# "CLIMATE CHANGE AND LONG-RUN DISCOUNT RATES: EVIDENCE FROM REAL ESTATE"

## Internet Appendix

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## A.1 Discounting: The Role of Risk and Horizon

How should policymakers decide whether a particular investment in climate change abatement is worth pursuing? A common approach is to conduct a cost-benefit analysis to determine the societal net present value (NPV) of an investment project that is costly *today* and provides a stream of potentially uncertain *future* benefits (cash flows), with positive NPVs indicating socially beneficial projects. Discount rates play a central role in determining NPVs, since even small changes in discount rates can dramatically alter the NPVs of investments with long horizons (see, e.g., Arrow et al., 2013; Dietz, Gollier and Kessler, 2015; Dreze and Stern, 1987; Moyer et al., 2014).

In this section, we review the basic theoretical concepts for our empirical and structural analysis in Section 2. Section A.1.1 describes how the appropriate rate for discounting a particular cash flow depends on both the riskiness and the maturity of that cash flow. Section A.1.2 highlights what this implies for learning about the appropriate discount rates for climate change policies from observable assets that pay cash flows with different riskiness and maturity. The main body of the paper uses insights from the term structure of discount rates for one particular asset, real estate, to guide the choice of appropriate discount rates for investments in climate change abatement.

To introduce our basic notation, let us represent an investment at time t as a claim to a stream of future benefits (cash flows),  $D_{t+k}$ , k=1,2...,n, where n is the final maturity of the cash flows. For example, an investment to avoid one ton of  $CO_2$  emissions today provides benefits in terms of mitigated climate change in each future period for hundreds of years. Each of these benefits,  $D_{t+k}$ , is stochastic and depends on the state of the world at time t+k. For example, the future benefits of reducing  $CO_2$  emissions today could depend on how much the economy grows in the future. We denote the state of the world at time t+k as  $\omega_{t+k} \in \Omega_{t+k}$  and stress the dependence of benefits on its stochastic realization with the notation  $D_{t+k}(\omega_{t+k})$ . The set  $\Omega_{t+k}$  includes all possible states of the world at time t+k, which can differ along many dimensions, including the health of

the aggregate economy and the degree of environmental damage. In what follows, we will sometimes refer to general assets with maturity n that could pay cash flows such as dividends or rents at any point in time up to their maturity; these will simply be referred to with superscript n. A subset of these assets is the set of claims to a single cash flow at a specific point in time, maturity n; we will refer to these with superscript (n).

#### A.1.1 The Value of a Single Cash Flow Investment

We begin our analysis by studying the value of an investment that pays only one cash flow, at a specific point in time: t+n. This cash flow is not predetermined: it might be different in different states of the world,  $\omega_{t+n} \in \Omega_{t+n}$ . We denote the present value of the claim to this benefit as  $P_t^{(n)}$ . A classic tenet of asset pricing is that, under the relatively mild assumptions of no arbitrage and the law of one price,  $P_t^{(n)}$  can be expressed as the weighted expected value of that cash flow across scenarios  $\omega_{t+n}$ , where a benefit paid in each scenario is weighted by the importance investors assign to benefits in that state (see Hansen and Richard (1987), and Cochrane (2005) for a textbook treatment). Let  $M_{t,t+n}(\omega_{t+n}) > 0$  denote the value that investors attach at time t to benefits in state  $\omega_{t+n}$ . An asset is considered more *risky* if it pays off primarily in states of the world in which investors value that payoff less. If investors value benefits paid out earlier more than benefits paid out later, the weighting  $M_{t,t+n}$  will also adjust for this time discounting. We can then write the value of an investment that yields  $D_{t+n}$  as:

$$P_{t}^{(n)} = \sum_{\omega_{t+n} \in \Omega_{t+n}} M_{t,t+n}(\omega_{t+n}) D_{t+n}(\omega_{t+n}) \pi_{t,t+n}(\omega_{t+n}) = E_{t} [M_{t,t+n} D_{t+n}], \quad (A.1)$$

where  $\pi_{t,t+n}(\omega_{t+n})$  is the conditional probability of state  $\omega_{t+n}$ . The object  $M_{t,t+n}$  is called the *stochastic discount factor* (SDF). In economic terms, the SDF reflects the marginal utility of a payoff in different states of the world. The value of the asset thus reflects both the physical properties of the asset (when and how much it pays in each state  $\omega_{t+n}$ ) and the preferences of investors (how much they value payoffs in each scenario  $\omega_{t+n}$ ).

An equivalent representation of  $P_t^{(n)}$ , which is more prevalent in policy analysis, is in terms of *discount rates*. The time and risk adjustments are then expressed using a perperiod discount rate  $\bar{r}_t^n$ :

$$P_t^{(n)} = E_t \left[ M_{t,t+n} D_{t+n} \right] = \frac{E_t \left[ D_{t+n} \right]}{(1 + \overline{r}_t^n)^n}. \tag{A.2}$$

Put differently, we can think of prices as the expected value of the cash flow discounted at

a per-period discount rate  $\bar{r}_t^n$ . The appropriate discount rate will differ across investments depending on which states of the world an investment pays benefits in, and the relative valuation of benefits across states of the world: more risky investments are valued less, and thus discounted at higher per-period discount rates.

#### A.1.2 The Importance of Horizon-Specific Risk Adjustments

We now consider a multi-period-payoff investment project that pays stochastic benefits at different points in time up to maturity n. Any such asset can be thought of as the combination of many single cash flow assets, each paying at specific points in time, t + 1, t + 2, ..., t + n. Therefore, the value of a multi-period-payoff investment project is the sum of the values of the individual single-period-payoff projects:

$$P_t^n = P_t^{(1)} + P_t^{(2)} + \dots + P_t^{(n)}$$

Since the two representations discussed above for the one-period case also apply to the multi-period case, the value  $P_t^n$  can be written as:

$$P_t^n = E_t \left[ M_{t,t+1} D_{t+1} + M_{t,t+2} D_{t+2} + \dots + M_{t,t+n} D_{t+n} \right]$$
 (A.3)

$$= \frac{E_t [D_{t+1}]}{1 + \overline{r}_t^1} + \frac{E_t [D_{t+2}]}{(1 + \overline{r}_t^2)^2} + \dots + \frac{E_t [D_{t+n}]}{(1 + \overline{r}_t^n)^n}. \tag{A.4}$$

These two representations differ from the valuation formula that is often applied in costbenefit analyses, which discounts each cash flow at the *same* per-period discount rate  $\bar{r}_t$ :

$$P_t^n = \frac{E_t [D_{t+1}]}{1 + \overline{r}_t} + \frac{E_t [D_{t+2}]}{(1 + \overline{r}_t)^2} + \dots + \frac{E_t [D_{t+n}]}{(1 + \overline{r}_t)^n}.$$
 (A.5)

Representations A.3 and A.4 are always correct and equivalent; the last one is *only* correct if the discount rate  $\bar{r}_t$  is chosen to match the risk and maturity of a particular asset. Therefore,  $\bar{r}_t$  can only be applied to value the benefits of a project with exactly the same risk characteristics and exactly the same maturity as the asset from which  $\bar{r}_t$  was derived in the first place. For the purpose of discounting the benefits of a project with different characteristics, the full term structure of discount rates  $\bar{r}_t^1, \bar{r}_t^2, ..., \bar{r}_t^n$  needs to be known and appropriately adjusted for differences in risk characteristics. We highlight the importance of this by considering the valuation of three different investment projects below: a project with the same risk and payoff horizon as those of an observed traded asset (whose average per-period discount rate is  $\bar{r}_t$ ); a project with the same risk properties but a different

payoff horizon; and a project with different risk properties but the same payoff horizon.

