	895	895	8950
		874				
875	Garbage and recyclable material collectors	875	875	972	972	9720
878	Machine feeders and offbearers	878	878	963	963	9630
885	Garage and service station related occupations	885	885	726	726	7260
				936	936	9360
887	Vehicle washers and equipment cleaners	887	887	961	961	9610
888	Packers and packagers by hand	888	888	964	964	9640
889	Laborers, freight, stock, and material handlers, n.e.c.	876	876	674	674	6740
		877	877	962	962	9620
		883	883			
		889	889			

References

Autor, D. and Dorn, D. (2013). The growth of low-skill service jobs and the polarization of the us labor market. *The American Economic Review*, 103(5):1553–1597.

Scopp, T. S. (2003). *The relationship between the 1990 census and census 2000 industry and occupation classification systems*. US Census Bureau Washington, DC.

Model Appendix

February 2017

1 Set Up

1.1 Production Function

Consider a competitive market consisting of a single firm employing two workers.[1] The workers perform a continuum of tasks indexed over the unit interval, $y(i)$ with $i \in [0,1]$, which are combined to produce a final good Y. The production function for the final good takes a Cobb-Douglas form:

$$Y = \exp[\int_0^1 \ln y(i) di]. \tag{1}$$

The production function for task i produced by worker j takes the form:

$$y_j(i) = A_j \alpha_j(i) l_j(i), \tag{2}$$

where A_j is a worker-specific productivity factor, $\alpha_j(i)$ is a worker- and task-specific productivity factor, and $l_j(i)$ is the quantity of labor devoted by worker j to task i.

For simplicity, assume that each worker supplies one unit of labor inelastically to the market:

$$\int_0^1 l_j(i) di = L_j = 1. \tag{3}$$

This model can therefore be interpreted as describing two *types* of workers, with each type having a unit mass.

1.2 Comparative Advantage Schedule

Define:

$$\gamma(i) \equiv \frac{A_1 \alpha_1(i)}{A_2 \alpha_2(i)} \tag{4}$$

to be the comparative advantage of worker 1 relative to worker 2 in task i. Assume that:

$$\gamma'(i) < 0, \tag{5}$$

[1] This model closely follows Dornbusch, Fischer and Samuelson (1977) and Autor and Acemoglu (2011).

so that tasks are indexed in order of decreasing comparative advantage for worker 1 ($\frac{\alpha_1(0)}{\alpha_2(0)} > \cdots > \frac{\alpha_1(i)}{\alpha_2(i)} > \cdots > \frac{\alpha_1(1)}{\alpha_2(1)}$). Furthermore, assume that the comparative advantage schedule takes the form:

$$\gamma(i) = \bar{A}\exp(\theta(1-2i)), \tag{6}$$

where $\bar{A} = A_1/A_2$ and $\theta \geq 0$. Thus θ indicates the steepness of the comparative schedule. A higher θ means that worker 1 has a stronger comparative advantage relative to worker 2 in low-i tasks and worker 2 has a stronger comparative advantage relative to worker 1 in high-i tasks. When $\theta = 0$, worker j's productivity reduces to A_j for all tasks, and there is no comparative advantage - only absolute advantage.

1.3 Productivity Distribution

The functional form for $\gamma(i)$ in 6 is consistent with the notion that a worker's productivity in a given task is is drawn from a log-normal distribution defined by parameters that are increasing functions of A_j and θ. Specifically, suppose that for worker j, productivity in task t is a random variable $a_j(t)$ with a log-normal distribution with a worker-specific mean but equal variance across workers:

$$a_j(t) \sim \ln\mathcal{N}(\mu_j, \sigma^2). \tag{7}$$

Imagine worker 1 and worker 2 making simultaneous, independent draws from their worker specific distributions for an infinite number of tasks.

Define $G(t) \equiv a_1(t)/a_2(t)$ to be the ratio of the productivity of worker 1 relative to the productivity of worker 2 in task t. Then $\ln G(t) = \ln a_1(t) - \ln a_2(t)$, so $G(t)$ also has a log normal distribution:

$$G(t) \sim \ln\mathcal{N}(\mu_G, \sigma_G^2), \tag{8}$$

where $\mu_G = \mu_1 - \mu_2$ and $\sigma_G^2 = 2\sigma^2$. Since the draws of $a_1(t)$ and $a_2(t)$ - and therefore of $G(t)$ - are independent and identically distributed, I omit t from here on.

The task index i indexes tasks in terms of their value of G in descending order, after productivity draws are realized for each worker. Specifically, define $F_G(\cdot)$ and $F_G^{-1}(\cdot)$ to be the cumulative distribution function and quantile function of G, respectively. Note that the log-normal distribution of G implies:

$$\begin{aligned} F_G(a) &= \frac{1}{2}\left[1 + \text{erf}\left(\frac{\ln a - \mu_G}{\sqrt{2}\sigma_G}\right)\right] \\ F_G^{-1}(q) &= \exp[\mu_G + \sqrt{2}\sigma_G \text{erf}^{-1}(2q-1)], \end{aligned} \tag{9}$$

where $\text{erf}(\cdot)$ is the error function, defined as:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt. \tag{10}$$

Furthermore, define:

$$\begin{aligned} \tilde{\gamma}(i) &\equiv F_G^{-1}(1-i) \\ &= \exp(\mu_G + \sqrt{2}(\sqrt{2}\sigma)\text{erf}^{-1}(1-2i)) \\ &= \frac{\exp(\mu_1)}{\exp(\mu_2)}\exp(2\sigma\text{erf}^{-1}(1-2i)). \end{aligned} \tag{11}$$

Thus $\tilde{\gamma}(i)$ is the value of G such that i percent of the distribution of the ratio of the productivity of worker 1 to worker 2 is above that point, assuming that G is distributed log-normally.

It is clear that $\tilde{\gamma}(i)$ is roughly comparable to $\gamma(i)$, with $A_1 \approx \exp(\mu_1)$, $A_2 \approx \exp(\mu_2)$, $\bar{A} \approx \exp(\mu_1 - \mu_2)$, and $\theta \approx 2\sigma$. In addition, $\text{erf}^{-1}(\cdot)$ is a monotonically increasing function. Thus I can argue that all the qualitative results for θ hold for the variance of the productivity distribution, while the results for A_1 and A_2 hold for the mean of the productivity distribution for worker 1 and worker 2, respectively.

2 Equilibrium with Costless Trade

2.1 Worker Objective Function

Workers seek to maximize output subject to the constraint that $\int_0^1 l_j(i)di = 1$. This is because wages are maximized when output is maximized (see section 2.6 below for a proof of this).

2.2 Task Threshold and Comparative Advantage

Let w_j be the wage paid to worker j in exchange for a unit of labor, and let $p_j(i)$ be the price of a unit of task i produced by worker j. The worker-specific price of task i is equal to the wage paid to worker j for a unit of labor divided by the quantity of task i that can be produced by worker j with a unit of labor:

$$p_j(i) = \frac{w_j}{A_j \alpha_j(i)}. \tag{12}$$

It is clear that the worker-specific price is decreasing in worker j's skill and task-specific productivity.

With costless task trade, the price that will actually obtain in the market is the lower of the two offered prices:

$$p(i) = \min\{p_1(i), p_2(i)\}. \tag{13}$$

In other words, each worker will purchase task i from the lowest-cost producer, including herself. Specifically, worker 1 will perform task i if:

$$\begin{aligned} p_1(i) &< p_2(i) \\ \frac{w_1}{A_1\alpha_1(i)} &< \frac{w_2}{A_2\alpha_2(i)} \\ \frac{w_1}{w_2} &< \frac{A_1\alpha_1(i)}{A_2\alpha_2(i)} \\ \omega &< \gamma(i), \end{aligned} \tag{14}$$

where $\omega \equiv w_1/w_2$.

Since $\gamma'(i) < 0$, it is clear that in equilibrium there will be a threshold task, i^*, such that:

$$\gamma(i^*) = \omega. \tag{15}$$

Worker 1 will perform all tasks in the interval $[0, i^*]$ and worker 2 will perform all tasks in the interval $[i^*, 1]$.

