Supplementary material for “Mortgages and Monetary Policy”

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The supplementary material covers (A) summary of equilibrium conditions, (B) computation, (C) data used in calibration, (D) estimates of mortgage debt servicing costs for the United States, (E) details of calibration, (F) details of optimal refinancing, (G) details of optimal mortgage choice.

**Appendix A: Equilibrium conditions**

This appendix lists the conditions characterizing the equilibrium defined in Section 3 for the basic FRM and ARM cases. We use these conditions to solve for the steady state (a situation in which all shocks are equal to their unconditional means and all real variables, interest rates, and the inflation rate are constant). The steady state is used to calibrate the model and serves as the point of a local approximation of the economy in our solution method described in Appendix B. Interested readers may also find these conditions useful if they wish to compute the equilibrium by log-linearization of the equilibrium conditions. Our computational method does not rely on approximating these conditions. Instead, it takes a linear-quadratic approximation of the Bellman equations within the recursive competitive equilibrium problem described in Section 3 and then solves the LQ problem. Details are provided in Appendix B. As is well known, the two methods yield the same approximate decision rules and pricing functions. To solve for the case of refinancing, the standard method needs to be modified, as explained in Appendix B. The steady state, however, is the same for the basic FRM case, the ARM case, refinancing, and the mortgage choice problem. This is because, in the steady state, \( i_t = i^F_t = R_t \). (As a result, steady-state refinancing and ARM share are parameterized, as noted in the text.) The conditions below can thus be used to solve for the steady state under all contract types considered in this paper.

Throughout, the notation is that, for instance, \( u_{ct} \) denotes the first derivative of the function \( u \) with respect to \( c \), evaluated in period \( t \). Alternatively, \( v_{2t} \), for instance, denotes the first derivative of the function \( v \) with respect to the second argument, evaluated in period \( t \).

**Capital owner’s optimality**

First-order conditions:

\[
1 = E_t \left\{ \beta \frac{u_{c,t+1}}{u_{ct}} \left[ 1 + (1 - \tau_K)(r_{t+1} - \delta_K) \right] \right\}, 
\]

(A1)

\[
1 = E_t \left\{ \beta \frac{u_{c,t+1}}{u_{ct}} \left[ \frac{(1 - \tau_b,t+1)(1 + i_t)}{1 + \pi_{t+1}} \right] \right\}, 
\]

(A2)

\[
1 = E_t \left\{ \beta \frac{\tilde{U}_{d,t+1}}{u_{ct}} + \beta \frac{U_{,\gamma,t+1}}{u_{ct}} \xi_{Dt}^* \left[ \kappa - (\gamma^*_t)^{\alpha} \right] + \beta \frac{U_{R,t+1}}{u_{ct}} \xi_{Dt}^* (i^F_t - R^*_t) \right\}. 
\]

(A3)

Note: the last equation (for \( l^*_t \)) applies only in the FRM case, as explained in the text.
Benveniste-Scheinkman conditions:

\[
\tilde{U}_{dt} = u_{ct} \frac{R_t^* + \gamma_t^*}{1 + \pi_t} + \beta \frac{1 - \gamma_t^*}{1 + \pi_t} E_t \left\{ \tilde{U}_{d,t+1} + \zeta_t^* \left[ (\gamma_t^*)^\alpha - \kappa \right] U_{\gamma,t+1} + \zeta_t^* (R_t^* - i_t^F) U_{R,t+1} \right\}, \quad (A4)
\]

\[
U_{\gamma t} = u_{ct} \left( \frac{\tilde{d}_t^*}{1 + \pi_t} \right) - \beta \left( \frac{\tilde{d}_t^*}{1 + \pi_t} \right) E_t \tilde{U}_{d,t+1} + \beta \left( \frac{\tilde{d}_t^*}{1 + \pi_t} \right) \zeta_t^* \left[ \kappa - (\gamma_t^*)^\alpha \right] + \frac{(1 - \gamma_t^*)\alpha(\gamma_t^*)^{\alpha - 1}}{1 - \gamma_t^*} \tilde{d}_t^* + \tilde{l}_t^*, \quad (A5)
\]

\[
U_{R_t} = u_{ct} \left( \frac{\tilde{d}_t^*}{1 + \pi_t} \right) + \beta \left( \frac{1 - \gamma_t^*}{1 + \pi_t} \tilde{d}_t^* \right) E_t U_{R,t+1}. \quad (A6)
\]

Constraints:

\[
c_t^* + k_{t+1} + \tilde{b}_{t+1}^* + \tilde{l}_t^* = [1 + (1 - \tau K)(r_t - \delta_K)] k_t + (1 - \tau b_t)(1 + i_{t-1}) \frac{\tilde{b}_t^*}{1 + \pi_t} + \tilde{m}_t^* + \tau_t^* + \frac{p_{Lt}}{1 - \Psi_t}, \quad (A7)
\]

\[
\tilde{m}_t^* = (R_t^* + \gamma_t^*) \frac{\tilde{d}_t^*}{1 + \pi_t}, \quad (A8)
\]

\[
\tilde{d}_{t+1}^* = \frac{1 - \gamma_t^*}{1 + \pi_t} \tilde{d}_t^* + \tilde{l}_t^*, \quad (A9)
\]

\[
\gamma_{t+1}^* = (1 - \phi_t^*) (\gamma_t^*)^\alpha + \phi_t^* \kappa, \quad (A10)
\]

\[
R_{t+1}^* = \begin{cases} (1 - \phi_t^*) R_t^* + \phi_t^* i_t^F, & \text{if FRM}, \\ i_t, & \text{if ARM}, \end{cases} \quad (A11)
\]

\[
x_K t = k_{t+1} - (1 - \delta_k) k_t. \quad (A12)
\]

**Homeowner’s optimality**

First-order conditions:

\[
v_{ct}(1 - \theta) p_{Ht} = \beta E_t \left\{ V_{h,t+1} + p_{Ht} \theta \left[ \tilde{V}_{d,t+1} + \zeta_t^* (\kappa - \gamma_t^* \kappa) V_{\gamma,t+1} + \zeta_t^* (i_{t+1}^M - R_t) V_{R,t+1} \right] \right\}, \quad (A13)
\]

\[
1 = E_t \left[ \beta \frac{v_{ct,t+1}^*}{v_{ct}} \left( \frac{1 + i_t + \gamma_t}{1 + \pi_{t+1}} \right) \right]. \quad (A14)
\]
Note: $i_{t+1}^M = i_t^F$ (FRM case), $i_{t+1}^M = i_t$ (ARM case).