Case 1: Same Risk, Same Horizon. Consider first an observable asset with maturity n and stochastic cash flows  $D_{t+1}$ ,  $D_{t+2}$ ,...,  $D_{t+n}$  (if the asset has infinite maturity as in the case of the stock market, then  $n = \infty$ ). Imagine we are able to observe the average discount rate of this asset,  $\bar{r}_t$ . Put differently, given an asset with maturity n and some risk profile,  $\bar{r}_t$  is defined as the constant discount rate consistent with the asset's price. Now consider the case in which an investment in climate change abatement pays cash flows  $\tilde{D}_{t+1}$ ,...,  $\tilde{D}_{t+n}$  that are different from the cash flows of the observed asset, but have the same risk characteristics (i.e., the same dependence on the state of the world  $\omega_{t+n}$ ). This is the only case in which cash flows from climate change abatement can be discounted at the *same* average rate as those from the observable asset. The value of the climate change investment,  $C_t^n$ , will be:

$$C_t^n = \frac{E_t \left[ \tilde{D}_{t+1} \right]}{1 + \overline{r}_t} + \frac{E_t \left[ \tilde{D}_{t+2} \right]}{(1 + \overline{r}_t)^2} + \dots + \frac{E_t \left[ \tilde{D}_{t+n} \right]}{(1 + \overline{r}_t)^n}.$$

Case 2: Same Risk, Different Horizon. Since the risk preferences captured by  $M_{t,t+k}$  potentially depend on the horizon, using average discount rates from one asset to discount cash flows from another investment is no longer valid if those cash flows materialize over different horizons. Take our example from above and assume that the asset's cash flows have the same riskiness as the cash flows from the investment in climate change abatement at each horizon. Assume further that the observable asset yields benefits in every period between time t and time t+n, while the investment in climate change abatement only yields benefits after maturity  $\underline{n} > 1$ . Since the riskiness of the cash flows of both investments is the same, one may be tempted to use the observed average discount rate  $\overline{r}_t$  from the observable asset to discount climate change project cash flows. This turns out to be incorrect, however. The correct price is obtained as below:

$$C_t^{(\underline{n},n)} = \frac{E_t \left[ \tilde{D}_{t+\underline{n}} \right]}{(1 + \overline{r}_t^{\underline{n}})^{\underline{n}}} + \frac{E_t \left[ \tilde{D}_{t+\underline{n}+1} \right]}{(1 + \overline{r}_t^{\underline{n}+1})^{\underline{n}+1}} + \dots + \frac{E_t \left[ \tilde{D}_{t+n} \right]}{(1 + \overline{r}_t^n)^n},$$

where each dividend is discounted at the horizon-specific discount rate,  $\bar{r}_t^n, \bar{r}_t^{n+1}, ..., \bar{r}_t^n$ . Since  $\bar{r}_t$  was obtained as the discount rate that applies to the observable asset, it reflects an average of *all* the horizon-specific discount rates  $\bar{r}_t^1, \bar{r}_t^2, ..., \bar{r}_t^n$ , including the ones for maturities up to  $\underline{n} - 1$ . Since the climate change project does not accrue benefits at those horizons, its value should not depend on the discount rates between t + 1 and  $\underline{n} - 1$ .

To see this more clearly, suppose that investors are only worried about the states of the world in which the relatively near cash flows are being paid out (horizons 1 to  $\underline{n}-1$ ), while they are not worried about risks for horizons higher than  $\underline{n}$ : for long maturities, investors only care about the expected payout from the asset, not the state of the world in which it is paid out. They will discount the short-term cash flows at high rates,  $\overline{r}_t^1, \overline{r}_t^2, ..., \overline{r}_t^{n-1}$ , but the longer-maturity cash flows at lower rates,  $\overline{r}_t^n, \overline{r}_t^{n+1}, ..., \overline{r}_t^n$ , reflecting their risk-neutrality at those horizons. The term structure of discount rates for this particular asset is thus downward-sloping. The claim to all cash flows may have a relatively high implied average discount rate, in particular if many of the cash flows accrue before  $\underline{n}$ . At the same time, if the benefits from a climate change investment had the same risk properties, but only accrued after  $\underline{n}$ , the correct present value for such an investment should *only* depend on the low discount rates  $\overline{r}_t^n, \overline{r}_t^{n+1}, ..., \overline{r}_t^n$ . It would thus be higher than under the relatively high *average* discount rate  $\overline{r}_t$ .

Case 3: Different Risk, Same Horizon. Beyond the timing of cash flows, a second potentially important difference between an observed asset's discount rates and those that apply to some investment project is the relative riskiness of the payoffs at the same horizon. As outlined before, riskiness here refers to whether an asset mostly pays in states of the world  $\omega_{t+k}$  where payments are least valuable for the investor. Consider our example from above again. Assume that the asset as well as the climate change investment project only pay a single cash flow in period t+n. Further assume that the observed asset's cash flow is riskier than the investment's cash flow: for example, equities generally pay off in states of the world where the economy is doing well, while investments that mitigate the impact of climate disasters would pay off in states of the world where the economy is not doing well. The discount rate implied by the observable price of the asset will then be different from the appropriate discount rate for the investment project.

For concreteness, assume that there are only two equally likely states of the world – a good one  $(\omega_{t+n}^G)$  and a bad one  $(\omega_{t+n}^B)$ . Assume that marginal utility in the good state of the world is lower than marginal utility in the bad state of the world, and assume that the observed asset pays out in the good state of the world only, while the investment project only pays out in the bad state of the world; both pay out the same amount if they pay out. This implies that  $E_t[M_{t,t+n}D_{t+n}] < E_t[M_{t,t+n}\tilde{D}_{t+n}]$ . It then follows from equation A.2 that the investment project should be discounted at a lower rate than the asset.

## A.2 Estimating the Climate Risk Exposure of Real Estate

In this section, we provide additional information related to our analysis of the climaterisk exposure of real estate in Section 1.1. We first provide additional information on the construction of the "Climate Attention Index" before discussing the hedonic regressions in more detail.

#### A.2.1 Construction of Climate Attention Index

To construct the Climate Attention Index, we conduct a textual analysis of the descriptions of properties in our for-sale listings data. First, we convert every word to lowercase letters before using the *stopwords* function of the *nltk* Python package to remove prepositions, articles, pronouns, and punctuation marks. We flag the listing of a property as "one" if it contains at least one of the climate-related words or bigrams from Table A.1 and as "zero" if none of them is used. More specifically, for single words, we simply check if any one of them matches with the textual description of the listing. For bigrams, we check whether the combination of two words in different orders matches with the description. For instance, for the bigram *sea level*, we check if either *sea level* or *level sea* appears in the sequence of words of the description as *level of sea* will be stripped of the preposition due to the textual analysis function. We also check whether the names of the deadliest and costliest hurricanes since 2000 appear in the description.

Appendix Tables A.2, A.3, A.4, and A.5 list the most common words indicating attention to climate change in each state. In Florida, the most common term is "hurricane", occurring in about 3.3% of all property listings, while in the other states the most common term is "storm." We next present a number of examples of property listings that would be flagged using our algorithm.

<u>Example 1:</u> Diamond in the Rough on water with pier and dock! **Owner holds letter of expemption from FEMA, stating high elevation, flood insurance may not be required,** minutes to area beaches, Close to Jacksonville and Wilmington.

<u>Example 2:</u> Adorable home in Archdale situated on 1.43 acres!! Features include vinyl replacement windows, large bonus room perfect for extra bedroom, den or game room, fenced backyard, large outbuilding and two driveways for extra parking room. Creek is in 500 year flood plain, left side of lot is in 100 year flood plain. House is not in a flood zone to our knowledge. Flood insurance has never been required.