The condition in 15 relates the relative wage to the task threshold through the comparative advantage schedule. Because the comparative advantage schedule is a decreasing function of i, this conditions implies that the threshold value (and thus the share of tasks performed by worker) is: 1) decreasing in worker 1's relative wage, ω, and 2) is increasing in worker 1's relative skill, $\bar{A}$.

2.3 Task Threshold and the Demand for Tasks

I can derive a second equation relating the task threshold and relative wages that depends on the demand for tasks. In the context of the model, demand is determined by the production function for the final good.

First, note that the relationship in 12 implies that worker j must be paid the same wage per unit of labor for all tasks performed in equilibrium:[2]

$$w_j = p(i)A_j\alpha_j(i). \tag{16}$$

Thus:

$$\begin{aligned} p(i)\alpha_1(i) &= p(i')\alpha_1(i') \equiv P_1 \text{ for any } i, i' < i^* \\ p(i)\alpha_2(i) &= p(i')\alpha_2(i') \equiv P_2 \text{ for any } i, i' > i^*. \end{aligned} \tag{17}$$

Second, note that the Cobb-Douglas production technology for the final product implies that the same share of output must be paid to each input task. Therefore for all tasks i, i' performed by worker j:

$$\begin{aligned} p(i)y_j(i) &= p(i')y_j(i') \\ p(i)A_j\alpha_j(i)l_j(i) &= p(i')A_j\alpha_j(i')l_j(i') \\ P_jl_j(i) &= P_jl_j(i') \\ l_j(i) &= l_j(i'). \end{aligned} \tag{18}$$

Therefore workers expend the same quantity of labor on all tasks performed in equilibrium. Specifically, workers will devote to each task the total quantity of labor (normalized to one unit) divided by the share of tasks performed:

$$\begin{aligned} l_1(i) &= \frac{L_1}{i^*} = \frac{1}{i^*} \\ l_2(i) &= \frac{L_2}{1-i^*} = \frac{1}{1-i^*}. \end{aligned} \tag{19}$$

Next, I can derive an expression for the relative wage, ω, as a function of the task threshold and labor supply. Note that at the task threshold, i^*, where $p_1(i^*) = p_2(i^*) = p(i^*)$, it must be that the same quantity of task output can be produced either by worker 1 or worker 2:[3]

$$\begin{aligned} p(i^*)y_1(i^*) &= p(i^*)y_2(i^*) \\ p(i^*)A_1\alpha_1(i^*)l_1(i^*) &= p(i^*)A_2\alpha_2(i^*)l_2(i^*) \\ \frac{A_1P_1}{i^*} &= \frac{A_2P_2}{1-i^*} \\ \frac{A_1P_1}{A_2P_2} &= \frac{i^*}{1-i^*}. \end{aligned} \tag{20}$$

[2]This is the "law of one price for skill" described in Autor & Acemoglu (2011).

[3]This is the "no arbitrage" condition in Autor & Acemoglu (2011). If one worker were able to produce more than the other at the threshold, then the firm could increase profits by assigning more tasks to the more productive worker, so the threshold would not be stable. Formally, 20 holds taking the left and right limit as $i \to i^*$, using the Cobb-Douglas equal factor shares condition.

Thus the relative wage (for any $i < i^*$ and $i' > i^*$) is:

$$\begin{aligned}\frac{w_1}{w_2} &= \frac{p(i)A_1\alpha_1(i)}{p(i')A_2\alpha_2(i')} \\ \omega &= \frac{A_1P_1}{A_2P_2} \\ \omega &= \frac{i^*}{1-i^*}. \end{aligned} \tag{21}$$

This condition implies that relative wages are increasing in the task threshold. In the trade context, the upward-sloping curve reflects the fact that worker 1's wages are increasing in the demand for tasks that worker 1 has a comparative advantage in producing, and decreasing in the quantity of labor supplied by worker 1 relative to worker 2. These comparative statics are not evident from 21 due to the simplifying assumptions, namely the Cobb-Douglas technology with identical exponents for each task and the fact that both workers supply a single unit of labor.[4]

2.4 Solving for the Task Threshold

I now have two equilibrium conditions for ω as a function of the task threshold: a downward sloping condition reflecting comparative advantage in 15 and an upward sloping condition reflecting demand for worker 1's labor in 21. I can solve directly for the task threshold by combining these two conditions:

$$\begin{aligned}\gamma(i^*) &= \frac{i^*}{1-i^*} \\ \bar{A}\exp(\theta(1-2i^*)) &= \frac{i^*}{1-i^*} \\ \bar{A}\exp(\theta(1-2i^*))(1-i^*) &= i^* \\ i^*[1+\bar{A}\exp(\theta(1-2i^*))] &= \bar{A}\exp(\theta(1-2i^*)) \\ i^* &= \frac{\bar{A}\exp(\theta(1-2i^*))}{1+\bar{A}\exp(\theta(1-2i^*))} \\ i^* &= \frac{A_1}{A_1+A_2\exp(\theta(2i^*-1))}. \end{aligned} \tag{22}$$

The two equations define a unique equilibrium threshold as a function of worker skills and the θ parameter that indexes comparative advantage. Note that if $A_1 = A_2$, then $i^* = \frac{1}{2}$ and $\omega = 1$.[5]

[4] The condition in 21 can also be derived by noting that trade must be balanced, i.e. the fraction of worker 1's income spent on tasks produced by worker 2 must be equal to the fraction of worker 2's income spent on tasks produced by worker 1: $w_1(1-i^*) = w_2 i^*$. Note again that the Cobb-Douglas technology implies that the fraction of income spent on tasks $i < i^*$ is equal to i^*.

[5] To show that this is the case:

$$\begin{aligned}i^* &= \frac{1}{1+\exp(\theta(2i^*-1))} \\ \frac{1}{2} &= \frac{1}{1+\exp(\theta(2(1/2)-1))} \\ \frac{1}{2} &= \frac{1}{2}\end{aligned}$$

2.5 Production Levels

Under autarky, no task trade is possible, and each worker uses only her own labor to produce the final good. Therefore the quantity of the final good produced by worker j under autarky is given by:

$$\begin{aligned} Y_j^A &= \exp[\int_0^1 \ln y_j(i)di] \\ &= \exp[\int_0^1 \ln[A_j\alpha_j(i)l_j(i)]di] \\ &= \exp[\int_0^1 \ln[A_j\alpha_i(i)]di], \end{aligned} \tag{23}$$

where I am using the fact that $i^* = 1$ implicitly under autarky, and so $l_j(i) = 1/i^* = 1$.

To derive an expression for production under task trade, I first note that worker j's budget to "purchase" tasks is:

$$\int_0^1 p(i)y_j(i)di = w_jL_j, \tag{24}$$

where again $p(i)$ is the lower of the two prices offered by worker 1 and worker 2. Since $L_j = 1$ and the Cobb-Douglas production technology implies that $p(i)y_j(i)$ is a constant:[6]

$$y_j(i) = \frac{w_j}{p(i)}. \tag{25}$$

Worker 1 will produce her own tasks in the interval $[0, i^*]$, and will purchase tasks from worker 2 in the interval $[i^*, 1]$ at a cost of $p(i) = w_2/(A_2\alpha_2(i))$. Therefore, worker 1's level of production under task trade is given by:[7]

$$\begin{aligned} Y_1^T &= \exp[\int_0^1 \ln y_1(i)di] \\ &= \exp[\int_0^{i^*} \ln[w_1/p_1(i)]di + \int_{i^*}^1 \ln[w_1/p_2(i)]di] \\ &= \exp[\int_0^{i^*} \ln[A_1\alpha_1(i)]di + \int_{i^*}^1 \ln[A_2\alpha_2(i)w_1/w_2]di]. \end{aligned} \tag{26}$$

Similarly, worker 2 will purchase tasks from worker 1 in the interval $[0, i^*]$ at a cost of $p(i) = w_1/(A_1\alpha_1(i))$, and will produce her own tasks in the interval $[i^*, 1]$. Therefore, worker 2's level of production under task

[6] To see that this is true, note that 25 implies:

$$\begin{aligned} p(i)y_j(i) &= w_jL_j \\ \int_0^1 p(i)y_j(i)di &= \int_0^1 w_jL_jdi \\ \int_0^1 p(i)y_j(i)di &= w_jL_j. \end{aligned}$$