Benveniste-Scheinkman conditions:

$$
\tilde{V}_{dt} = -v_{ct} \frac{R_t + \gamma_t}{1 + \pi_t} + \beta \frac{1 - \gamma_t}{1 + \pi_t} E_t \left[ \tilde{V}_{d,t+1} + \zeta_t (\gamma_t^\alpha - \kappa) V_{\gamma,t+1} + \zeta_t (R_t - i_{t+1}^M) V_{R,t+1} \right],
$$

(A15)

$$
V_{\gamma,t} = -v_{ct} \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) - \beta \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) E_t \tilde{V}_{d,t+1}
+ \beta \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) \left[ \zeta_t (\kappa - \gamma_t^\alpha) + \frac{(1 - \gamma_t) \alpha \gamma_t^{\alpha-1}}{1 + \pi_t} \frac{l_t}{d_t + l_t} \right] E_t \tilde{V}_{d,t+1}
+ \beta \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) \zeta_t (i_{t+1}^M - R_t) E_t V_{R,t+1},
$$

(A16)

$$
V_{R,t} = -v_{ct} \left( \frac{\tilde{d}_t}{1 + \pi_t} \right) + \beta \left( \frac{1 - \gamma_t}{1 + \pi_t} \frac{\tilde{d}_t}{d_t + l_t} \right) E_t V_{R,t+1},
$$

(A17)

$$
V_{ht} = v_{ht} + \beta (1 - \delta_H) E_t V_{h,t+1}.
$$

(A18)

Constraints

$$
\tilde{l}_t = \theta p_{Ht} x_{Ht},
$$

(A19)

$$
x_{Ht} = h_{t+1} - (1 - \delta_H) h_t.
$$

(A20)

Production

First-order conditions:

$$
r_t = A_t f_1 \left( (1 - \Psi) k_t, \Psi n \right),
$$

(A21)

$$
 w_t = A_t f_2 \left( (1 - \Psi) k_t, \Psi n \right),
$$

(A22)

$$
 Y_t = A_t f \left( (1 - \Psi) k_t, \Psi n \right).
$$

(A23)

PPF curvature:

$$
q_t = \eta q (\Psi x_{St}).
$$

(A24)

Homebuilding

First-order conditions (after imposing land market clearing $X_{Lt} = 1$):

$$
x_{St} = \frac{1}{\Psi} \left( \Psi x_{Ht} \right)^{1/\nu},
$$

(A25)
\[ p_{Ht} = q_t \frac{(\psi x_{St})^\varphi}{1 - \varphi}, \]  
(A26)  
\[ p_{Lt} = p_{Ht} \varphi (\psi x_{St})^{1-\varphi}. \]  
(A27)

**Monetary policy**

\[ i_t = (i_t - \bar{\pi} + \bar{\pi}_t) + \nu_t (\pi_t - \bar{\pi}_t). \]  
(A28)

**Government budget constraint**

\[ G + (1 - \Psi) r_t^* = \tau_k (r_t - \delta_k)(1 - \Psi)k_t + \tau_n (w_t \Psi n - \Psi \tau) + \Psi \tau + \Xi_t. \]  
(A29)

**Market clearing**

\[ (1 - \Psi)c_t^* + \Psi c_t + (1 - \Psi)x_{Kt} + q_t \Psi x_{St} + G = Y_t, \]  
(A30)  
\[ (1 - \Psi)\tilde{b}_t^* + \Psi \tilde{b}_t = 0, \]  
(A31)  
\[ (1 - \Psi)\tilde{d}_t^* = \Psi \tilde{d}_t. \]  
(A32)

**Aggregate consistency**

\[ (1 - \Psi)\tilde{d}_t^* = \Psi \tilde{d}_t, \]  
(A33)  
\[ \gamma_t^* = \gamma_t, \]  
(A34)  
\[ R_t^* = R_t. \]  
(A35)

**Equation count:** 35 equations (FRM), 34 equations (ARM). Note: the homeowner’s budget constraint holds by Walras’ law.

**Endogenous variables**

35 variables (FRM), 34 variables (ARM):

- **Allocations:** \( c_t^*, x_{Kt}, k_{t+1}, c_t, x_{Ht}, h_{t+1}, x_{St}, Y_t \). 8 variables
- **Prices:** \( \pi_t, i_t, i^F_t \) (FRM only), \( r_t, w_t, q_t, p_{Lt}, p_{Ht} \). 8 vars (FRM), 7 vars (ARM)
- **Mortgages:** \( \tilde{l}_t^*, m_t^*, \tilde{d}_t^{*+1}, \gamma_{t+1}^*, R_{t+1}^*, \tilde{l}_t, \tilde{d}_t^{+1}, \gamma_t^{+1}, R_t^{+1} \). 9 variables
- **Bonds:** \( \tilde{b}_t^{*+1}, \tilde{b}_t^{+1} \). 2 variables
- **Transfer:** \( \tau_t^* \). 1 variable
- **Value function:** \( \tilde{U}_{dt}, U_{\gamma_t}, U_{Rt}, \tilde{V}_{dt}, V_{\gamma_t}, V_{Rt}, V_{ht} \). 7 variables
Shocks

\[ \log A_{t+1} = (1 - \rho_A) \log A + \rho_A \log A_t + \epsilon_{A,t+1}, \]
\[ \pi_{t+1} = (1 - \rho_\pi) \pi_t + \rho_\pi \pi_t + \epsilon_{\pi,t+1}, \]
\[ \tau_{b,t+1} = \rho_b\tau_{b,t} + \epsilon_{b,t+1}, \]
\[ \eta_{t+1} = (1 - \rho_q) + \rho_q \eta_t + \epsilon_{q,t+1}, \]
\[ \epsilon_{A,t+1} \sim iidN(0, \sigma_A). \]
\[ \epsilon_{\pi,t+1} \sim iidN(0, \sigma_\pi). \]
\[ \epsilon_{b,t+1} \sim iidN(0, \sigma_b). \]
\[ \epsilon_{q,t+1} \sim iidN(0, \sigma_q). \]

Transformation of variables used in (A1)-(A35)