Example 3: SUPERIOR CONSTRUCTION, UPGRADES GALORE & STUNNING BAY VIEWS SET THIS HOME A PART FROM THE OTHERS! You'll have a hard time finding a higher quality constructed home in Destin. In addition, because of its construction and location on high & dry ground (17-20 FT ABOVE SEA LEVEL), IT'S HOMEOWNERS INSURANCE & FLOOD INSURANCE COSTS ARE SOME OF THE LOWEST IN THE AREA! AL-THOUGH FLOOD INSURANCE IS NOT REQUIRED (HOME IS IN ZONE X), THIS HOME IS NOT IN THE COBRA ZONE AND IS ONE OF THE FEW BAY FRONT HOMES IN DESTIN ELIGIBLE FOR \$348 PER YEAR FEDERAL FLOOD INSURANCE. ALL OF THE HOMES IN KELLY PLAN-TATION, REGATTA BAY, EMERALD LAKES, EMERALD BAY AND MOST OF DESTINY ARE IN THE COASTAL BARRIER ZONE AND ARE NOT ELIGIBLE FOR FEDERAL FLOOD INSURANCE. This is a huge benefit because private flood insurance for homes in those neighborhoods can cost between \$8,000 and \$20,000 per year! This home's Insulated Concrete Form (ICF) construction provides superior storm protection, is resistant to mold & termites, can reduce this home's heating & cooling bills up to 50%, & delivers LOW homeowners insurance.

Example 4: Looking for a family home that's ready to move in and only 6 years old? This 4 bedroom 2 1/2 bath plus office/hobby room is in a great neighborhood in the award-winning Carolina Forest school district and is priced to sell! A new home in 2011, it has a private office area away from the upstairs bedrooms, a fireplace, a screened porch, and even a low HOA with a community pool! Loads of storage space, a 2 car garage, an upstairs laundry room, and an open living area with lots of natural light add to the value of this beauty. All the items in the garage convey- such as lawn mower, freezer, safe, hurricane coverings for windows, edger, etc. **Not in a flood zone, it's high and dry!** Close to Coastal University, Carolina Forest, Tanger outlets, and Hwy 31. A quick back route available using Hwy 544 when Hwy 501 is too busy. Only 15 minutes to the beach. **Not in a flood zone.** Great for a family residence, or a good investment for long term rentals. Come and see!

Example 5: Now selling just 6 lots left EMERALD COAST Yacht Club; **FLOOD ZONE X:** This beautiful neighborhood faces West for the most spectacular sunsets; **21 feet above sea level**; This is a rare find in the Panhandle WOW! **No** 

Flood insurance this will save you \$5-\$6000 annually. Underground utilities on site; All permits for dock had been received however some have expired for dock and 14'x30' Boat Slips. All Neighborhood HOA Documentation will convey with sale. Permits and plans attached. Should HOA members decide to proceed with Dock construction, all funding for permit resubmittals and construction must be agreed to and paid for by HOA members.

Importantly, most of these property listings include descriptions that highlight that a specific property is *less* exposed to climate risk. We believe that this is sensible: if you were selling a property with particular exposure to climate risk, for example because it sits in a flood zone, you would not highlight this negative feature in a property listing. However, if you are selling a house that is *not* exposed to climate risk, this is something worth highlighting in a property listing, in particular in areas and at times when potential buyers pay more attention to these risks.

After identifying all listings that suggest particular attention is paid to climate risks, the Climate Attention Index is then constructed as the share of listings with these climate-related texts at the ZIP-code-quarter and ZIP-code-year level. To explore how this Climate Attention Index varies across regions, Figures A.1, A.2, and A.3 show heatmaps that are similar as that in Panel A of Figure 2. As before, the Climate Attention Index is particularly high in those ZIP-codes near the coast line.

#### A.2.2 Coefficients on Control Variables in Hedonic Regression

In our main hedonic regression specification, equation 1, we control for a large number of property characteristics that could affect the value of the property. While the coefficients on these characteristics are not of primary interest for our work, in this section we discuss the relationship between each control variable and transaction prices (Figure A.4) and rental prices (Figure A.5), controlling for the other hedonic characteristics.

We see a consistently increasing positive impact of a larger finished square footage, lot size, and number of bathrooms on the transaction price and rental price. The effect of the number of bedrooms on transaction values and rental values is an inverted U-shape. This is consistent with our empirical understanding of the real estate market in which, at some point, home buyers prefer having a larger common area to having more bedrooms for the same property size: for most households, having six tiny bedrooms and a small living room is less desirable than having four larger bedrooms and a larger living room. We also observe an increasing negative effect of older remodel ages on prices. For property age, we see an initial negative impact as age increases, but after a certain point property age

impacts prices positively. We find it reasonable that people prefer a mid-century house to a house built in the 1990s (especially holding the remodeling age fixed) that is neither new nor old enough for its age to be appealing.

Overall, these relationships are highly consistent with those estimated in the literature (see, e.g., Stroebel, 2016), which highlights the quality of our transaction and property characteristics data.

## A.3 Details on the Riskiness of Housing

#### A.3.1 The Riskiness of Housing – Details on Main Analyses

This section provides the details underlying the analysis carried out in Section 1.3. Section A.3.2 will provide additional evidence for the riskiness of housing.

Table A.7 reports the availability of house price data and the associated financial crises and rare disasters. The first column in Table A.7 shows the time coverage of house price indices for each country. For some countries, we can go far back in time; for example, we sourced data as far back as 1819 for Norway, 1890 for the U.S., and 1840 for France. The second and third columns report the dates of any banking crises or rare consumption disasters that occur in each country over the time period provided in the first column. Banking crises dates for all countries, except Singapore, Belgium, Finland, New Zealand, South Korea, and South Africa, are from Schularick and Taylor (2012). Banking crises dates for the countries not covered by Schularick and Taylor (2012) are from Reinhart and Rogoff (2009). Rare disaster dates in the last column indicate the year of the trough in consumption during a consumption disaster as reported by Barro and Ursua (2008).

For each country, we obtained the longest continuous and high-quality time series of house price data available. To make the data comparable across countries and time periods, we focus on real house prices at an annual frequency. Finally, to increase historical comparability across time within each time series, we report each index for the unit of observation, for instance a city, for which the longest possible high quality time series is available. For example, since a house price index for France is only available since 1936, but a similar index is available for Paris since 1840, we focus on the Paris index for the entire history from 1840-2012. We stress, however, that for each index and country we have carried out an extensive comparison with alternative indices, in particular with indices available for the most recent time period, in order to ensure that we are observing

<sup>&</sup>lt;sup>1</sup>For this second set of countries and dates, we have also consulted Bordo et al. (2001), who confirm all dates in Reinhart and Rogoff (2009), except 1985 for South Korea and 1989 for South Africa.

consistent patterns in the data. In the following, we detail the sources for each of the 20 countries in our sample:

- **Australia**: Real annual house price indices are from Stapledon (2012). For our analysis, we use the arithmetic average of the indices (rebased such that 1880 = 100) for Melbourne and Sydney.
- Belgium, Canada, Denmark, Finland, Germany, Japan, Italy, New Zealand, South Africa, South Korea, and Spain: Real annual house price indices are from the Federal Reserve Bank of Dallas.<sup>2</sup> The sources and methodology are described in Mack and Martínez-García (2011).
- France: Nominal annual house price index and CPI are available from the Conseil Général de l'Environnement et du Développement Durable (CGEDD).<sup>3</sup> We obtain the real house price index by deflating the nominal index using the CPI. For our analysis, we use the longer time series available for the Paris house price index.
- Netherlands: Nominal annual house price index for Amsterdam and CPI for the Netherlands are available from Eichholtz (1997) and Ambrose, Eichholtz, and Lindenthal (2013).<sup>4</sup> We obtain the real house price index by deflating the nominal index using the CPI.
- **Norway**: Nominal annual house price index and CPI are from the Norges Bank.<sup>5</sup> We obtain the real house price index by deflating the nominal index using the CPI.
- **Singapore**: Nominal annual house price index for the whole island is from the Urban Redevelopment Authority (http://www.ura.gov.sg). CPI is from Statistics Singapore. We obtain the real house price index by deflating the nominal index using the CPI.
- **Sweden**: Nominal house price index for one-or-two-dwelling buildings and CPI are from Statistics Sweden. We obtain the real house price index by deflating the nominal index by CPI.