[7] Note that worker 1 produces additional quantities of tasks in the interval $[0, i^*]$ that she then trades to worker 2 rather than using herself in production.

trade is given by:

$$\begin{aligned}
Y_2^T &= \exp[\int_0^1 \ln y_2(i)di] \\
&= \exp[\int_0^{i^*} \ln[w_2/p_1(i)]di + \int_{i^*}^1 \ln[w_2/p_2(i)]di] \\
&= \exp[\int_0^{i^*} \ln[A_1\alpha_1(i)w_2/w_1]di + \int_{i^*}^1 \ln[A_2\alpha_2(i)]di]. \qquad (27)
\end{aligned}$$

2.6 Wage Levels

Because I am assuming a competitive market, the price of the final good, P^*, will pin down the wage. Under autarky, for each worker j the firm will maximize:

$$\begin{aligned}
&\max_{w_j} \quad P^*Y_j^A - w_jL_j \\
&\max_{w_j} \quad P^*\exp[\int_0^1 \ln y_j(i)di] - w_j \\
&\max_{w_j} \quad P^*\exp[\int_0^1 \ln\left[\frac{w_j}{p_j(i)}\right]di] - w_j \\
&\max_{w_j} \quad P^*w_j\exp[-\int_0^1 \ln p_j(i)di] - w_j. \qquad (28)
\end{aligned}$$

The first order condition with respect to w_j is:

$$\begin{aligned}
P^*\exp[-\int_0^1 \ln p_j(i)di] &= 1 \\
P^* &= \exp[\int_0^1 \ln p_j(i)di] \\
\ln P^* &= \int_0^1 \ln p_j(i)di \\
\ln P^* &= \int_0^1 \ln\left[\frac{w_j}{A_j\alpha_j(i)}\right]di \\
\ln w_j &= \ln P^* + \ln A_j + \int_0^1 \ln\alpha_j(i)di \\
w_j &= P^*A_j\exp[\int_0^1 \ln\alpha_j(i)di] \qquad (29)
\end{aligned}$$

Thus wages are determined by skills, task productivities, and the output price.

Note that I can re-write production under autarky as:

$$\begin{aligned}
Y_j^A &= \exp[\int_0^1 \ln[A_j\alpha_j(i)]di] \\
&= A_j\exp[\int_0^1 \ln[\alpha_j(i)]di], \qquad (30)
\end{aligned}$$

so the wage level is equal to the output level scaled by the output price.

With task trade, the firm will maximize:

$$
\begin{aligned}
\max_{w_1,w_2} \quad & P^*(Y_1^T+Y_2^T)-w_1-w_2\\
\max_{w_1,w_2} \quad & P^*\big(\exp[\int_0^{i^*}\ln[w_1/p_1(i)]di+\int_{i^*}^1\ln[w_1/p_2(i)]di]\\
& +\exp[\int_0^{i^*}\ln[w_2/p_1(i)]di+\int_{i^*}^1\ln[w_2/p_2(i)]di]\big)-w_1-w_2\\
\max_{w_1,w_2} \quad & P^*\big(w_1\exp[-\int_0^{i^*}\ln p_1(i)di-\int_{i^*}^1\ln p_2(i)di]\\
& +w_2\exp[-\int_0^{i^*}\ln p_1(i)di-\int_{i^*}^1\ln p_2(i)di]\big)-w_1-w_2. \qquad (31)
\end{aligned}
$$

The first order condition with respect to w_1 (which is identical to the first order condition with respect to w_2) is:

$$
\begin{aligned}
P^*\exp[-\int_0^{i^*}\ln p_1(i)di-\int_{i^*}^1\ln p_2(i)di] &= 1\\
P^* &= \exp[\int_0^{i^*}\ln p_1(i)di+\int_{i^*}^1\ln p_2(i)di]\\
P^* &= \exp[\int_0^{i^*}\ln\left[\frac{w_1}{A_1\alpha_1(i)}\right]di+\int_{i^*}^1\ln\left[\frac{w_2}{A_2\alpha_2(i)}\right]di]\\
w_1^{i^*}w_2^{1-i^*} &= P^*A_1^{i^*}A_2^{1-i^*}\exp[\int_0^{i^*}\ln\alpha_1(i)di+\int_{i^*}^1\ln\alpha_2(i)di] \qquad (32)
\end{aligned}
$$

Note that I have shown in 22 that I can solve directly for the threshold value, i, as a function of A_1, A_2 and θ (or more generally, as a function of A_1, A_2 and the comparative advantage schedules). I can then solve for ω as a function of i^*, using 21. Thus I can use 32 to express w_1 as a function of ω, A_1, A_2 and the comparative advantage schedules, as follows:

$$
\begin{aligned}
w_1^{i^*}\omega^{-(1-i^*)} &= w_1^{-(1-i^*)}P^*A_1^{i^*}A_2^{1-i^*}\exp[\int_0^{i^*}\ln\alpha_1(i)di+\int_{i^*}^1\ln\alpha_2(i)di]\\
w_1 &= P^*A_1^{i^*}(A_2\omega)^{1-i^*}\exp[\int_0^{i^*}\ln\alpha_1(i)di+\int_{i^*}^1\ln\alpha_2(i)di] \qquad (33)
\end{aligned}
$$

Similarly, I can solve for w_2 as a function of ω, A_1, A_2 and the comparative advantage schedules:

$$
\begin{aligned}
\omega^{i^*}w_2^{1-i^*} &= w_2^{-i^*}A_1^{i^*}A_2^{1-i^*}\exp[\int_0^{i^*}\ln\alpha_1(i)di+\int_{i^*}^1\ln\alpha_2(i)di]\\
w_2 &= P^*A_2^{1-i^*}(A_1\omega^{-1})^{i^*}\exp[\int_0^{i^*}\ln\alpha_1(i)di+\int_{i^*}^1\ln\alpha_2(i)di]. \qquad (34)
\end{aligned}
$$

Note that I can re-write production under trade as:

$$
\begin{aligned}
Y_1^T &= \exp[\int_0^{i^*} \ln[A_1\alpha_1(i)]di + \int_{i^*}^1 \ln[A_2\alpha_2(i)w_1/w_2]di] \\
&= A_1^{i^*}\left(A_2\omega\right)^{1-i^*} \exp[\int_0^{i^*} \ln\alpha_1(i)di + \int_{i^*}^1 \ln\alpha_2(i)di] \qquad (35)\\
Y_2^T &= \exp[\int_0^{i^*} \ln[A_1\alpha_1(i)w_2/w_1]di + \int_{i^*}^1 \ln[A_2\alpha_2(i)]di] \\
&= A_2^{1-i^*}(A_1\omega^{-1})^{i^*}\exp[\int_0^{i^*} \ln\alpha_1(i)di + \int_{i^*}^1 \ln\alpha_2(i)di]. \qquad (36)
\end{aligned}
$$

Thus again, wages are equal to the output level scaled by the output price.

2.7 Gains from Trade

The proportional gains from task trade for worker 1 are:

$$
\begin{aligned}
\Delta Y_1 &= \frac{Y_1^T}{Y_1^A} \\
&= \frac{\exp(\int_0^{i^*} \ln[A_1\alpha_1(i)]di + \int_{i^*}^1 \ln[A_2\alpha_2(i)w_1/w_2]di)}{\exp(\int_0^1 \ln[A_1\alpha_1(i)]di)} \\
&= \frac{\exp(\int_0^{i^*} \ln[A_1\alpha_1(i)]di)\exp(\int_{i^*}^1 \ln[A_2\alpha_2(i)w_1/w_2]di])}{\exp(\int_0^{i^*} \ln[A_1\alpha_1(i)]di)\exp(\int_{i^*}^1 \ln[A_1\alpha_1(i)]di)} \\
&= \exp(\int_{i^*}^1 (\ln[A_2\alpha_2(i)w_1/w_2] - \ln[A_1\alpha_1(i)])di) \\
&= \exp\left(\int_{i^*}^1 \ln\left[\frac{\alpha_2(i)}{\alpha_1(i)}\frac{A_2w_1}{A_1w_2}\right]di\right) \\
&= \exp\left(\int_{i^*}^1 \ln\left[\frac{\gamma(i^*)}{\gamma(i)}\right]di\right). \qquad (37)
\end{aligned}
$$

This quantity is positive because i^* is chosen to satisfy the condition in 14, which is that $\gamma(i) < \omega = \gamma(i^*)$ in the interval $[i^*, 1]$, where worker 2 has a comparative advantage.