To ensure stationarity:

\[ \tilde{U}_{dt} \equiv p_t - 1 U_{dt}, \]
\[ \tilde{m}_t^* \equiv m_t^*/p_t, \]
\[ \tilde{d}_t^* \equiv d_t^*/p_{t-1}, \]
\[ \tilde{l}_t^* \equiv l_t^*/p_t, \]
\[ \tilde{b}_t^* \equiv b_t^*/p_{t-1}, \]
\[ \tilde{V}_{dt} \equiv p_t - 1 V_{dt}, \]
\[ \tilde{d}_t \equiv d_t/p_{t-1}, \]
\[ \tilde{l}_t \equiv l_t/p_t, \]
\[ \tilde{b}_t \equiv b_t/p_{t-1}. \]

Auxiliary notation:

\[ \zeta_{lt}^* \equiv \frac{\tilde{l}_t^*}{\left( \frac{1 - \gamma^*}{1 + \pi_t} \tilde{d}_t^* + \tilde{l}_t^* \right)^2} \in (0, 1), \]
\[ \zeta_{Dt}^* \equiv \frac{1 - \gamma^*}{1 + \pi_t} \tilde{d}_t \left( \frac{1 - \gamma^*}{1 + \pi_t} \tilde{d}_t + \tilde{l}_t \right)^2 \in (0, 1), \]
\[ \zeta_{Dt} \equiv \frac{1 - \gamma^*}{1 + \pi_t} \tilde{d}_t \left( \frac{1 - \gamma^*}{1 + \pi_t} \tilde{d}_t + \tilde{l}_t \right)^2 \in (0, 1), \]
\[ \zeta_{lt} \equiv \frac{\tilde{l}_t}{\left( \frac{1 - \gamma}{1 + \pi_t} \tilde{d}_t + \tilde{l}_t \right)^2} \in (0, 1), \]
\[ \phi_t^* \equiv \frac{\tilde{l}_t^*}{\tilde{d}_{t+1}^*} \in (0, 1), \]
\[ \Xi_t \equiv (1 - \Psi) \tau_{Bt}(1 - \tau_{t-1}) \tilde{b}_t^*/(1 + \pi_t). \]
\[ \gamma_t = \gamma \left( -\tilde{b}_{t+1} \right). \]

Residual variable

\[ \tilde{m}_t = (R_t + \gamma_t) \frac{d_t}{1 + \pi_t} \quad \text{or} \quad \tilde{m}_t = \frac{1 - \Psi}{\Psi} \tilde{m}_t. \]

Connecting the GE model with the simple PE model

Mortgage pricing

Notice that for a once-and-for-all mortgage loan (\( l_t^* = l^* \) in period \( t \) and \( l_t^* = 0 \) thereafter) and no outstanding mortgage debt (\( d_t^* = 0 \) in period \( t \)), we have \( \zeta_{Dt}^* = 0 \) and \( \zeta_{t, t+j}^* = 0 \), for \( j = 1, 2, \ldots \). In this case, the first-order condition for \( l_t^* \) (A3) and the BS condition for \( \tilde{U}_{dt} \) (A4) simplify, as the terms related to \( U_{yt} \) and \( U_{Rt} \) drop out. Once combined, the two optimality conditions result in an equation that is a straightforward infinite-horizon extension of the mortgage-pricing equation (1) in the two-period mortgage example of Section 2:

\[ 1 = E_t \left[ Q_{1t}^* (f_t^* + \gamma_{t+1}^*) + Q_{2t}^* (f_t^* + \gamma_{t+2}^*) (1 - \gamma_{t+1}^*) + \ldots \right], \]

where

\[ Q_{jt}^* \equiv \prod_{j=1}^{J} \frac{u_{c,t+j} - u_{c,t+j-1}}{1 + \pi_{t+j}} \frac{1}{1 + \pi_{t+j}} \quad J = 1, 2, \ldots \]

The terms related to \( U_{yt} \) and \( U_{Rt} \) in the general form of the optimality conditions arise because the mortgage payment \( m_t^* \) entering the budget constraint of the capital owner pertains to payments on the entire outstanding mortgage debt, not just the new loan. In this case, the terms related to \( U_{yt} \) and \( U_{Rt} \) capture the marginal effect of \( l_t^* \) on the average interest and amortization rates of the outstanding debt, and thus the marginal effect of \( l_t^* \) on the mortgage payments on the outstanding debt.

The wedge

Rearranging the first-order condition for \( x_{Ht} \) (A13) yields

\[ v_{ct} p_{Ht} (1 + \tau_{Ht}) = \beta E_t V_{h,t+1}, \]

where the wedge \( \tau_{Ht} \) is given by

\[ \tau_{Ht} \equiv -\theta E_t \left[ 1 + \beta \frac{\tilde{V}_{dt,t+1}}{v_{ct}} + \zeta_{Dt} (\kappa - \gamma_t^*) \frac{V_{t+1}}{v_{ct}} + \zeta_{Dt} (i_{t+1}^M - R_t) \frac{V_{R,t+1}}{v_{ct}} \right]. \]
For the same reasons as above, the wedge is more complicated than in the case of the two-period mortgage. It becomes a straightforward infinite-horizon extension of equation (3) in Section 2 if the housing investment decision is once-and-for-all and there is no outstanding mortgage debt (i.e., \( \zeta_{Dt} = 0 \) and \( \zeta_{t+j} = 0 \), for \( j = 1, 2, \ldots \)):

\[
\tau_{Ht} \equiv -\theta E_t \left\{ 1 - \left[ Q_{1t} \left( \frac{M_{t+1}}{1 + \gamma_{t+1}} + \gamma_{t+1} \right) - Q_{2t} \left( \frac{M_{t+2}}{1 + \gamma_{t+2}} \right) \right] \right\},
\]

where

\[
Q_{jt} \equiv \prod_{j=1}^{J} \beta \frac{v_{c,t+j}}{1 + \pi_{t+j}}.
\]

**Appendix B: Computation**

**Overview**

The equilibrium is computed using a linear-quadratic (LQ) approximation method for distorted economies with exogenously heterogenous agents (see Hansen and Prescott, 1995), adjusted along the lines of Benigno and Woodford (2006). Further modification, described below, is made to handle refinancing.

In a nutshell, the method approximates the recursive competitive equilibrium problem of the original economy, specified in Section 3, with a corresponding linear-quadratic problem. The method involves iteration on quadratic Bellman equations (one for each agent type) subject to linear individual, aggregate, and market clearing constraints. The fixed point of the iteration is a pair of (quadratic) value functions for the homeowner and capital owner, \( \hat{P}_H(z, S) \) and \( \hat{P}_C(z, S) \), and a set of (linear) aggregate decisions rules and pricing functions, \( \hat{W}(z, S) \), where \( z \) is a vector of exogenous state variables and \( S \) is a vector of endogenous aggregate state variables, as specified in Section 3. This function is accompanied by a resulting set of linear laws of motion for the endogenous state variables, \( S' = \Omega(S, z) \).