<sup>&</sup>lt;sup>2</sup>The data are available at: http://www.dallasfed.org/institute/houseprice/, last accessed February 2018.

<sup>&</sup>lt;sup>3</sup>http://www.cgedd.developpement-durable.gouv.fr/les-missions-du-cgedd-r206.html, last accessed February 2014.

<sup>&</sup>lt;sup>4</sup>Part of the data are available on Eicholtz' website at: http://www.maastrichtuniversity.nl/web/Main/Sitewide/Content/EichholtzPiet.htm, last accessed February 2014.

<sup>&</sup>lt;sup>5</sup>http://www.norges-bank.no/en/price-stability/historical-monetary-statistics/, last accessed February 2014.

- **Switzerland**: Nominal house price index for Switzerland is available from Constantinescu and Francke (2013). Among the various indices the authors estimate, we focus on the local linear trend (LLT) index. The data are available for the period 1937-2007. We update the index for the period 2007-2012 by using the percentage growth of the house price index for Switzerland available from the Dallas Fed.<sup>6</sup> The CPI index for Switzerland is from the Office fédéral de la statistique (OFS). We obtain the real house price index by deflating the nominal index using the CPI.
- U.K.: Annual nominal house price data are from the Nationwide House Price Index. We divide the nominal index by the U.K. Office of National Statistics "long term indicator of prices of consumer goods and services" to obtain the real house price index. The Nationwide index has a missing value for the year 2005, for that year we impute the value based on the percentage change in value of the house price index produced by the England and Wales Land Registry.
- **U.S.**: Real annual house price data are originally from Shiller (2000). Updated data are available on the author's website.<sup>7</sup>

For all countries, the real annual consumption data are from Barro and Ursua (2008) and available on the authors' website.<sup>8</sup>

Figure 3 is produced by combining the time series of house prices and consumption described in the previous subsection with the dates for banking crises and rare disasters in Table A.7. When taking averages across countries in Panel A of Figure 3 for the 6 year windows around a banking crisis, the following countries have missing observations for the house price series: France data are unavailable for the year 2011 following the 2008 crisis, Netherlands data are unavailable for the years 2010 and 2011 following the 2008 crisis, and South Africa data are unavailable for the year 1974 before the 1977 crises. In these cases, the crises are still included in the sample but the average reported in the figure excludes these missing country-year observations.<sup>9</sup>

## A.3.2 The Riskiness of Housing – Additional Evidence

In this section, we provide additional details and evidence for the riskiness of real estate to complement the analysis in Section 1.3.

<sup>&</sup>lt;sup>6</sup>This source is described in the second bullet point above.

<sup>&</sup>lt;sup>7</sup>Available at: http://aida.wss.yale.edu/~shiller/data.htm, last accessed February 2018.

<sup>&</sup>lt;sup>8</sup>Available at: https://scholar.harvard.edu/barro/data\_sets, last accessed February 2018.

<sup>&</sup>lt;sup>9</sup>In unreported results, we have verified that the result is essentially unchanged if we exclude the 2008 crisis for France and the Netherlands and the 1975 crisis for South Africa from the data.

Figure A.9 plots the growth rates of rents and personal consumption expenditures (PCE) in the U.S. since 1929. In periods of falling PCE, in particular the Great Depression, rents also fell noticeably. The bottom panel shows a (weak) positive relationship between the growth rates of rents and personal consumption expenditures. This suggests that housing rents tend to increase when consumption increases and marginal utility of consumption is low. Figure A.10 indicates that rents in London are positively correlated with house prices in London, but more volatile.

## A.4 Details on Average Returns to Residential Real Estate

This section describes the methodology and data used to compute average real returns and rent growth for residential properties for the price-rent approach and for the balance-sheet approach presented in Section 1.3.1.

#### A.4.1 Details on the Price-Rent Approach

**United States.** For the U.S., we calculate returns between 1953 and 2016. For consistency with the balance-sheet approach, we use Q4-indices. We follow Favilukis, Ludvigson, and Van Nieuwerburgh (2017) and use the house price index from Shiller (2000), which combines data from two sources in its current version: the home purchase component of the U.S. CPI from 1953 to 1975, and the *S&P CoreLogic Case-Shiller Home Price Index* thereafter.<sup>10</sup>

For rent growth, we also follow Favilukis, Ludvigson, and Van Nieuwerburgh (2017) and use the shelter index from the BLS (the component of CPI related to shelter, item *CUUR0000SAH1* from the Federal Reserve Bank of St. Louis). However, for the period up until 1985, we substitute for the BLS shelter index with the adjusted rental index from Crone, Nakamura and Voith (2010). We do not use the BLS shelter index up until 1985 for two reasons: First, as documented by Gordon and vanGoethem (2007) and Crone, Nakamura and Voith (2010), there appears to be a significant downward bias in all rental CPIs published by the BLS up until the mid-1980s, most likely due to a non-response bias for units that were vacant or where tenants changed. Second, while we are interested in housing as an unlevered asset, mortage interest rates directly affected BLS rental indices until 1982 (see, e.g., Reed, 2014). For the same reasons, we use the *CPI for All Urban Consumers* excluding shelter as our inflation measure before 1986 (item *CUUR0000SA0L2*), and including shelter (item *CPIAUCNS*) thereafter. The downside of this substitution is

<sup>10</sup>http://www.econ.yale.edu/~shiller/data.htm, last accessed February 2018.

that the rental index of Crone, Nakamura and Voith (2010) does not include (imputed) owner-occupied rents, but at least since the BLS shelter index methodology has been updated with regard to the treatment of mortgage interest in the mid-80s, the BLS shelter index and the BLS rent index have tracked each other closely.

Unlike Favilukis, Ludvigson, and Van Nieuwerburgh (2017), we choose 2012 as a baseline year for our rent-to-price ratio, which we estimate to be 10%; the choice of the baseline year is motivated by the availability of high-quality data obtained from real estate portal Trulia that allows us to directly estimate rent-to-price ratios for the U.S. Figure A.6 shows the distribution of rent-to-price ratios across the 100 largest MSAs provided by Trulia and Figure A.7 suggests that these rent-to-price ratios are close to their long-run average.<sup>11</sup>

In robustness checks, we use several complementary time series. First, our benchmark rent-to-price ratio from Trulia might include rental properties where some utilities are covered by the monthly rent. Using data from the balance-sheet approach (described in detail in Section A.4.2), the "utilities yield" for water and gas for all residential real estate was 0.6% in 2012. In a robustness check, we therefore reduce our gross rental yield estimate by this amount. Alternatively, we use the 2012 gross rent-to-price ratio implied by our preferred balance sheet approach specification that adjusts for revaluation-implied housing stock growth as discussed in Section A.4.2. This estimate is slightly lower than our Trulia estimate, at 8.6%.