Similarly, the gains from task trade for worker 2 are given by:

$$\begin{aligned}
\Delta Y_2 &= \frac{Y_2^T}{Y_2^A} \\
&= \frac{\exp(\int_0^{i^*} \ln[A_1\alpha_1(i)w_2/w_1]di + \int_{i^*}^1 \ln[A_2\alpha_2(i)]di)}{\exp(\int_0^1 \ln[A_2\alpha_2(i)]di)} \\
&= \frac{\exp(\int_{i^*}^1 \ln[A_2\alpha_2(i)]di)\exp(\int_0^{i^*} \ln[A_1\alpha_1(i)w_2/w_1]di)}{\exp(\int_{i^*}^1 \ln[A_2\alpha_2(i)]di)\,\exp(\int_0^{i^*} \ln[A_2\alpha_2(i)]di)} \\
&= \exp(\int_0^{i^*} (\ln[A_1\alpha_1(i)w_2/w_1] - \ln[A_2\alpha_2(i)])di) \\
&= \exp\left(\int_0^{i^*} \ln\left[\frac{\alpha_1(i)}{\alpha_2(i)}\frac{A_1 w_2}{A_2 w_1}\right]di\right) \\
&= \exp\left(\int_0^{i^*} \ln\left[\frac{\gamma(i)}{\gamma(i^*)}\right]di\right). \qquad (38)
\end{aligned}$$

This quantity is positive, as well, because $\gamma(i) > \gamma(i^*)$ in the interval $[0, i^*]$, where worker 1 has a comparative advantage.

Using the specification in 6 for the comparative advantage schedule, I can express the gains from trade as a function of the θ parameter:

$$\begin{aligned}
\Delta Y_1 &= \exp\left(\int_{i^*}^{1} \ln\left[\frac{\gamma(i^*)}{\gamma(i)}\right] di\right) \\
&= \exp\left(\int_{i^*}^{1} \ln\left[\frac{\bar{A}\exp(\theta(1-2i^*))}{\bar{A}\exp(\theta(1-2i))}\right] di\right) \\
&= \exp(\int_{i^*}^{1} \ln[\exp[\theta - 2\theta i^* - \theta + 2\theta i] di) \\
&= \exp(\int_{i^*}^{1} 2\theta[i - i^*] di) \\
&= \exp\left(2\theta\left[\frac{i^2}{2} - i^* i\right]\Big|_{i^*}^{1}\right) \\
&= \exp\left(2\theta\left[\frac{1}{2} - i^* - \frac{i^{*2}}{2} + i^{*2}\right]\right) \\
&= \exp\left(2\theta\left[\frac{i^{*2}}{2} - i^* + \frac{1}{2}\right]\right) \\
&= \exp(\theta[i^{*2} - 2i^* + 1]) \\
&= \exp(\theta[i^* - 1]^2) \qquad (39) \\
\Delta Y_2 &= \exp\left(\int_{0}^{i^*} \ln\left[\frac{\gamma(i)}{\gamma(i^*)}\right] di\right) \\
&= \exp\left(\int_{0}^{i^*} \ln\left[\frac{\bar{A}\exp(\theta(1-2i))}{\bar{A}\exp(\theta(1-2i^*))}\right] di\right) \\
&= \exp(\int_{0}^{i^*} \ln[\exp[\theta - 2\theta i - \theta + 2\theta i^*]] di) \\
&= \exp(\int_{0}^{i^*} 2\theta[i^* - i] di) \\
&= \exp\left(2\theta\left[i^* i - \frac{i^2}{2}\right]\Big|_{0}^{i^*}\right) \\
&= \exp\left(2\theta\left[i^{*2} - \frac{i^{*2}}{2}\right]\right) \\
&= \exp\left(2\theta\left[\frac{i^{*2}}{2}\right]\right) \\
&= \exp(\theta i^{*2}). \qquad (40)
\end{aligned}$$

It is clear from 39 and 40 that the gains from trade are increasing in θ.

3 Equilibrium with Trading Costs

3.1 Defining Social Skills as Trading Costs

In section 2, trading tasks was costless. Now trading tasks requires costly coordination among workers, and I define *social skill* as a worker-specific reduction in coordination costs. Specifically, define S_j as a worker-specific depreciation factor, and define $S_{j,k} = S_j * S_k$ as a depreciation factor applied to all task trades between worker j and worker k. Let $S_{j,j} = 1$, $\forall j$, so that workers can trade costlessly with themselves. With only two workers, the trade cost is symmetric, and I can define $S^* = S_1 * S_2$.

When worker 1 purchases $y_1(i)$ units of task i from worker 2, she pays $p(i)y_1(i)$ but receives only $S^* y_1(i)$ units of task i. Therefore the gross price of task i produced by worker j and purchased by worker k (where $j \neq k$), including trading costs, becomes:

$$\begin{aligned} p_j^S(i) &= \frac{p(i)}{S^*} \\ p_j^S(i) &= \frac{w_j}{S^* A_j \alpha_j(i)} \end{aligned} \tag{41}$$

It is clear that workers with high levels of social skill pay a lower coordination cost to engage in trask trades with other workers.

3.2 Equilibrium Task Thresholds with Social Skills

With these adjusted costs, worker 1 will prefer to produce her own tasks rather than purchase them from worker 2 if:

$$\begin{aligned} p_1(i) &< p_2^S(i) \\ \frac{w_1}{A_1 \alpha_1(i)} &< \frac{w_2}{S^* A_2 \alpha_2(i)} \\ \omega &< \frac{\gamma(i)}{S^*}. \end{aligned} \tag{42}$$

Worker 2 will prefer to produce her own tasks rather than purchase them from worker 1 if:

$$\begin{aligned} p_2(i) &< p_1^S(i) \\ \frac{w_2}{A_2 \alpha_2(i)} &< \frac{w_1}{S^* A_1 \alpha_1(i)} \\ \omega &> S^* \gamma(i). \end{aligned} \tag{43}$$

Thus in equilibrium there will be two task thresholds, defined by:

$$\gamma(i^H) = S^* \omega \tag{44}$$

$$\gamma(i^L) = \frac{\omega}{S^*}. \tag{45}$$

Since $\gamma'(i) < 0$ and $S^* < 1$, it is clear that $i^H > i^* > i^L$.

Worker 1 will produce all tasks $i < i^H$ and worker 2 will produce all tasks $i > i^L$. In other words, tasks in the interval $[0, i^L]$ will be produced exclusively by worker 1, tasks in the interval $[i^L, i^H]$ will be non-traded (produced by both workers for their own use), and tasks in the interval $[i^H, 1]$ will be produced exclusively by worker 2. If S^* is low enough, trade will not be worthwhile for any i, and production will revert to autarky (i.e. $i^L = 0$ and $i^H = 1$).

Note that 45 gives us two equations for ω. Setting these equal, I can derive an equation for $i^H - i^L$, as follows:

$$\begin{aligned}
\frac{\gamma(i^H)}{S^*} &= S^*\gamma(i^L) \\
\frac{\gamma(i^H)}{\gamma(i^L)} &= (S^*)^2 \\
\frac{\exp(\theta(1-2i^H))}{\exp(\theta(1-2i^L))} &= (S^*)^2 \\
\exp(2\theta(i^L - i^H)) &= (S^*)^2 \\
2\theta(i^L - i^H) &= 2\ln S^* \\
i^H - i^L &= -\frac{\ln S^*}{\theta} \qquad (46)
\end{aligned}$$

It is clear that $i^H - i^L$ is decreasing in S^*. In other words, as S^* rises, i^H approaches i^* from above and i^L approaches i^* from below. When $S^* = 1$, the model reverts to the case with no trading costs.