The centering point of the LQ approximation is the steady state, satisfying the equilibrium conditions listed in Appendix A, and the LQ approximation of the original Bellman equations is computed using numerical derivatives. All variables in the approximation are either in percentage deviations or percentage point deviations (for interest, inflation, and amortization rates) from the steady state. Before computing the equilibrium, the model is made stationary by expressing all nominal variables in real terms and replacing ratios of price levels with the inflation rate, as in Appendix A (as a result, \( p_t \) is replaced with \( \pi_t \) in the vector \( W_t \), specified in Section 3, and \( p_{t-1} \) drops out of the vector \( z_t \)).

**Return functions in Bellman equations**

The Hansen-Prescott method needs to be modified along the lines of Benigno and Woodford (2006) because the laws of motion for the mortgage variables are nonlinear. This means that these equations cannot be substituted out into the per-period utility functions, as is normally done in LQ approximation procedures. The modification involves forming a Lagrangian for each agent, consisting of the per-period utility function and the respective laws of motion for the mortgage variables. The Lagrangian is then used as the return function in the Bellman
equation being approximated. This adjustment is necessary to ensure that second-order cross-derivatives of the utility function and the constraints are taken into account in the LQ approximation. This modification, as applied to the homeowner, is described in detail by Kydland, Rupert, and Šustek (2016). The specification for the capital owner is analogous. We therefore refer the reader to that paper for details.

An alternative procedure would be to log-linearize the equilibrium conditions in Appendix A and use a version of the Blanchard-Kahn method, or a method of undetermined coefficients, to arrive at the equilibrium decision rules and pricing functions. As is well known, these methods yield the same linear equilibrium decision rules and pricing functions as the modified LQ approximation method.

**Certainty equivalence**

The linear equilibrium decision rules and pricing functions computed as described above are characterized by certainty equivalence. That is, the coefficients of the linear functions $\hat{W}(z, S)$ loading onto $z$ and $S$ depend only on the conditional means of future values of the exogenous state variables, implied by their AR(1) processes. Not on higher moments.

**Computing business cycle moments and impulse-responses**

The business cycle moments used to cross-validate the model are based on artificial data from 150 runs of the model. In a single run, a sequence of innovations $[\epsilon_{At}, \epsilon_{Pt}, \epsilon_{Mt}, \epsilon_{qt}]$, for $t = 1, \ldots, 150$, is drawn from their respective iid normal distributions. The AR(1) processes for the exogenous state variables are then used to generate a sequence of the vector $z$ (for $t = 1, \ldots, 150$), which is then fed into the endogenous functions $\hat{W}(z, S)$ and $S' = \hat{\Omega}(S, z)$ to recursively generate a sequence of the endogenous variables of interest.

When computing impulse-responses, either to cross-validate the model or in the main computational experiments, all exogenous state variables except the one of our interest are held constant at their unconditional means. A path for the exogenous state variable of our interest is generated by setting its innovation equal to a particular value in period $t = 1$ and to zero in all subsequent periods. The path of the state variable is then generated using the variable’s AR(1) process and this path is fed into the endogenous functions $\hat{W}(z, S)$ and $S' = \hat{\Omega}(S, z)$ to recursively generate a sequence of the endogenous variables. Because the model is stationary, after the initial impulse in $t = 1$ that moves the economy away from the steady state, all variables converge back to the steady state.

**Handling refinancing**

The computational method needs to be modified in the case of refinancing. The adjustments we introduce are inspired by the method proposed by Guerrieri and Iacoviello (2015). Their ideas are inset here in the general computation procedure described so far. Essentially, with refinancing, the economy can operate under two regimes: one in which $i_t^F \geq R_t$ and $\varrho_t = \varrho$ and another in which $i_t^F < R_t$ and $\varrho_t = \tilde{\varrho}_t$. We therefore compute two sets of approximate equilibrium decision rules and pricing functions, $\hat{W}_1(z, S)$ for the first regime and $\hat{W}_2(z, S)$ for the second regime (and two sets of the corresponding laws of motion for $S$). The point of approximation is the steady state, which (as explained in Appendix A) is common
to both regimes. Given that the economy can operate under two regimes, the endogenous linear functions $\hat{W}_1(z, S)$ and $\hat{W}_2(z, S)$ have to take into account the fact that the economy can switch between them. This is relatively straightforward to implement in the case of the experiments of Section 5.2. There are two reasons for this. First, after the initial decline, in response to the negative $\pi_t$ shock, $i^F_t$ converges monotonically back to the steady state. Second, from the law of motion for the interest rate on outstanding debt follows that $i^F_t$ is a marginal rate of $R_t+1$. Therefore, if $i^F_t$ falls below the steady state and converges back monotonically, $i^F_t$ and $R_t$ cross paths only once. That is, the economy switches from the regime $i^F_t < R_t$ (which occurs on impact of the shock) to the regime $i^F_t \geq R_t$ only once along the convergence path. Therefore, once the economy is in the latter regime, it stays in it until it converges to the steady state. Further, the point in which it switches is deterministic. It occurs some $N$ periods after the initial impact of the shock, where $N$ is endogenous.

To compute the equilibrium decision rules and pricing functions for the impulse-responses under refinancing, we proceed in two steps. First, we compute the approximation $\hat{W}_1(z, S)$ for the case $i^F_t \geq R_t$. This involves the usual procedure described above, as the economy stays in this regime once it is in it. Denote the pair of the fixed-point quadratic value functions (one for each agent type) that result from this step as $\hat{P}_1(z, S)$. To compute the equilibrium decision rules and pricing functions for the case of $i^F_t < R_t$, we modify the usual procedure in two ways. First, we use as a starting point of the iterative procedure on the Bellman equations the pair of value functions $\hat{P}_1(z, S)$. Second, we do not iterate until a fixed point, but only $N$ times. After obtaining $\hat{W}_1(z, S)$ and $\hat{W}_2(z, S)$ this way, we generate impulse-response functions and check whether the $N$ we have used in computing $\hat{W}_2(z, S)$ is consistent with the resulting impulse-responses; i.e., whether $i^F_t$ and $R_t$ cross paths $N$ periods after the shock. If not, we change $N$ and repeat the steps until consistency is obtained. For the calibrated persistence of the shock, $N = 60$. It turns out that for $N$ this far in the future, the responses in the first 40 periods we are looking at obtained for the two-regime economy are almost identical to the responses obtained under ‘naive’ decision rules and pricing functions that do not take into account the fact that the economy will switch to the second regime 60 periods after the shock (the naive decision rules and pricing functions are obtained as a fixed point of the usual iterative procedure assuming that the economy is always in the $i^F_t < R_t$ regime).