Second, we use the FHFA house price index (formerly OFHEO house price index, item *USSTHPI* from the Federal Reserve Bank of St. Louis) for the period since 1975. The FHFA house price index differs on four main dimensions from the Case-Shiller House Price Index: While the latter is only based on purchase prices, the former also includes refinance appraisals. While the Case-Shiller HPI relies on transaction information obtained from county assessor and recorder offices, the FHFA HPI relies on data from conforming mortgages provided by Fannie Mae and Freddie Mac. Moreover, while the Case-Shiller HPI is value-weighted, the FHFA HPI is equal-weighted. Finally, the FHFA's geograhic coverage includes all U.S. states, while the Case-Shiller HPI does not.<sup>12</sup>

In a third robustness check, we use the BEA price index for personal consumption expenditure on housing (NIPA Table Table 7.4.4. Line 1) in place of the BLS shelter index

 $<sup>^{11}</sup>$ We thank Jed Kolko and Trulia for providing these data. Trulia observes a large set of both for-sale and for-rent listings. The rent-to-price ratio is constructed using an MSA-level hedonic regression of ln(price) on property attributes, ZIP-code fixed effects, and a dummy for whether the unit is for sale or for rent. The rent-to-price ratio is constructed by taking the exponent of the coefficient on this dummy variable.

<sup>12</sup> For additional details, see https://www.fhfa.gov/Media/PublicAffairs/Pages/Housing-Price-Index-Frequently-Asked-Questions.aspx#quest11, last accessed February 2018.

for the years since 1985; for internal consistency, we use the BEA price index for personal consumption expenditure to deflate nominal returns (*NIPA Table 2.3.4. Line 1*) instead of the BLS CPI for this time period in this specification. While the BLS price indices are the most widely known inflation measures, the Federal Reserve states its goal for inflation in terms of the PCE price index. Both measures follow similar trends, but differ along four key dimensions: First, CPI weights are based on a survey of what households are buying, while PCE price index weights are based on surveys of what businesses are selling. Second, the CPI only includes out-of-pocket expenditures, while the PCE price index also includes expenditures indirectly paid for, e.g. insurance payments through employer-provided medical insurance. Finally, the PCE price index reflects substitution between goods when relative prices change, while the CPI does not.<sup>13</sup>

We assume a property tax impact of 0.67% for a representative household for the price-rent approach. Property taxes in the U.S. are levied at the state level and, while there is variation across states, are generally around 1% of house prices. Property taxes, however, are deductible from federal income tax. We assume that the deductibility reflects a marginal U.S. federal income tax rate of 33%. The net impact is therefore (1-0.33)\*0.01=0.67%.

**United Kingdom.** For the U.K., we calculate returns between 1988 and 2016.<sup>14</sup> For consistency with the balance-sheet approach, we use Q4-indices. We use the house price index from the U.K. Land Registry (series *K02000001*) to compute price appreciation. This new house price index has been introduced in 2016 in an effort to provide a "single definitive House Price Index (HPI)" that replaces the previously and separately published house price indices by the Land Registry and the Office of National Statistics (ONS).<sup>15</sup> It addresses a number of limitations of the previous house price indices, namely: It has increased coverage and is therefore more representative of the overall U.K. housing market, it is less sensitive to extreme prices, and it is internally consistent and therefore fully comparable across time.<sup>16</sup>

<sup>&</sup>lt;sup>13</sup>For additional details, see https://www.clevelandfed.org/newsroom-and-events/publications/economic-trends/2014-economic-trends/et-20140417-pce-and-cpi-inflation-whats-the-difference.aspx, last accessed February 2018.

<sup>&</sup>lt;sup>14</sup>The data provided by the ONS would allow us to include 1987 as well, but housing returns were extraordinary high in that year – in fact the inclusion of this one year would increase our return estimates by almost a full percentage point. We therefore decided to drop it from our sample.

<sup>&</sup>lt;sup>15</sup>For more details, see https://www.ons.gov.uk/economy/inflationandpriceindices/methodologies/developmentofasingleofficialhousepriceindex, last accessed February 2018.

<sup>&</sup>lt;sup>16</sup>Overall, growth rates are mostly comparable with the two outdated house price indices from the Land Registry and the ONS. For more details, see https://www.ons.gov.uk/economy/inflationandpriceindices/articles/

To compute rent growth, we combine three rental indices from the ONS: For the years before 1996, we use the *RPI Component Housing Rent* (series *DOBP*). For the years between 1996 and 2005, we use the *CPI Component Actual Rents for Housing* (series *D7CE*). For the years since 2005, the ONS has included owner-occupied housing into its CPI measures and calls these enriched series CPIH. For this period, we combine the *CPIH Component Actual Rents for Housing* (series *L536*) with the *CPIH Component Owner Occupiers' Costs for Housing* (series *L5P5*) following the methodology outlined by the ONS in its *Consumer Price Indices Technical Manual* ONS (2014). In particular, we calculate weighted arithmetic means using the relevant COICOP weights for the respective current year (series *L5E5*, *L5PA*) at the monthly level, and average across quarters to get to quarterly indices. <sup>17</sup> We use the *CPI for All Items* for the period before 2005 (series *D7BT*) and the *CPIH for All Items* for the period since 2005 (series *L522*) to adjust for inflation.

For the baseline rent-to-price ratio, we rely on estimates for matched properties that are both sold and rented out within six months in London from Bracke (2015), who finds a median rent-to-price ratio of 5% between 2006 and 2012. In our setting, this translates into a rent-to-price ratio of 5.2% in 2012. Since properties in city centers tend to have lower rent-to-price ratios on average, we consider this a conservative estimate for the average U.K. housing stock. Nevertheless, it is close to the rent-to-price ratio of 5.3% that we estimate using the balance-sheet approach for 2012. Since U.K. rents typically do not cover any utilities (see Bracke, 2015), we do not correct gross rent-to-price ratios for utilities.

**Singapore.** For Singapore, we calculate returns between 1990 and 2016. For consistency with the U.S. and the U.K., we use Q4-indices. We obtain time series of price and rental indices for the whole island from the Urban Redevelopment Authority (the government's official housing arm: ura.gov.sg). Both series are published by the Department of Statistics Singapore (series *M212261* and *M212311*). We obtain the CPI from the same source (series *M212191*). To estimate the baseline rent-to-price ratio, we use data from for-sale and forrent listings provided by iProperty.com, Asia's largest online property listing portal in 2012. We observe approximately 105,000 unique listings from 2012, about 46% of which are for-rent listings. To estimate the rent-to-price ratio, we run the following regression, which pools both types of listings. The methodology is similar to the one used to construct

explainingtheimpactofthenewukhousepriceindex/may2016, last accessed February 2018.

<sup>&</sup>lt;sup>17</sup>COICOP stands for "Classification of Individual Consumption according to Purpose"; it is a reference published by the U.N. Statistics Division and followed by the ONS that divides individual consumption expenditures into standardized divisions and groups. *Housing, Water, Electricity, Gas and Other Fuels* is Division (04), and *Actual Rentals for Housing* and *Imputed Rentals for Housing* are groups (04.1) and (04.2).

rent-to-price ratios for the U.S. in Figure A.6:

$$\ln\left(ListingPrice\right)_{i,t} = \alpha + \beta_i ForRent_i + \gamma Controls_{i,t} + \epsilon_{i,t}$$
(A.6)

The dependent variable, *ListingPrice*, is equal to the list-price in "for-sale" listings, and equal to the annual rent in "for-rent" listings. *ForRent*<sub>i</sub> is an indicator variable that is equal to one if the listing is a for-rent listing. The results are reported in Table A.6. In column 1, we control for postal code by quarter fixed effects. The estimated coefficient on  $\beta_i$  suggests a rent-to-price ratio of  $e^{\beta_i} = 4.5\%$ . In columns 2–4, we also control for other characteristics of the property, such as the property type, the number of bedrooms, the number of bathrooms as well as the size, age, and floor of the building. In columns 3 and 4, we tighten fixed effects to the month by postal code level and the month by postal code by number of bedrooms level respectively. In all specifications, the estimated rent-to-price ratio for 2012 is 4.4% or 4.5%.