To derive the "demand" equation relating ω, i^L and i^H, comparable to 21, let us go directly to the trade balance condition described in footnote 4. Specifically, it must be the case that the fraction of worker 1's income spent on traded goods produced by worker 2 must equal the fraction of worker 2's income spent on traded goods produced by worker 1:

$$\begin{aligned}
w_1(1 - i^H) &= w_2 i^L \\
\omega &= \frac{i^L}{1 - i^H}. \qquad (47)
\end{aligned}$$

Thus combining 44, 45 and 47, the equilibrium thresholds with trading costs are defined by the following series of equalities:

$$\frac{\gamma(i^H)}{S^*} = S^*\gamma(i^L) = \frac{i^L}{1 - i^H}. \qquad (48)$$

Re-arranging:

$$\begin{aligned}
\frac{\gamma(i^H)}{S^*} &= S^*\gamma(i^L) = \frac{i^L}{1 - i^H} \\
\frac{\gamma(i^H)(1 - i^H)}{S^* i^L} &= \frac{S^*\gamma(i^L)(1 - i^H)}{i^L} = 1 \\
\frac{\bar{A}\exp(\theta(1-2i^H))(1 - i^H)}{S^* i^L} - 1 &= \frac{\bar{A}S^*\exp(\theta(1-2i^L))(1 - i^H)}{i^L} - 1 = 0. \qquad (49)
\end{aligned}$$

This gives us two functions with two unknowns (i^H and i^L) and three parameters ($\bar{A}$, S^* and θ). Plotting these two implicit functions in the (i^L, i^H) space shows that their intersection defines the unique equilibrium values of i^H and i^L.

It can be verified numerically that, comparable to the case with no trading costs, when $A_1 = A_2$ so that $\bar{A} = 1$, then $i^L = 1 - i^H$ and $\omega = 1$. In this case, I can solve for the thresholds by setting 44 equal to 45, as follows:

$$\begin{aligned}
\frac{\gamma(i^H)}{S^*} &= S^*\gamma(i^L) \\
\frac{\bar{A}\exp(\theta(1-2(1-i^L))}{S^*} &= S^*\bar{A}\exp(\theta(1-2i^L)) \\
\exp(\theta-2\theta+2\theta i^L-\theta+2\theta i^L) &= (S^*)^2 \\
4\theta i^L-2\theta &= 2\ln S^* \\
2\theta i^L &= \ln S^*+\theta \\
i^L &= \frac{\ln S^*}{2\theta}+\frac{1}{2}. \qquad (50)
\end{aligned}$$

Since $\ln S^* < 0$, it is clear that i^L is increasing in θ. Also:

$$\begin{aligned}
i^H &= 1-i^L \\
&= -\frac{\ln S^*}{2\theta}+\frac{1}{2}. \qquad (51)
\end{aligned}$$

This special case makes it very clear that for some values of S^* and θ - specifically, low values of both parameters - trade is not worthwhile (i.e. the implied level of i^L is less than zero) and autarky will prevail. This is in contrast to case with costless trade - equation 22 implies that some level of trade is always optimal, although i^* may approach arbitrarily close to zero or one depending on θ, A_1 and A_2.

3.3 Production Levels with Social Skills

With trading costs, worker 1 will produce:

$$\begin{aligned}
Y_1^S &= \exp(\int_0^1 \ln y_1(i)di) \\
&= \exp(\int_0^{i^H} \ln[w_1/p_1(i)]di + \int_{i^H}^1 \ln[w_1/p_2^S(i)]di) \\
&= \exp(\int_0^{i^H} \ln[A_1\alpha_1(i)]di + \int_{i^H}^1 \ln[S^*A_2\alpha_2(i)w_1/w_2]di). \qquad (52)
\end{aligned}$$

Worker 2 will produce:

$$\begin{aligned}
Y_2^S &= \exp(\int_0^1 \ln y_2(i)di) \\
&= \exp(\int_0^{i^L} \ln[w_2/p_1^S(i)]di + \int_{i^L}^1 \ln[w_2/p_2(i)]di) \\
&= \exp(\int_0^{i^L} \ln[S^*A_1\alpha_1(i)w_2/w_1]di + \int_{i^L}^1 \ln[A_2\alpha_2(i)]di). \qquad (53)
\end{aligned}$$

3.4 Wage Levels with Social Skills

With trading costs, the firm will maximize:

$$\begin{aligned}
\max_{w_1,w_2} \quad & P^*(Y_1^S + Y_2^S) - w_1 - w_2 \\
\max_{w_1,w_2} \quad & P^*\big(\exp[\int_0^{i^H} \ln[w_1/p_1(i)]di + \int_{i^H}^1 \ln[w_1/p_2^S(i)]di] \\
& + \exp[\int_0^{i^L} \ln[w_2/p_1^S(i)]di + \int_{i^L}^1 \ln[w_2/p_2(i)]di]\big) - w_1 - w_2 \\
\max_{w_1,w_2} \quad & P^*\big(w_1 \exp[-\int_0^{i^H} \ln p_1(i)di - \int_{i^H}^1 \ln[p_2(i)/S^*]di] \\
& w_2 \exp[-\int_0^{i^L} \ln[p_1(i)/S^*]di - \int_{i^L}^1 \ln p_2(i)di]\big) - w_1 - w_2. \qquad (54)
\end{aligned}$$

The first order condition with respect to w_1 is:

$$\begin{aligned}
P^* &= \exp[\int_0^{i^H} \ln p_1(i)di + \int_{i^H}^1 \ln[p_2(i)/S^*]di] \\
P^* &= \exp\left[\int_0^{i^H} \ln\left[\frac{w_1}{A_1\alpha_1(i)}\right]di + \int_{i^H}^1 \ln\left[\frac{w_2}{S^*A_2\alpha_2(i)}\right]di\right] \\
w_1^{i^H} w_2^{1-i^H} &= P^*(S^*)^{1-i^H} A_1^{i^H} A_2^{1-i^H} \exp[\int_0^{i^H} \ln\alpha_1(i)di + \int_{i^H}^1 \ln\alpha_2(i)di]. \qquad (55)
\end{aligned}$$

The first order condition with respect to w_2 is:

$$\begin{aligned}
P^* &= \exp[\int_0^{i^L} \ln[p_1(i)/S^*]di + \int_{i^L}^1 \ln p_2(i)di] \\
P^* &= \exp\left[\int_0^{i^L} \ln\left[\frac{w_1}{S^*A_1\alpha_1(i)}\right]di + \int_{i^L}^1 \ln\left[\frac{w_2}{A_2\alpha_2(i)}\right]di\right] \\
w_1^{i^L} w_2^{1-i^L} &= P^*(S^*)^{i^L} A_1^{i^L} A_2^{1-i^L} \exp[\int_0^{i^L} \ln\alpha_1(i)di + \int_{i^L}^1 \ln\alpha_2(i)di]. \qquad (56)
\end{aligned}$$

I solve for w_1 using 55, as follows:

$$\begin{aligned}
w_1^{i^H} \omega^{-(1-i^H)} &= w_1^{-(1-i^H)} P^*(S^*)^{1-i^H} A_1^{i^H} A_2^{1-i^H} \exp[\int_0^{i^H} \ln\alpha_1(i)di + \int_{i^H}^1 \ln\alpha_2(i)di] \\
w_1 &= P^* A_1^{i^H} (S^* A_2 \omega)^{1-i^H} \exp[\int_0^{i^H} \ln\alpha_1(i)di + \int_{i^H}^1 \ln\alpha_2(i)di] \qquad (57)
\end{aligned}$$

I solve for w_2 using 56, as follows:

$$\begin{aligned}
\omega^{i^L} w_2^{1-i^L} &= w_2^{-i^L} P^*(S^*)^{i^L} A_1^{i^L} A_2^{1-i^L} \exp[\int_0^{i^L} \ln\alpha_1(i)di + \int_{i^L}^1 \ln\alpha_2(i)di] \\
w_2 &= P^*(S^* A_1 \omega^{-1})^{i^L} A_2^{1-i^L} \exp[\int_0^{i^L} \ln\alpha_1(i)di + \int_{i^L}^1 \ln\alpha_2(i)di]. \qquad (58)
\end{aligned}$$

Again, I can re-write with expressions for production with trading costs as follows:

$$
\begin{aligned}
Y_1^S &= \exp(\int_0^{i^H} \ln[A_1\alpha_1(i)]di + \int_{i^H}^1 \ln[S^* A_2\alpha_2(i)w_1/w_2]di) \\
&= \exp(i^H \ln A_1 + (1-i^H)\ln[S^* A_2 w_1/w_2] + \int_0^{i^H} \ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di) \\
&= A_1^{i^H}(S^* A_2\omega)^{1-i^H}\exp(\int_0^{i^H} \ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di) \qquad (59) \\
Y_2^S &= \exp(\int_0^{i^L} \ln[S^* A_1\alpha_1(i)w_2/w_1]di + \int_{i^L}^1 \ln[A_2\alpha_2(i)]di) \\
&= \exp(i^L \ln[S^* A_1 w_2/w_1] + (1-i^L)\ln A_2 + \int_0^{i^L} \ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di) \\
&= (S^* A_1\omega^{-1})^{i^L} A_2^{1-i^L}\exp(\int_0^{i^L} \ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di) \qquad (60)
\end{aligned}
$$

Thus wages are once more equal to the output level scaled by the output price. Finally, I can see that if S^* is sufficiently low such that that trade is not worthwile, so $i^H = 1$ and $i^L = 0$, then the wage equations reduce to 29, the expression for wages under autarky.