Appendix C: Data counterparts to variables

This appendix describes the data used to calculate the aggregate ratios employed in calibrating the model. Adjustments to official data are made to ensure that the data correspond conceptually more closely to the variables in the model. To start, for reasons discussed by Gomme and Rupert (2007), the following expenditure categories are taken out of GDP: gross housing value added, compensation of general government employees, and net exports. In addition, we also exclude expenditures on consumer durable goods (as our ‘home capital’ includes only housing) and multifamily structures (which are owned by corporate entities and rented out to households mainly in the 1st and 2nd quintiles of the wealth distribution). With these adjustments, the data counterparts to the expenditure components of output in the model are constructed from BEA’s NIPA tables as follows: consumption ($C$) = the sum of expenditures on nondurable goods and services less gross housing value added; capital in-
vestment ($X_K$) = the sum of nonresidential structures, equipment & software, and the change in private inventories; housing structures ($X_S$) = residential gross fixed private investment less multifamily structures; and government expenditures ($G$) = the sum of government consumption expenditures and gross investment less compensation of general government employees. Our measure of output ($Y = C + X_K + X_S + G$) accounts, on average (1958-2006), for 74% of GDP. BEA’s Fixed Assets Tables and Census Bureau’s M3 data provide stock counterparts to capital and housing investment: capital stock ($K$) = the sum of private nonresidential fixed assets and business inventories; housing stock ($H$) = residential assets less 5+ unit properties.¹

Appendix D: Estimation of mortgage debt servicing costs

A key measurement for calibrating the model concerns the mortgage debt servicing costs of homeowners. Unfortunately, such information for the United States is not readily available. Four different procedures are therefore used to arrive at an estimate. The four procedures exploit the notion that the homeowners in the model correspond to the 3rd and 4th quintiles of the U.S. wealth distribution. Some of these estimates arguably overestimate the debt servicing costs, while others underestimate it. Nevertheless, all four procedures yield estimates in the ballpark of 18.5% of pre-tax income, the value used to calibrate the model.

1. Estimate based on income from the Survey of Consumer Finances

The first procedure, for FRM (1972-2006) and ARM (1984-2006), combines data on income from the Survey of Consumer Finances (SCF) and the model’s expression for debt servicing costs. Suppose that all mortgage debt is FRM.² The model’s expression for steady-state debt-servicing costs, $(R + \gamma)D/(pwN - p\tau\Psi)$, can then be used to compute the average debt-servicing costs of homeowners. The various elements of this expression are mapped into data in the following way: $D/(pwN - p\tau\Psi)$ corresponds to the average ratio of mortgage debt (for 1-4 unit structures) to the combined personal income (annual, pre-tax) of the 3rd and 4th quintiles, equal to 1.56; $R$ corresponds to the average FRM annual interest rate for a conventional 30-year mortgage, equal to 9.31%; and $\gamma$ corresponds to the average amortization rate over the life of the mortgage, equal to 4.7% per annum. This yields debt servicing costs of 22%. This estimate is likely an upper bound as some of the outstanding mortgage debt in the data is owed by the 5th quintile (the 1st and 2nd quintiles are essentially renters) and the effective interest rate on the stock in the data is likely lower than the average FRM rate due to refinancing. When all mortgage debt is assumed to be ARM, this procedure yields 17.5% (based on the average Treasury-indexed 1-year ARM rate for a conventional 30-year mortgage).

¹Separate stock data on 2-4 unit properties are not available, but based on completions data from the Census Bureau’s Construction Survey, 2-4 unit properties make up only a tiny fraction of the multifamily housing stock.

²Federal Reserve’s Flow of Funds Accounts provide data on mortgages and we equalize mortgage debt in the model ($D$) with the stock of home mortgages for 1-4 family properties. The Flow of Funds data, however, include mortgage debt issued for purchases of existing homes, second mortgages, and home equity loans. In contrast, the model speaks only to first mortgages on new housing. The data thus provide an upper bound for $D$ in the model.
2. Estimate based on Financial Obligation Ratios

The second estimate is based on Federal Reserve’s Financial Obligation Ratios (FOR) for mortgages (1980-2006). FOR report all payments on mortgage debt (mortgage payments, homeowner’s insurance, and property taxes) as a fraction of NIPA’s share of disposable income attributed to homeowners. For our purposes, the problem with these data is that members of the 5th quintile of the wealth distribution are also counted as homeowners in the data (as long as they own a home), even though they do not represent the typical homeowner in the sense of Campbell and Cocco (2003). To correct for this, we apply the share of the aggregate SCF personal income, attributed to the union of the 3rd, 4th, and 5th quintiles of the wealth distribution, to aggregate disposable income from NIPA. This gives us an estimate of NIPA disposable income attributed to these three quintiles. This aggregate is then multiplied by the financial obligation ratio to arrive at a time series for total mortgage payments. Assuming again that all mortgage payments are made by the 3rd and 4th quintiles, the total mortgage payments are divided by NIPA personal (pre-tax) income attributed to just these two quintiles (calculated by applying the SCF shares). This procedure yields average debt-servicing costs of 20%.

3. Estimate based on wealth quintiles from the Survey of Consumer Finances

Third, we use the ratio of all debt payments to pre-tax family income for the 50-74.9 percentile of the wealth distribution, reported in SCF for 1989-2007. The average ratio is 19%. About 80% of the payments are classified as residential by the purpose of debt, yielding an average ratio of 15.2%. A key limitation of this procedure is that the data exclude the 1970s and most of the 1980s—periods that experienced almost twice as high mortgage interest rates, on average, than the period covered by the survey. Another issue is that the information reported in the survey is not exactly for the 3rd and 4th quintiles.