We calibrate the property tax impact to be 0.6%. Before 2003, Singapore levied a 10% annual tax on the estimated rental income of the property. A lower tax rate of 4% applied to owner-occupied properties. Starting in 2011 for owner-occupiers and in 2014 for landlords, Singapore has introduced increasingly progressive tax schemes that start at 0% and cap out at 16% for owner-occupiers, and start at 10% and cap out at 20% for landlords since 2015. Even though homeownership rates are around 90% during our sample period (numbers based on series M810401 -  $Resident\ Households\ By\ Tenancy$  as published by Statistics Singapore on its website http://www.singstat.gov.sg), we use the more conservative (higher) rate of 10% for rental properties as it has prevailed for most of our sample period. The tax impact on returns is the tax rate times the average rent-price ratio, estimated at around 6%. Hence,  $\tau = 0.1*0.06 = 0.6\%$ . <sup>18</sup>

## A.4.2 Details on the Balance-Sheet Approach

**United States.** For the U.S., we calculate returns between 1953 and 2016. We focus on owner-occupied housing and tenant-occupied housing in the nonfinancial noncorporate sector, the most representative sectors of the U.S. housing market (both sectors accounted for more than 90% of the value of residential housing on average during our sample period, and for roughly 95% towards the end of our sample period). Our data for the U.S. come from two main sources, the *Financial Accounts of the United States (FAUS)* for housing

 $<sup>^{18}</sup> For\ details,\ see\ https://www.iras.gov.sg/irashome/Property/Property-owners/Working-out-your-taxes/Property-Tax-Rates-and-Sample-Calculations/, last accessed February 2018.$ 

wealth (published by the Federal Reserve Board, FRB), and the *National Income and Product Accounts (NIPA)* for rents (published by the Bureau of Economic Analysis, BEA). In total, we calculate six objects: (1) the value of the housing stock, (2) net rents, (3) depreciation, (4) maintenance and other intermediate inputs, (5) taxes, and (6) varying measures of the physical housing stock. We add (3) and (4) to get a measure of depreciation gross of maintenance. For each object, we describe in detail below how we perform the sectoral match between NIPAs and FAUS to ensure that our rental yields are internally consistent. The details are as follows:<sup>19</sup>

- Value of the Housing Stock: For the value of the housing stock, we use data obtained from the FAUS. In particular, we sum *Owner-Occupied Real Estate at Market Value* (FL155035013) and *Residential Tenant-Occupied Real Estate at Market Value* in the *Nonfinancial Noncorporate Business* sector (FL115035023).
- Net Rents: To calculate net rents, we start from Mayerhauser and Reinsdorf (2006), and sum *Rental Income of Persons with Capital Consumption Adjustments* of owner-occupied housing (NIPA Table 7.12. Line 164) less mobile homes (NIPA Table 7.9. Line 12) and of tenant-occupied housing in the nonfinancial noncorporate sector with *Proprietor's Income with Inventory Valuation and Capital Consumption Adjustments* (NIPA Table 7.4.5. Line 20),<sup>20</sup> where "with capital consumption adjustment" means after depreciation. For rental income in the tenant-occupied nonfinancial noncorporate sector, we subtract rental income of persons with capital consumption adjustments for owner-occupiers and for nonprofits (NIPA Table 7.9. Line 14) from all rental income of persons with capital consumption adjustments in the housing sector (NIPA Table 7.4.5. Line 21).

Since we are interested in housing as an unlevered asset, we also add back mortgage interest. For owner-occupied housing, we follow Piketty and Zucman (2014) and use *Monetary Interest Paid* (*NIPA Table 7.11. Line 16*).<sup>21</sup> For tenant-occupied nonfinancial noncorporate housing, we compute monetary interest paid in two steps. First,

<sup>&</sup>lt;sup>19</sup>Some time series that are used in a supportive function to derive key objects have missing data points for the first few years in our sample. In those cases, we extrapolate back using a decadal trailing average of yearly growth rates. All results are robust to setting these values to zero instead.

<sup>&</sup>lt;sup>20</sup>(Nonfarm) proprietors are unincorporated (nonfarm) businesses that are included in the nonfinancial noncorporate sector in the FAUS (for details, see Bond et al. (2007) and Bureau of Economic Analysis (2017), Chapter 11).

<sup>&</sup>lt;sup>21</sup>We effectively assume that the share of mobile homes corresponds to its share in owner-occupied structure values based on FAUS data (*FL155012013* for mobile homes and *FL155012665* for all other owner-occupied structures; the FAUS only publish structure values for mobile homes), and reduce the resulting series accordingly. Mobile homes accounted for less than 3.5% of owner-occupied structure values over our sample period and all results are robust to including mobile homes in our capital stock.

note that the NIPA table for the housing sector lists *Net Interest* (*NIPA Table 7.4.5. Line 18*) instead of monetary interest paid.<sup>22</sup> We calculate net interest for tenant-occupied housing as the difference between all net interest in the housing sector (*NIPA Table 7.4.5. Line 18*) and net interest paid by owner-occupiers (*NIPA Table 7.12. Line 160*). Second, to calculate monetary interest paid by tenant-occupied housing, we assume that the percent difference between total interest and net interest is the same for owner-occupied and tenant-occupied housing.<sup>23</sup> To infer the share of nonfinancial noncorporate tenant-occupied housing among all tenant-occupied housing, we use its share in mortgages on tenant-occupied housing using data from the FAUS (*FL113165105*, *FL113165405*, *FL153165105*, *FL893065105*, *FL893065405*).

Finally, to arrive at net operating surplus, i.e. our measure of net rents, we add back *Current Transfer Payments*, which mainly consist of insurance settlements (see Mayerhauser and Reinsdorf, 2006).<sup>24</sup> For owner-occupiers, we use *NIPA Table 7.12*. *Line 163.*<sup>25</sup> For nonfinancial noncorporate tenant-occupied housing, we calculate current transfer payments to all tenant-occupied housing as the difference between current transfer payments to all housing (*NIPA Table 7.4.5*. *Line 19*) and current transfer payments to owner-occupied housing, and infer the share of nonfinancial noncorporate tenant-occupied housing among all tenant-occupied housing based on its share in tenant-occupied housing wealth using data from the FAUS again (*FL105035023*, *FL115035023*, *FL1165035023*).<sup>26</sup>

<sup>&</sup>lt;sup>22</sup>The difference between monetary interest paid and net interest is imputed interest. In the housing sector, imputed interest essentially stems from mortgage borrowing and property insurance: Homeowners and landlords that have financed their homes with mortgages are consuming financial intermediation services. These are called "Financial Services Furnished Without Payments" in the NIPAs. They are treated as intermediate inputs (i.e., maintenance and other intermediate inputs) instead of interest, and are typically imputed as the margin between mortgage interest rates and a reference rate at which the lender refinances itself. In a similar spirit, insurance premiums paid by homeowners and landlords are often supplemented through interest earned as insurers invest these premiums, called "Premium Supplements for Property and Casualty Insurance", which is treated as earned interest in the NIPAs.

<sup>&</sup>lt;sup>23</sup>The percent difference of total interest and net interest was 7% for owner-occupiers during our sample period.

<sup>&</sup>lt;sup>24</sup>Since our measure of maintenance and other intermediate inputs includes incurance payments, we symmetrically include insurance settlements as a benefit accruing to the homeowner. It is by far the smallest of the above items and all results are robust to its removal.

<sup>&</sup>lt;sup>25</sup>As before, we effectively assume that the share of mobile homes corresponds to its share in owner-occupied structure values and reduce the resulting series accordingly.

<sup>&</sup>lt;sup>26</sup>Note that while NIPA housing flows include the government sector, the FAUS' residential wealth measures do not. To correct for this, we use data from the BEA *Fixed Assets Accounts (FAA)* to scale up FAUS values for the non-profit sector using the ratio of non-profit to government sector values in the FAAs. For example, for the value of real estate, we use the *Current-Cost Net Stock of Residential Fixed Assets (FAA Table 5.1, Lines 6 & 8)*, and for depreciation we use *Current-Cost Depreciation of Residential Fixed Assets (FAA Table 5.4, Lines 6 & 8)*. These allocations affect our measures of rents only marginally.