3.5 Complementarity of Cognitive and Social Skills

In this section, I want to show that cognitive skill defined as A_j and social skill S^* are complements in production. For simplicity, consider the special case where $A_1 = A_2 = A$, so $\bar{A} = \omega = 1$. I will derive comparative statics to determine the impact on output of raising the level of cognitive skill, A.

Note that in this case, the expressions for production in 59 and 60 become:

$$
\begin{aligned}
Y_1^S &= A(S^*)^{1-i^H}\exp(\int_0^{i^H} \ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di) \qquad (61) \\
Y_2^S &= A(S^*)^{i^L}\exp(\int_0^{i^L} \ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di) \qquad (62)
\end{aligned}
$$

Note also that in this case the expressions for the thresholds in terms of comparative advantage, 44 and 45, become:[8]

$$
\begin{aligned}
\gamma(i^H) &= S^* \qquad (63) \\
\gamma(i^L) &= \frac{1}{S^*}. \qquad (64)
\end{aligned}
$$

The derivatives of production with respect to cognitive skill are thus:

[8] These expressions imply that the derivatives of the thresholds with respect to S^* are:

$$
\begin{aligned}
\frac{di^H}{dS^*} &= \frac{1}{\gamma'(i^H)} \\
\frac{di^L}{dS^*} &= -\frac{1}{\gamma'(i^L)(S^*)^2}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{dY_1^S}{dA} &= (S^*)^{1-i^H}\exp(\int_0^{i^H}\ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di) \\
&\quad + \frac{di^H}{dA}A(S^*)^{1-i^H}\exp(\int_0^{i^H}\ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di)\ln\left(\frac{\alpha_1(i^H)}{S^*\alpha_2(i^H)}\right) \\
&= (S^*)^{1-i^H}\exp(\int_0^{i^H}\ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di) \\
&\quad + \frac{di^H}{dA}A(S^*)^{1-i^H}\exp(\int_0^{i^H}\ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di)\ln\left(\frac{\gamma(i^H)}{\gamma(i^H)}\right) \\
&= (S^*)^{1-i^H}\exp(\int_0^{i^H}\ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di) \qquad (65) \\
\frac{dY_2^S}{dA} &= (S^*)^{i^L}\exp(\int_0^{i^L}\ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di) \\
&\quad + \frac{di^L}{dA}A(S^*)^{i^L}\exp(\int_0^{i^L}\ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di)\ln\left(\frac{S^*\alpha_1(i^L)}{\alpha_2(i^L)}\right) \\
&= (S^*)^{i^L}\exp(\int_0^{i^L}\ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di) \\
&\quad + \frac{di^L}{dA}A(S^*)^{i^L}\exp(\int_0^{i^L}\ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di)\ln\left(\frac{\gamma(i^L)}{\gamma(i^L)}\right) \\
&= (S^*)^{i^L}\exp(\int_0^{i^L}\ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di). \qquad (66)
\end{aligned}
$$

Next, I take the second derivative with respect to own cognitive skill and S^*:

$$
\begin{aligned}
\frac{d^2Y_1^S}{dAdS^*} &= (1-i^H)(S^*)^{-i^H}\exp(\int_0^{i^H}\ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di) \\
&\quad + \frac{di^H}{dS^*}(S^*)^{1-i^H}\exp(\int_0^{i^H}\ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di)\ln\left(\frac{\alpha_1(i^H)}{S^*\alpha_2(i^H)}\right) \\
&= (1-i^H)(S^*)^{-i^H}\exp(\int_0^{i^H}\ln[\alpha_1(i)]di + \int_{i^H}^1 \ln[\alpha_2(i)]di) \qquad (67) \\
\frac{d^2Y_2^S}{dAdS^*} &= i^L(S^*)^{i^L-1}\exp(\int_0^{i^L}\ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di) \\
&\quad + \frac{di^L}{dS^*}\exp(\int_0^{i^L}\ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di)\ln\left(\frac{S^*\alpha_1(i^L)}{\alpha_2(i^L)}\right) \\
&= i^L(S^*)^{i^L-1}\exp(\int_0^{i^L}\ln[\alpha_1(i)]di + \int_{i^L}^1 \ln[\alpha_2(i)]di). \qquad (68)
\end{aligned}
$$

Both of these expressions are positive.

Thus it is clear that in this simple case, cognitive and social skill are complements in production.

3.6 Complementarity of θ and Social Skill in Gains from Trade

In this section, I show that social skill S^* and θ are complements in the gains from trade.

With trading costs, worker 1's gains from trade will be:

$$\begin{aligned}
\Delta Y_1^S &= \frac{Y_1^S}{Y_1^A} \\
&= \frac{\exp(\int_0^{i^H} \ln[A_1\alpha_1(i)]di + \int_{i^H}^1 \ln[S^* A_2\alpha_2(i)w_1/w_2]di)}{\exp(\int_0^1 \ln[A_1\alpha_1(i)]di)} \\
&= \exp\bigg(\int_{i^H}^1 \ln\bigg[\frac{S^* A_2\alpha_2(i)}{A_1\alpha_1(i)}\frac{w_1}{w_2}\bigg]di\bigg) \\
&= \exp\bigg(\int_{i^H}^1 \ln\bigg[\frac{S^*\gamma(i^*)}{\gamma(i)}\bigg]di\bigg) \\
&= \exp\bigg(\int_{i^H}^1 [2\theta(i-i^*) + \ln S^*]di\bigg) \\
&= \exp\bigg(2\theta\bigg[\frac{i^2}{2} - i^* i\bigg] + \ln S^* i\bigg|_{i^H}^1\bigg) \\
&= \exp\bigg(2\theta\bigg[\frac{1}{2} - i^* - \frac{(i^H)^2}{2} + i^* i^H\bigg] + \ln S^*(1-i^H)\bigg) \\
&= \exp(\theta[1-(i^H)^2 - 2i^*(1-i^H)] + \ln S^*(1-i^H)) \\
&= \exp(\theta[(1+i^H)(1-i^H) - 2i^*(1-i^H)] + \ln S^*(1-i^H)) \\
&= \exp((1-i^H)[\theta([1-i^*] + [i^H - i^*]) + \ln S^*]) \qquad (69)
\end{aligned}$$

Worker 2's gains from trade will be:

$$\begin{aligned}
\Delta Y_2^S &= \frac{Y_2^S}{Y_2^A} \\
&= \frac{\exp(\int_0^{i^L} \ln[S^* A_1\alpha_1(i)w_2/w_1]di + \int_{i^L}^1 \ln[A_2\alpha_2(i)]di)}{\exp(\int_0^1 \ln[A_2\alpha_2(i)]di)} \\
&= \exp\bigg(\int_0^{i^L} \ln\bigg[\frac{S^* A_1\alpha_1(i)}{A_2\alpha_2(i)}\frac{w_2}{w_1}\bigg]di\bigg) \\
&= \exp\bigg(\int_0^{i^L} \ln\bigg[\frac{S^*\gamma(i)}{\gamma(i^*)}\bigg]di\bigg) \\
&= \exp(\int_0^{i^L} [2\theta(i^*-i) + \ln S^*]di) \\
&= \exp\bigg(2\theta\bigg[i^* i - \frac{i^2}{2}\bigg] + \ln S^* i\bigg|_0^{i^L}\bigg) \\
&= \exp\bigg(2\theta\bigg[i^* i^L - \frac{(i^L)^2}{2}\bigg] + \ln S^* i^L\bigg) \\
&= \exp(i^L[\theta(2i^* - i^L) + \ln S^*]). \qquad (70)
\end{aligned}$$

