A medical problem of cobblestones and pavements

Anyone walking through the streets of cities, towns and villages, both in the UK and abroad will be aware of how painful on the feet these experiences often are. This is due to the unevenness of the surfaces. This analysis is designed to point out to local public health advisors and local Council members the precise nature of the problem. I am afraid a certain amount of elementary mathematics is necessary, but those members who cannot understand it will no doubt be able to find someone who can explain it to them.

Let $D$ be a measure of distress, i.e. pain in the feet, scaled from zero (perfectly comfortable to one (agonizing). Obviously a number of factors have to be considered. First, the heavier you are, the more painful for the feet ($W$—in kg—indicates weight). Secondly, the thicker the shoe soles, the less painful ($T$ in cm is thickness of the sole). Thirdly, the bigger feet you have got, the more the load is spread, so area of sole in square centimetres which correlates with shoe size has to be included. It is clearly necessary to include the size of the surface elements; pure fine grained sand is going to be entirely comfortable to walk on, but as the elements get bigger, i.e. approaching cobble size, walking gets progressively less comfortable. However, as the size of the elements continues to increase, they gradually approach paving stone type, which then become more comfortable again. In other words, the effect on pain of the size of surface elements is biphasic—it starts low and reaches a maximum, from which point it becomes more comfortable again, and as the element size approaches infinity, it becomes entirely comfortable again; this is shown in the graph below, for which the data points were invented by myself to describe the situation. Infinity is, of course the state of affairs when the surface is consists of well-laid smooth tarmac.

This curve can be fitted by the following equation:

$$D = 1.9(k/\lambda),(x/\lambda)^{k-1}.e^{-(x/\lambda)^k},$$

where, in this case, $k = 1.5$ and $\lambda = 1$.

This is an example of the Weibull distribution, doubtless well-known to readers of the QJM. If, by any unlikely chance, there are those who, like the present author, were unfamiliar with Waloddi Weibull and his distribution until now, they can look it up in Wikipedia. The ‘1.9’ at the beginning of the right hand side of the equation is a composite scaling factor, taking $W$, $T$ and $A$ (see above) into account.

The above shows that you should not overeat (vain hope) and that you should remember to buy thick-soled shoes next time, while considering whether to sue your Local Council for the distress caused by the state of their pavements! It also demonstrates the lengths to which retired academics have to go to prevent cerebral atrophy. More seriously, I once had a patient with total lipodystrophy, whose most distressing problem was extreme discomfort in the feet, even when walking on a smooth surface, because of the lack of padding; what it must have been like for him to walk around towns with the current state of our pavements, one can scarcely imagine.

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