- **Depreciation**: To calculate depreciation, we rely on data for the consumption of fixed capital from the FAUS. For owner-occupiers, we use *FU156320063*, and for nonfinancial noncorporate tenant-occupied housing, we use *FU116320065*.
- Maintenance and Other Intermediate Inputs: To calculate maintenance and other intermediate inputs, we start from Mayerhauser and Reinsdorf (2006) as before. For owner-occupiers, we use Intermediate Goods and Services Consumed (NIPA Table 7.12. Line 155). We infer the share of mobile homes among all owner-occupied housing by assuming that the ratio of depreciation plus maintenance and other intermediate inputs over housing wealth is constant across owner-occupied housing sectors, and reduce the series by that share. For nonfinancial noncorporate tenantoccupied housing, we proceed in two steps. First, we calculate intermediate goods and services consumed by all tenant-occupied housing as the difference between intermediate goods and services consumed by all housing (NIPA Table 7.4.5. Line 6) and intermediate goods and services consumed by owner-occupied housing. We then infer the share of nonfinancial noncorporate tenant-occupied housing among all tenant-occupied housing by assuming that the ratio of depreciation plus maintenance and other intermediate inputs over housing wealth is constant across tenantoccupied housing sectors. To do so, we calculate tenant-occupied housing wealth as described above and tenant-occupied consumption of fixed capital using data from the FAUS (FU106320065, FU116320065, FU166320063). Second, we add the cost for compensation of employees to arrive at maintenance and other intermediate inputs for nonfinancial noncorporate tenant-occupied housing. We start by assuming that all compensation of employees (NIPA Table 7.4.5. Line 14) is paid by tenant-occupied housing, and allocate the share of nonfinancial noncorporate tenant-occupied housing among all tenant-occupied housing based on its share in tenant-occupied housing wealth again. Finally, we remove the imputed interest that we added to net rents above from both, owner-occupied and nonfinancial noncorporate tenant-occupied maintenance and other intermediate inputs.
- Taxes: We calculate net taxes of owner-occupiers as *Taxes on Production and Imports* (*NIPA Table 7.12. Line 158*) minus *Subsidies* (*NIPA Table 7.12. Line 159*).<sup>27</sup> For nonfinancial noncorporate tenant-occupied housing, we conservatively assume that the remainder of all housing-related taxes is paid by tenant-occupied for-profit sectors, and calculate these taxes as the difference between all taxes on production and

<sup>&</sup>lt;sup>27</sup>As before, we effectively assume that the share of mobile homes corresponds to its share in owner-occupied structure values and reduce the resulting series accordingly.

imports to housing (NIPA Table 7.4.5. Line 15), and taxes on production and imports to owner-occupied housing. To calculate net taxes, we assume that the ratio of taxes to subsidies is constant across owner-occupied and for-profit tenant-occupied sectors (the implied assumption is that a large share of subsidies accrues to the non-profit sector). Finally, we infer net taxes for nonfinancial noncorporate tenant-occupied housing based on its share in for-profit tenant-occupied housing wealth based on data from the FAUS again.

• Housing Stock: We adjust for the growth in the housing stock in various ways and rely on a variety of data sources: *Population* estimates are based on U.S. Census data and sourced from the Federal Reserve Bank of St. Louis (item *POP*). *Housing Unit* estimates are based on Moura, Smith and Belzer (2015) for the years before 2010 and on the Census Housing Vacancy Survey Supplement of the Current Population Survey (CPS/HVS) otherwise. *Floor Space* estimates are inferred from Moura, Smith and Belzer (2015).<sup>28</sup> *Holding Period Gains* are taken from the FAUS Revaluation Accounts. We use *FR155035013* for owner-occupiers and *FR115035023* nonfinancial noncorporate tenant-occupied housing.<sup>29</sup> *Quantity indices* are taken from Davis and Heathcote (2007). To be consistent with our price-rent approach, we use the quantity indices derived from the Case-Shiller-Weiss price index for the period after 1975.<sup>30</sup>

We use the BEA price index for personal consumption expenditure to deflate nominal returns (*NIPA Table 2.3.4. Line 1*) to be consistent with our housing consumption source data. Consistent with our yearly flow and stock data, all price and quantity indices are Q4-indices.

**United Kingdom.** For the U.K., we calculate returns between 1988 and 2016.<sup>31</sup> We focus on the *Household Sector (S.14)*, the most representative sector of the U.K. housing market (it accounted for close to 90% of residential housing on average during our sample period). Since the U.K. National Accounts are based on the *System of National Accounts (SNA)*, the household sector includes activities associated with tenant-occupied housing (these are

<sup>&</sup>lt;sup>28</sup>We extrapolate using a decadal trailing average of yearly growth rates for the years after 2011.

<sup>&</sup>lt;sup>29</sup>We thank Eric Nielsen from the FRB for clarifying details of the Revaluation Accounts for us.

<sup>&</sup>lt;sup>30</sup>Quantity indices are a widely used concept in national accounts and aim to capture changes in the value of an asset that are not driven by (constant-quality) price changes. Quality changes are treated as changes in quantity in such a decomposition. See Bureau of Economic Analysis (2017) for more details.

<sup>&</sup>lt;sup>31</sup>The data provided by the ONS would allow us to include 1987 as well, but housing returns were extraordinarily high in that year – in fact the inclusion of this one year would increase our return estimates by almost a full percentage point. We therefore decided to drop it from the sample. As we can see from Figure A.8, our net return series for the U.K. starts around its long-run mean in 1988.

included in the nonfinancial noncorporate business sector in the U.S.).<sup>32</sup> Moreover, since all our data (except some measures of the physical housing stock) are based on the U.K. National Accounts as published by the Office of National Statistics (ONS), ensuring a sectoral match between housing wealth and rental flows is more straightforward for the U.K. than for the U.S. However, for the period before 1995, the ONS does not provide separate statistics for the household sector, but combined statistics for the *Household & Nonprofit Institutions Serving Households (HH & NPISH) Sector (S.14 & S.15)*, which we use to extrapolate levels from the household sector backwards (between 1995 and 2015, the household sector accounted for around 98% of housing wealth in the combined sector).<sup>33</sup> Overall, we calculate five objects: (1) the value of the housing stock, (2) net rents, (3) depreciation, (4) maintenance and other intermediate inputs, and (5) varying measures of the physical housing stock. We add (3) and (4) to get a measure of depreciation gross of maintenance. Since there is no property tax in the U.K., we set taxes to zero. The details are as follows:

- Value of the Housing Stock: For the period since 1995, we add the value of residential structures, (*Dwellings*, *E46V*) and the value of residential land, (*Land*, *E44N*), to calculate the total value of residential housing in the household sector. For the period before 1995, the ONS only reported a combined value of structures and land for residential housing for the HH & NPISH sector (series *CGRI*), which we use to extrapolate levels backwards.
- **Net Rents**: To calculate net rents, we follow Piketty and Zucman (2014) and start from *Gross Operating Surplus* in the household sector (series *HABM*).<sup>34</sup> Note that gross operating surplus includes net interest, so we do not need to add it back as we do for the U.S. However, following Piketty and Zucman (2014) again, to fully correct our measure of net rents for mortgage-related interest payments, we need to add back imputed interest, called *Financial Intermediation Services Indirectly Measured (FISIM)* in the U.K. National Accounts.<sup>35</sup> But, unlike Piketty and Zucman

<sup>&</sup>lt;sup>32</sup>See Bond et al. (2007) for details.