As in section 3.5, I consider the special case where $A_1 = A_2 = A$ and $\bar{A} = \omega = 1$. In this case, i^L and i^H

are defined as in 50 and 51, and $i^* = \frac{1}{2}$. Thus 69 and 70 become:

$$\Delta Y_1^S = \exp\Bigg(\Bigg(1 + \frac{\ln S^*}{2\theta} - \frac{1}{2}\Bigg)\Bigg[\theta\Bigg(\Bigg[1 - \frac{1}{2}\Bigg] + \Bigg[-\frac{\ln S^*}{2\theta} + \frac{1}{2} - \frac{1}{2}\Bigg]\Bigg) + \ln S^*\Bigg]\Bigg)$$

$$= \exp\Bigg(\Bigg(\frac{\ln S^*}{2\theta} + \frac{1}{2}\Bigg)\Bigg[\theta\Bigg(-\frac{\ln S^*}{2\theta} + \frac{1}{2}\Bigg) + \ln S^*\Bigg]\Bigg) \tag{71}$$

$$\Delta Y_2^S = \exp\Bigg(\Bigg(\frac{\ln S^*}{2\theta} + \frac{1}{2}\Bigg)\Bigg[\theta\Bigg(1 - \frac{\ln S^*}{2\theta} - \frac{1}{2}\Bigg) + \ln S^*\Bigg]\Bigg)$$

$$= \exp\Bigg(\Bigg(\frac{\ln S^*}{2\theta} + \frac{1}{2}\Bigg)\Bigg[\theta\Bigg(-\frac{\ln S^*}{2\theta} + \frac{1}{2}\Bigg) + \ln S^*\Bigg]\Bigg). \tag{72}$$

Note that the gains from trade are identical for the two workers, due to the assumption of equal cognitive skill (i.e. that $\bar{A} = 1$).

The derivatives of the gains from trade with respect to S^* are given by:

$$\frac{d\Delta Y_1^S}{dS^*} = \Delta Y_1^S\Bigg(\frac{1}{2\theta S^*}\Bigg[\theta\Bigg(-\frac{\ln S^*}{2\theta} + \frac{1}{2}\Bigg) + \ln S^*\Bigg] + \Bigg(\frac{\ln S^*}{2\theta} + \frac{1}{2}\Bigg)\Bigg[-\frac{1}{2S^*} + \frac{1}{S^*}\Bigg]\Bigg)$$

$$= \Delta Y_1^S\Bigg(\frac{1}{2\theta S^*}\Bigg[-\frac{\ln S^*}{2} + \frac{\theta}{2} + \ln S^*\Bigg] - \frac{\ln S^*}{4\theta S^*} + \frac{\ln S^*}{2\theta S^*} - \frac{1}{4S^*} + \frac{1}{2S^*}\Bigg)$$

$$= \Delta Y_1^S\Bigg(\frac{1}{2\theta S^*}\Bigg[\frac{\ln S^* + \theta}{2} - \frac{\ln S^*}{2} + \ln S^* - \frac{\theta}{2} + \theta\Bigg]\Bigg)$$

$$= \Delta Y_1^S\Bigg(\frac{\ln S^* + \theta}{2\theta S^*}\Bigg) \tag{73}$$

$$\frac{d\Delta Y_2^S}{dS^*} = \Delta Y_2^S\Bigg(\frac{\ln S^* + \theta}{2\theta S^*}\Bigg) \tag{74}$$

Note that $\frac{d\Delta Y_j^S}{dS^*} > 0$ since $\frac{\ln S^* + \theta}{2\theta S^*} = \frac{1}{S^*}\Bigg(\frac{\ln S^*}{2\theta} + \frac{1}{2}\Bigg) = \frac{i^L}{S^*}$, by equation 50. Thus the gains from trade are increasing in social skill.

Finally, I take the second derivative of 73 and 74 with respect to S^* and θ:

$$
\begin{aligned}
\frac{d^2\Delta Y_1^S}{dS^* d\theta} &= \Delta Y_1^S \left(\frac{\text{ln}S^* + \theta}{2\theta S^*}\right)\left(-\frac{\text{ln}S^*}{2\theta^2}\left[\theta\left(-\frac{\text{ln}S^*}{2\theta} + \frac{1}{2}\right) + \text{ln}S^*\right] + \right. \\
&\left. \left(\frac{\text{ln}S^*}{2\theta} + \frac{1}{2}\right)\left[\left(-\frac{\text{ln}S^*}{2\theta} + \frac{1}{2}\right) + \theta\frac{\text{ln}S^*}{2\theta^2}\right]\right) + \Delta Y_1^S \left(\frac{1}{2\theta S^*} - \frac{\text{ln}S^* + \theta}{4\theta^2 (S^*)^2} 2S^*\right) \\
&= \Delta Y_1^S \left(\frac{\text{ln}S^* + \theta}{2\theta S^*}\right)\left(\frac{(\text{ln}S^*)^2}{4\theta^2} - \frac{\text{ln}S^*}{4\theta} - \frac{(\text{ln}S^*)^2}{2\theta^2} - \frac{(\text{ln}S^*)^2}{4\theta^2} + \right. \\
&\left. \frac{\text{ln}S^*}{4\theta} + \frac{(\text{ln}S^*)^2}{4\theta^2} - \frac{\text{ln}S^*}{4\theta} + \frac{1}{4} + \frac{\text{ln}S^*}{4\theta}\right) + \Delta Y_1^S \left(\frac{\theta - \text{ln}S^* - \theta}{2\theta^2 S^*}\right) \\
&= \Delta Y_1^S \left(\frac{\text{ln}S^* + \theta}{2\theta S^*}\right)\left(\frac{(\text{ln}S^*)^2}{4\theta^2} - \frac{(\text{ln}S^*)^2}{2\theta^2} + \frac{1}{4}\right) + \Delta Y_1^S \left(-\frac{\text{ln}S^*}{2\theta^2 S^*}\right) \\
&= \Delta Y_1^S \left[\left(\frac{\text{ln}S^* + \theta}{2\theta S^*}\right)\left(-\frac{(\text{ln}S^*)^2}{4\theta^2} + \frac{1}{4}\right) - \frac{\text{ln}S^*}{2\theta^2 S^*}\right] \\
&= \Delta Y_1^S \left[\frac{1}{2\theta S^*}\left(-\frac{(\text{ln}S^*)^3}{4\theta^2} + \frac{\text{ln}S^*}{4} - \frac{(\text{ln}S^*)^2}{4\theta} + \frac{\theta}{4} - \frac{\text{ln}S^*}{\theta}\right)\right] \\
&= \Delta Y_1^S \left[\frac{1}{2\theta S^*}\left(\frac{-(\text{ln}S^*)^3 + \theta^2 \text{ln}S^* - \theta(\text{ln}S^*)^2 + \theta^3 - 4\theta \text{ln}S^*}{4\theta^2}\right)\right] \\
&= \Delta Y_1^S \left[\frac{-(\text{ln}S^*)^3 + \theta^2 \text{ln}S^* - \theta(\text{ln}S^*)^2 + \theta^3 - 4\theta \text{ln}S^*}{8\theta^3 S^*}\right] && (75) \\
\frac{d^2\Delta Y_2^S}{dS^* d\theta} &= \Delta Y_2^S \left[\frac{-(\text{ln}S^*)^3 + \theta^2 \text{ln}S^* - \theta(\text{ln}S^*)^2 + \theta^3 - 4\theta \text{ln}S^*}{8\theta^3 S^*}\right]. && (76)
\end{aligned}
$$