4. Estimate based on Consumer Expenditure Survey

The fourth procedure is based on the Consumer Expenditure Survey (CEX), 1984-2006. This survey reports the average income and mortgage payments (interest and amortization) of homeowners with a mortgage. To the extent that homeowners without a mortgage are likely to belong to the 5th quintile of the wealth distribution—they have 100% of equity in their home and thus have higher net worth than homeowners with a mortgage—the survey’s homeowners with a mortgage should closely correspond to the notion of homeowners used in this paper (CEX does not contain data on wealth). The resulting average, for the available data period, for mortgage debt servicing costs of this group (pre-tax income) is 15%. Given that the data do not cover the period of high mortgage rates of the late 1970s and early 1980s, like the third estimate, this estimate probably also underestimates the debt servicing costs for the period used in calibrating the model.

Taken together, the four procedures lead us to use 18.5%, a value in the middle of the range of the estimates, as a target in calibration.
Appendix E: Details of calibration

As most of the required historical data are readily available for the United States, the calibration is based on U.S. data, even though the mechanism applies more generally. One period in the model corresponds to one quarter.

A particular challenge arises due to the need to match debt-servicing costs—mortgage payments to income ratio—of homeowners. This requires the model to be consistent with the cross-sectional distribution of income, in addition to the standard aggregate targets: $X_K/Y = 0.156$, $X_S/Y = 0.054$, $K/Y = 7.06$, $H/Y = 5.28$, and $rK/Y = 0.283$, all averages for 1958-2006. Official data for mortgage debt servicing costs are not published for the United States. Estimates, however, can be obtained from different sources (see Appendix D), resulting in long-run averages in the ballpark of 18.5% of homeowners’ pre-tax income. The model’s steady-state counterpart to this ratio is $\tilde{M}/(wN - \Psi \tau)$, where $\tilde{M} = (R + \gamma)\tilde{D}/(1 + \pi)$, with $\tilde{D}$ being the debt-to-output ratio.

Consistency with the observed cross-sectional distribution of income is achieved through $\tau$. Recall that homeowners in the model are an abstraction for the 3rd and 4th quintiles of the wealth distribution, while capital owners are an abstraction for the 5th quintile. In the data, while the 5th quintile get substantial income from capital, they also receive income from transfers and labor. Thus, if the only source of income of capital owners in the model was capital, and given that the model is required to match the observed average capital share of output ($rK/Y = 0.283$), capital owners would account for too small fraction of aggregate income (28.3% in the model v.s. 48% in the data; SCF 1998), while homeowners’ share would be too large. As a result, the steady-state debt-servicing costs would be too low (or the required debt-to-output ratio would have to be too high, making it inconsistent with the observed $X_S/Y$ ratio, the land share in new housing, the loan-to-value ratio, and the amortization schedule). The parameter $\tau$ adjusts for this discrepancy, transferring some of the homeowners’ labor income to capital owners.

The parameter values are listed in Table 1, where they are organized into eight categories: $\Psi$ (population); $\beta$, $\xi$ (preferences); $\delta_K$, $\delta_H$, $\varsigma$, $\zeta$, $\varphi$ (technology); $\tau_K$, $\tau_N$, $G$, $\tau$ (fiscal); $\theta$, $\alpha$, $\kappa$ (mortgage contracts); $\vartheta$ (bond market); $\Pi$, $\nu_\pi$ (monetary policy); and $\rho_A$, $\sigma_A$, $\rho_\pi$, $\sigma_\pi$, $\rho_b$, $\sigma_b$, $\rho_q$, $\sigma_q$ (stochastic processes). Wherever possible, calibration targets are for the period 1958-2006. The calibration is mainly based on steady-state relations and unconditional first moments of the data. Most parameters can be assigned values without solving a system of steady-state equations, three parameters ($\xi$, $\tau_K$, $\tau$) have to be obtained jointly, and two parameters ($\varsigma$, $\vartheta$) are assigned values together with the parameters of the stochastic processes by matching unconditional second moments of the data. Calibration of the three sets of parameters is described in turn.

First, the share of homeowners $\Psi$ is set equal to 2/3. The capital share $\varsigma$ is set equal

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3 Parameters specific to refinancing and mortgage choice are dealt with separately in Sections 5.2 and 5.3, respectively. Because in steady state interest rates are constant and the short rate is equal to the FRM rate, the values of the parameters related to refinancing and mortgage choice do not affect the values of the parameters calibrated from steady state relations described here. (As discussed below, second moments used to calibrate some parameters are found to be broadly invariant to FRM vs. ARM, and thus should not be affected by mortgage choice or refi either.) The additional parameters related to refi and mortgage choice can therefore be chosen separately and the model does not need to be recalibrated once these additional parameters are introduced.

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to 0.283, an estimate from National Income and Product Accounts (NIPA) for aggregate output close to our measure of output (see Appendix C). The share of land in new housing ϕ is set equal to 0.1, an estimate reported by Davis and Heathcote (2005). The depreciation rates δ_K and δ_H are set equal to 0.02225 and 0.01021, respectively, to be consistent with the average flow-stock ratios for capital and housing, respectively. The labor income tax rate is derived from NIPA using a procedure of Mendoza, Razin, and Tesar (1994), yielding τ_N = 23.5%. The role of this parameter is to map the observed debt-servicing costs, which are based on gross income, into net income. The parameter G is measured to be equal to 0.138 (see Appendix C).

We focus on conventional mortgages and therefore set θ equal to 0.6. This is based on the observation that conventional loans make up, on average, 80% of single family newly-built home mortgages (Construction Survey, 1973-2006) and the average cross-sectional mean of their loan-to-value ratio is 0.76 (Freddie Mac’s Monthly Interest Rate Survey, 1973-2006). As in Kydland et al. (2016), the amortization parameters are κ = 0.00162 and α = 0.9946, approximating the amortization schedule of a 30-year mortgage. The weight on inflation in the monetary policy rule ν_π is set equal to a fairly standard value of 1.5. The steady-state inflation rate π is set equal to 0.0113, the average (1972-2006) quarterly inflation rate. In steady state, the first-order condition for t^i restricts i to equal to i. The first-order condition for b_t then relates i and π to β. The above value of π and i^F = 9.31% per annum (the 1972-2006 average for 30-year FRM rate) imply β = 0.9883.

Given the above parameter values, the second set of parameters (ξ, τ_K, τ) is calibrated by matching the observed average K/Y ratio, H/Y ratio, and debt-servicing costs. The relationship between the three parameters and the targets is given by the steady-state versions of the first-order conditions for x_Kt and x_Ht and the expression for steady-state debt-servicing costs noted above (see Appendix A for the first-order conditions). These restrictions yield ξ = 0.5003, τ_K = 0.3362, and τ = 0.4693.