Note that ΔY_1^S and ΔY_2^S are always positive, as is $8\theta^3 S^*$. Therefore, I can show that $\frac{d^2\Delta Y_j^S}{dS^* d\theta} > 0$ by showing that the numerator of the term in brackets in 75 and 76 is positive, using the definitions of i^L, i^H and $(i^H - i^L)$ in 50, 51 and 46, as follows:

$$
\begin{aligned}
-(\text{ln}S^*)^3 + \theta^2 \text{ln}S^* - \theta(\text{ln}S^*)^2 + \theta^3 - 4\theta \text{ln}S^* &> 0 \\
-2\theta(\text{ln}S^*)^2\left[\frac{\text{ln}S^*}{2\theta} + \frac{1}{2}\right] + 2\theta^3\left[\frac{\text{ln}S^*}{2\theta} + \frac{1}{2}\right] - 4\theta \text{ln}S^* &> 0 \\
i^L 2\theta[\theta^2 - (\text{ln}S^*)^2] - 4\theta \text{ln}S^* &> 0 \\
i^L 2\theta(\theta - \text{ln}S^*)(\theta + \text{ln}S^*) - 4\theta \text{ln}S^* &> 0 \\
i^L 8\theta^3\left(\frac{1}{2} - \frac{\text{ln}S^*}{2\theta}\right)\left(\frac{1}{2} + \frac{\text{ln}S^*}{2\theta}\right) - 4\theta \text{ln}S^* &> 0 \\
8\theta^3 (i^L)^2 i^H - 4\theta \text{ln}S^* &> 0 \\
2\theta (i^L)^2 i^H - \frac{\text{ln}S^*}{\theta} &> 0 \\
2\theta (i^L)^2 i^H + (i^H - i^L) &> 0. && (77)
\end{aligned}
$$

Thus S^* and θ are complements in the gains from trade with trading costs, in the special case where $\bar{A} = 1$.

Note also that the gains from trade with social skills are increasing in θ:

$$\begin{aligned}
\frac{d\Delta Y_1^S}{d\theta} &= \Delta Y_1^S\bigg(- \frac{\ln S^*}{2\theta^2}\Big[\theta\Big(- \frac{\ln S^*}{2\theta} + \frac{1}{2}\Big) + \ln S^*\Big] + \Big(\frac{\ln S^*}{2\theta} + \frac{1}{2}\Big)\Big[\Big(- \frac{\ln S^*}{2\theta} + \frac{1}{2}\Big) + \theta\frac{\ln S^*}{2\theta^2}\Big]\bigg) \\
&= \Delta Y_1^S\bigg(\frac{(\ln S^*)^2}{4\theta^2} - \frac{\ln S^*}{4\theta} - \frac{(\ln S^*)^2}{2\theta^2} - \frac{(\ln S^*)^2}{4\theta^2} + \frac{\ln S^*}{4\theta} + \frac{(\ln S^*)^2}{4\theta^2} - \frac{\ln S^*}{4\theta} + \frac{1}{4} + \frac{\ln S^*}{4\theta}\bigg) \\
&= \Delta Y_1^S\bigg(\frac{(\ln S^*)^2}{4\theta^2} - \frac{(\ln S^*)^2}{2\theta^2} + \frac{1}{4}\bigg) \\
&= \Delta Y_1^S\bigg(- \frac{(\ln S^*)^2}{4\theta^2} + \frac{1}{4}\bigg) \\
&= \Delta Y_1^S\Big(\frac{1}{2} - \frac{\ln S^*}{2\theta}\Big)\Big(\frac{1}{2} + \frac{\ln S^*}{2\theta}\Big) \\
&= \Delta Y_1^S i^H i^L > 0 \\
\frac{d\Delta Y_2^S}{d\theta} &= \Delta Y_2^S i^H i^L > 0. \qquad (78)
\end{aligned}$$

3.7 Complementarity of θ and Social Skill in Production

In this section, I consider the situation where there are two sectors, each employing two workers. Sector 1 has a higher value of θ and therefore more dispersion in productivity (a steeper comparative advantage schedule) compared with sector 2: $\theta_1 > \theta_2$. Also imagine indexing all possible sets of two workers by S^*. Define set A to be the set containing the two workers with the highest value of S^* (i.e. the two workers with the highest individual values of S_j), and set B to be the set containing the remaining two workers: $S_A^* > S_B^*$. All workers have identical ability A (so $\bar{A} = 1$ and $\omega = 1$).

I want to show that - while wages in both sectors will be higher for set A workers than set B workers - wages for set A workers in sector 1 will be higher than in sector 2. This is equivalent to showing that the second derivative of wages with respect to S^* and θ is positive. Since wages are identical to output scaled by a constant, I can take the derivatives with respect to output rather than wages.

To derive theses expressions, I use the result from 3.6 that S^* and θ are complements in the gains from trade. Note that:

$$Y_j^S = \frac{Y_j^S}{Y_j^A} Y_j^A = \Delta Y_j^S Y_j^A. \qquad (79)$$

In other words, production under task trade with trading costs is eqivalent to the gains from trade with social skills multiplied by production under autarky.

Taking the derivative of production with trading costs with respect to social skill using 79 yields:

$$\begin{aligned}
\frac{dY_1^S}{dS^*} &= \frac{d\Delta Y_1^S}{dS^*} Y_1^A + \Delta Y_1^S \frac{dY_1^A}{dS^*} \\
&= \frac{d\Delta Y_1^S}{dS^*} Y_1^A > 0 \qquad (80) \\
\frac{dY_2^S}{dS^*} &= \frac{d\Delta Y_2^S}{dS^*} Y_2^A + \Delta Y_2^S \frac{dY_2^A}{dS^*} \\
&= \frac{d\Delta Y_2^S}{dS^*} Y_2^A > 0. \qquad (81)
\end{aligned}$$

Note that I have shown in 73 and 74 that the gains from trade, ΔY_j^S, are increasing in S^*. In addition, $\frac{dY_j^A}{dS^*} = 0$ because S^* does not appear in the expression for production under autarky (23). Thus production is increasing in S^* for both workers.

Next, I take the second derivative of 80 and 81 with respect to S^* and θ:

$$\frac{d^2Y_1^S}{dS^*d\theta} = \frac{d^2\Delta Y_1^S}{dS^*d\theta}Y_1^A + \frac{d\Delta Y_1^S}{dS^*}\frac{dY_1^A}{d\theta} > 0 \tag{82}$$
$$\frac{d^2Y_2^S}{dS^*d\theta} = \frac{d^2\Delta Y_2^S}{dS^*d\theta}Y_2^A + \frac{d\Delta Y_2^S}{dS^*}\frac{dY_2^A}{d\theta} > 0. \tag{83}$$

I have shown in 75, 76 and 77 that $\frac{d^2\Delta Y_j^S}{dS^*d\theta} > 0$, and in 73 and 74 that $\frac{d\Delta Y_j^S}{dS^*} > 0$.

Thus all that remains to verify the claim in 82 and 83 that $\frac{d^2Y_j^S}{dS^*d\theta} > 0$ is to show that $\frac{dY_j^A}{d\theta} > 0$. In section 1.3, I argued that the functional form for $\gamma(i)$ is consistent with a log-normal distribution for the underlying task productivities. If productivity in task t for worker j is $a_j(t) \sim \ln\mathcal{N}(\mu_j, \sigma^2)$, as in 7, then mean productivity for worker j is $\exp(\mu_j + \sigma^2/2)$, which increasing in σ^2. Since $\theta \approx 2\sigma$, mean productivity for worker j should be increasing in θ, as well. Under autarky, production is an increasing function of worker j's productivity in each task (23). Thus an increase in θ should raise worker j's mean productivity level and hence the worker's total productivity under autarky. Note that the inequalities in 82 and 83 hold even if $\frac{dY_j^A}{d\theta} = 0$; the only necessary condition is that $\frac{dY_j^A}{d\theta}$ not be negative.

I can use the same approach to show that production (and therefore wages) are increasing in θ when $\bar{A} = 1$:

$$\frac{dY_1^S}{d\theta} = \frac{d\Delta Y_1^S}{d\theta}Y_1^A + \Delta Y_1^S\frac{dY_1^A}{d\theta} \tag{84}$$
$$\frac{dY_2^S}{d\theta} = \frac{d\Delta Y_2^S}{d\theta}Y_2^A + \Delta Y_2^S\frac{dY_2^A}{d\theta}. \tag{85}$$

I have shown in 78 that $\frac{d\Delta Y_j^S}{d\theta} > 0$.