Finally, given the values of the first and second set of parameters, ζ and ϑ are calibrated, together with the parameters of the stochastic processes, by minimizing an equally-weighted distance between the following second moments of the data and their simulated model counterparts: (i) the standard deviations and autocorrelations of the 10-year government bond yield (a longer series used as a proxy for the 30-year mortgage rate) and of the long-short spread (10-year minus 3-month), (ii) the standard deviation and autocorrelation of output, (iii) the standard deviation and autocorrelation of house prices, and (iv) the standard deviations of housing and capital investment. These are 10 targets for 10 parameters. The resulting parameter values are reported in Table 1. Here, instead of going over the values, we briefly discuss how the targets map into the parameters. Targets (i) help pin down the parameters of the inflation target and bond market shocks, as explained in Section 3; targets (ii) help pin down the parameters of the TFP shock; targets (iii) help pin down the parameters of the shock to the marginal rate of transformation; and targets (iv) help pin down the parameter controlling the curvature of PPF and the parameter of the cost function for homeowners’ participation in the bond market.

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4Having government expenditures in the model ensures a realistic expense of the tax revenues, which otherwise would have to go into transfers, thus affecting the distribution of income.

5Both the U.S. and simulated data are expressed as percentage (percentage point, for interest rates) deviations from HP-filter trend.

6Specifically, the bond market cost function controls the extent to which homeowners can indirectly invest in capital, thus affecting the volatility of capital investment. The curvature of PPF controls the volatility of
cyclical properties of the model are similar under FRM and ARM, in line with cross-country data. The parameter values, therefore, are not particularly sensitive to which contract is used to simulate the model to match the moments (the reported values are based on FRM).

**Appendix F: Details of optimal refinancing**

As described in Section 3.1.4., when \( i_t^F \geq R_t \) the homeowner sets \( \bar{\theta}_t = \Delta > 0 \), which is a parameter capturing exogenous reasons to refinance, such as moving home. However, when \( i_t^F < R_t \), the homeowner sets \( \bar{\theta}_t = \tilde{\bar{\theta}}_t \), where \( \tilde{\bar{\theta}}_t \) is chosen optimally, weighting the benefits and costs of refinancing. Here we characterize this decision.

In Section 3.1.4, we described the refi cost function as \( \Gamma(\bar{\theta}_t)'' > 0 \) with a minimum \( \Gamma(\bar{\theta}_t) = 0 \) at \( \tilde{\bar{\theta}}_t = \Delta \). Recall that the cost takes the form of a utility loss \( \chi_t \) in the homeowner’s utility function. To make the cost function operative, assume a quadratic parametric form \( \tilde{\bar{\theta}}_t = \Delta \) \( \tilde{\bar{\theta}}_t \), with \( \bar{\theta}_t \) in the value function

\[ \tilde{\bar{\theta}}_t = \frac{(1-\gamma_t)\tilde{d}_t}{(1+\pi_t)\tilde{d}_{t+1}} \left( -\beta E_t V_{R,t+1} \right) \left( R_t - i_t^F \right). \]  

(F1)

In equation (F1), \( \tilde{d}_t \equiv d_t/p_{t-1} \) and \( V_{R,t} < 0 \) is the derivative of the homeowner’s value function with respect to \( \bar{\theta}_t \), representing the marginal life-time gain of reducing the effective interest rate on outstanding debt by one unit. The derivative of the value function is given recursively by a Benveniste-Scheinkman condition as \( V_{R,t+1} = \frac{\beta (1-\gamma_{t+1}) (1-\gamma_t) \tilde{d}_{t+1}}{(1+\pi_{t+1}) \tilde{d}_{t+2}} E_t V_{R,t+2} \). By refinancing, the homeowner trades off the current marginal refi cost against the expected marginal life-time benefit. The further is \( i_t^F \) below \( R_t \), the larger is the gain from refinancing and thus the larger is the fraction of debt that gets refinanced. The second derivative of the objective function with respect to \( \tilde{\bar{\theta}}_t \) is equal to \( -2\bar{\theta}_t + \frac{(1-\gamma_t)\tilde{d}_t}{(1+\pi_t)\tilde{d}_{t+1}} (i_t^F - R_t) \) \( E_t V_{R,R,t+1} < 0 \), where the inequality follows from \( V_{R,R,t+1} < 0 \), which can be established by taking the derivative of \( V_{R,t+1} \), given above, with respect to \( R_{t+1} \):

\[ V_{R,R,t+1} = \frac{d_{t+1}}{1+\pi_{t+1}} \left( \frac{d_{t+1}}{1+\pi_{t+1}} \right)^2 \beta \left[ \frac{(1-\gamma_{t+1}) (1-\gamma_t) \tilde{d}_{t+1}}{(1+\pi_{t+1}) \tilde{d}_{t+2}} \right]^2 E_t V_{R,R,t+2}. \]

Recursion, \( V_{R,R,t+1} < 0 \) as \( v_{cc,t+1} < 0 \) \( \forall t \), a standard assumption. Condition (F1) thus characterizes a unique maximum with respect to \( \tilde{\bar{\theta}}_t \).

Observe from equation (F1) that as \( R_t \) converges to \( i_t^F \), \( \tilde{\bar{\theta}}_t \) converges to \( \Delta \), when the ratios in equation (F1) are bounded. The numerical solution of the model described in Appendix B yields a stationary state space representation of the equilibrium (i.e., the highest eigenvalue housing investment.

\[ \text{Recall that the law of motion for debt (19) boils down to the law of motion under no refi (11) once we substitute in for } l_t \text{ from equation (18). } \tilde{\bar{\theta}}_t \text{ thus does not affect } d_{t+1}. \]
of the state space system is less than one in absolute value). Thus, after a shock that moves \(i_t^F\) below \(R_t\), and absent any further shocks, the ratios in equation (F1) converge to a constant and the two interest rates converge back to their steady-state value equal to the parameter \(i\). As a result, \(\bar{\varrho}_t\) converges to \(\Delta\). When \(i_t^F = R_t\), there are no gains from refinancing and thus only the costless exogenous refinancing activity, \(\varrho_t = \Delta\), takes place.

The two parameters related to refinancing, \(\Delta\) and \(\upsilon\), are calibrated as follows. The parameter \(\Delta\) is set equal to 0.02, which is the long-run average fraction of outstanding debt that is refinanced per quarter. This fraction is reported by Chen, Michaux, and Roussanov (2013) and corresponds to a long-run share of refi loans in newly originated loans equal to 0.4, which is consistent with observations from the Freddie Mac’s Primary Mortgage Market Survey (1987-2007). The parameter \(\upsilon\) is set equal to 12, so as to match, in equilibrium, the local elasticity of the share of refi loans in new loans with respect to the FRM rate. Specifically, we identify periods of declines in the FRM rate (from a local peak to a local trough) accompanied with increases in refi activity and calculate the average percentage point increase in the refi share per one percentage point decline in the FRM rate during these periods. This calculation gives approximately a 32 percentage point increase in the refi share of new loans for a one percentage point decline in the 30-year FRM rate.\(^8\)

### Appendix G: Details of optimal mortgage choice

Campbell and Cocco (2003) provide a thorough theoretical analysis of mortgage choice at the household level. They show that mortgage choice is sensitive to short-run movements in interest rates (and thus in the long-short spread) when households have limited ability to smooth out fluctuations in real mortgage payments and are thus affected by their timing. Such a friction is present in our model and the timing of real mortgage payments matters, as shown in Section 2.

As in the case of refinancing, the mortgage choice cost function is made operative by assuming a quadratic form, \(\Phi(.) = \omega(l_{2t}/lt - \Lambda)^2\), where \(l_{2t}/lt\) is the fraction of ARM loans in newly originated loans and \(\Lambda\) is a parameter capturing the institutional norm of the ARM share in the economy. To characterize the optimal mortgage choice, it is again convenient to work with the recursive formulation of the homeowner’s problem (25). Observe that \(l_{1t}\) and \(l_{2t}\) appear only in the constraint (22) and the respective laws of motion (11)-(13). New ARM loans, \(l_{2t}\), appear also in the quadratic cost function, where \(l_t = \theta_{pHpH_t}x_{H_t}\).

Let \(\lambda_t\) denote the Lagrange multiplier associated with the constraint (22). Substituting the laws of motion into the value function and taking the first derivatives with respect to \(l_{1t}\) and \(l_{2t}\) and setting them equal to zero yields

\[
-\lambda_t + E_t \left[ \beta \bar{V}_{d1,t+1} + \zeta D_{d1}(\kappa - \gamma_{1t}) \beta V_{1,t+1} + \zeta D_{d1}(i_t^F - R_{1t}) \beta V_{R1,t+1} \right] = 0,
\]

\[
-\frac{\partial}{\partial l_{2t}} \Phi(l_{2t}/lt) - \lambda_t + E_t \left[ \beta \bar{V}_{d2,t+1} + \zeta D_{d2}(\kappa - \gamma_{2t}) \beta V_{2,t+1} + \zeta D_{d2}(i_t^F - R_{2t}) \beta V_{R2,t+1} \right] = 0,
\]

\(^8\)The average refi share in the subsample used for this calculation is close to the average refi share in the whole sample. The elasticity will necessarily be smaller in periods in which the share of refi loans in new loans approaches the upper bound of 100%. The elasticity we estimate is thus “local” in the sense that it holds around the average level of the refi share.
where \( \zeta_{Djt} \equiv \left( \frac{1-\gamma_{jt}}{1+\pi_t} \bar{d}_{jt} \right) / \left( \frac{1-\gamma_{jt}}{1+\pi_t} \bar{d}_{jt} + \bar{l}_{jt} \right)^2 \in (0, 1) \), for \( j = 1, 2 \). Eliminating \( \lambda_t \), using the functional form for \( \Phi'(l_{2t}/l_t) \), and carrying out some algebra yields

\[
l_{2t}/l_t - \Lambda = \frac{v_{ct}P_{H1t}x_{H1t}}{2\omega} (\tau_{H1t} - \tau_{H2t}), \tag{F2}
\]

where \( \tau_{H1t} \) and \( \tau_{H2t} \) are the wedges capturing the price effect, as discussed in Section 2 (for the full model, the wedges are derived in Appendix A, but they are straightforward generalizations of the wedges in the simple model of Section 2). Recall, that the wedges capture the costs of the respective mortgages (FRM vs. ARM) to the homeowner. The optimality condition (F2) thus has an intuitive interpretation. It states that the ARM share in new loans \( l_{2t}/l_t \) increases when the homeowner perceives ARM loans as cheaper than FRM loans; i.e., \( \tau_{H1t} > \tau_{H2t} \).

Observe that if the ratio \( (v_{ct}P_{H1t}x_{H1t})/(2\omega) \) in equation (F2) is bounded, \( l_{2t}/l_t \) converges to \( \Lambda \) as \( \tau_{H1t} \) and \( \tau_{H2t} \) converge to each other. In steady state, \( \tau_{H1t} = \tau_{H2t} = 0 \) and thus there are no benefits of deviating away from the institutional norm \( \Lambda \). The numerical solution, described in Appendix B, produces a stationary state space representation of the equilibrium, with all eigenvalues less than one in absolute value. Thus, after a shock, the above ratio converges to a constant, the wedges converge back to their steady state, and the ARM share \( l_{2t}/l_t \) therefore converges to \( \Lambda \).

The parameter \( \Lambda \) is set equal to 0.3, the long-run average of ARM share in the U.S. economy, and the cost parameter \( \omega \) is set so as to replicate the empirical elasticity of the ARM share to the long-short spread, equal to 7.0 (i.e., a seven percentage point increase in the ARM share in response to a one percentage point increase in the long-short spread). This elasticity implies \( \omega = 0.0055 \).

\[\text{9The second-order condition is difficult to establish analytically. A simple condition such as the one in the refi case is not available. We therefore instead check whether the second-order condition holds for the approximate economy computed numerically. In the approximate economy, the value function has a quadratic form and the associated matrix is negative definite. This establishes that the first-order condition characterizes a unique maximum.}\]

\[\text{10As discussed in the paper, because in the steady state homeowners have the same discount factor as capital owners, } \beta, \text{ both agents price future mortgage payments as equal to one unit of current consumption in present value terms. Thus, } \tau_{H1t} = \tau_{H2t} = 0 \text{ in the steady state.}\]

\[\text{11This value is much smaller than that for the cost parameter in the case of refinancing. This is because the potential gain from refinancing is much larger than the gain from mortgage choice as refinancing applies to the stock, whereas mortgage choice applies only to new loans. The fact that in the data only a small fraction of the stock is refinanced in response to a percentage fall in the mortgage rate implies a relatively large refi cost in the model.}\]
References