<sup>&</sup>lt;sup>33</sup>The ONS started to report values for the household sector separately with the 2017 edition of the Blue Book, following guidelines of the European System of Accounts 2010 (ESA 2010); most time series we are interested in were updated back to 1995 only.

<sup>&</sup>lt;sup>34</sup>Gross operating surplus in the household sector is essentially gross rents minus intermediate consumption and payment of employees. Indeed, the depreciation we record for housing in the household sector is higher than the depreciation allocated to gross operating surplus in the household sector in the U.K. National Accounts. If anything, this suggests that we may miss some of the surplus generated by housing in the household sector.

<sup>&</sup>lt;sup>35</sup>See our discussion for the U.S. on imputed interest, i.e., the difference between net interest and monetary interest paid.

(2014), we take a more conservative approach and consider that secured debt should command lower interest rates than unsecured debt. Therefore, instead of calculating the share of financial liabilities secured on dwellings amongst all financial liabilities in the household sector and allocating FISIM back proportionally, we use data on household-mortgage-related FISIM published by the ONS on its website for the years since 2005. Between 2005 and 2016, the difference between FISIM markups on loans secured on dwellings and all other household debt was fairly constant at 3.6 percentage points on average. We assume that this difference also holds for the years before 2005 and calculate household-mortgage-related FISIM using data on all household-debt-related FISIM (series *CRNB*), all household debt (series *NIWI*), and household loans secured on dwellings (series *NIWV*) accordingly. Finally, we subtract depreciation as calculated below to arrive at net rents.

- **Depreciation**: To calculate depreciation rates, we use data on the *Consumption of Fixed Capital* (series *MJX9*) in the household sector. For the period before 1995, we combine the corresponding series for the consumption of fixed capital (series *CIHB*) with housing wealth (series *CGRI*) in the HH & NPISH sector. For the time period where both series overlap (1995 to 2009), depreciation was around 0.5 percentage points higher in the updated data series. Therefore, we conservatively increase our depreciation estimates for the years before 1995 by 0.5 percentage points each year.
- Maintenance and Other Intermediate Inputs: To calculate *Maintenance and Other Running Costs*, we subtract net rents, depreciation, and net taxes from total personal consumption expenditure in the household sector. We proceed in two steps to calculate personal consumption expenditure on housing in the household sector: First, we calculate total personal consumption expenditure on housing across all

<sup>&</sup>lt;sup>36</sup>We use the non-risk-adjusted FISIM allocated to households as owners of dwellings underlying Figure 19 of the article "Financial intermediation services indirectly measured (FISIM) in the UK revisited", retrieved from <a href="https://www.ons.gov.uk/economy/grossdomesticproductgdp/articles/financialintermediationservicesindirectlymeasuredfisimintheukrevisited/2017-04-24">https://www.ons.gov.uk/economy/grossdomesticproductgdp/articles/financialintermediationservicesindirectlymeasuredfisimintheukrevisited/2017-04-24</a>, last accessed February 2018.

<sup>&</sup>lt;sup>37</sup>We calculate FISIM markups on loans secured on dwellings as household-mortgage-related FISIM divided by loans secured on dwellings. FISIM markups on all other household debt are calculated as the difference between all household-debt-related FISIM and household-mortgage-related FISIM, divided by the difference between all household debt and loans secured on dwellings. While fairly constant overall, the difference between household-mortgage-related FISIM markups and other household-debt FISIM markups tends to vary somewhat with overall FISIM markup levels between 2005 and 2016 (at an average markup of 3.6 percentage points, the standard deviation was 0.5 percentage points). However, since the average overall FISIM markup before 2005 is lower than the average overall FISIM markup since 2005 (if slightly at 1.4 vs. 1.9 percentage points) and much less volatile (with a standard deviation of 0.2 vs. 0.7 percentage points), our estimate for household-mortgage-related FISIM before 2005 should be slightly conservative if anything.

sectors as the sum of *Actual Rentals for Housing* (series *ADFT*), *Imputed Rentals for Housing* (series *ADFU*), and *Maintenance and Repair of the Dwelling* (series *ADFV*). Second, we allocate this total across all sectors based on the fraction of housing wealth in the household sector vs. all remaining sectors.<sup>38</sup>

• **Housing Stock**: We adjust for the growth in the housing stock in various ways and rely on a variety of data sources again: *Population* estimates are retrieved from the ONS (series *UKPOP*). *Housing Unit* estimates are based on dwelling stock data from the DCLG for the U.K. until 2013 (*Table 101*) and for England thereafter (*Table 104*). *Quantity Indexes* are derived following the baseline methodology outlined in Davis and Heathcote (2007); that is, we discount the value of the housing stock with a (constant-quality) house price index.<sup>39</sup>

To be consistent with our housing consumption source data and our approach for the U.S., we use the ONS price index for personal consumption expenditure to deflate nominal returns (series *CRXB*). Consistent with our yearly flow and stock data, all price and quantity indices are Q4-indices.

#### A.4.3 Consistency Across Rent-Price and Balance-Sheet Approaches

Figure A.8 plots the net housing returns for the balance-sheet and the price-rent approach for the U.S. and the U.K. (top row), the correlation between net housing returns from the balance-sheet and the price-rent approach for the U.S. and the U.K. (middle row), and housing depreciation (gross of maintenance) and tax yields from the balance-sheet approach for the U.S. and the U.K. (bottom row; there are no taxes in the U.K.). The U.S. results are based on specifications 2 and 9 in Table 4. The U.K. results are based on specifications 11 and 15 in the same table. We can see that both approaches yield similar net return time series that are highly correlated. Moreover, depreciation and taxes as derived from the balance-sheet approach have been fairly stable and trending around their long-run averages over our sample periods, which verifies our constant-adjustment approach for depreciation and taxes for the price-rent approach.

<sup>&</sup>lt;sup>38</sup>We think that this is a conservative assumption, since rental yields tend to be higher in the household sector than in other sectors. Note however that this assumption has no impact on our results for net rents as we derive these directly from gross operating surplus.

<sup>&</sup>lt;sup>39</sup>See our discussion of data sources for the price-rent approach for details on the U.K. house price index.

# A.5 Price and Quantity of Risk Across the Term Structure

Section 1 showed empirically that the term structure of discount rates for real estate – a risky asset – is steeply downward-sloping. In this section, we apply asset pricing theory to discuss a decomposition of this term structure into its building blocks: risk and return across the term structure. This decomposition will help us understand the forces that drive discount rates at different horizons, and provides a link between discount rates observed on tradable assets and investments in climate change abatement.

#### A.5.1 Per-Period Discount Rates and Expected One-Period Returns

Most of the insights of asset pricing theory are clearest to interpret when thinking about one-period expected returns, independent of the maturity of the asset. Since any asset can be bought, held for one period, and then sold at the end of that period (before its maturity), looking at the one-period return is one way to reduce assets of different maturities to a common horizon. This allows us to compare their risk and return properties.

We start by introducing our main notation and by linking together the concepts of returns to maturity and one-period returns. In what follows, we will sometimes refer to general assets with maturity n that could pay cash flows such as dividends or rents at any point in time up to maturity; these will simply be denoted with superscript n. A subset of these assets is the set of claims to single cash flows at a specific point in time, maturity n; we will denote these with superscript (n).

Define the one-period (gross) return per dollar spent on any security with maturity n as the total amount obtained from buying the security and liquidating it after one period:

$$R_{t,t+1}^n \equiv \frac{P_{t+1}^{n-1} + D_{t+1}}{P_t^n},$$

Note that at the time the asset is sold, its maturity has shortened to n-1. The return to holding the security over multiple periods (and reinvesting all intermediate cash flows) can be found by compounding the one-period returns. For example, the return to maturity of any investment with maturity n is

$$R_{t,t+n}^n = \prod_{k=0}^{n-1} R_{t+k,t+1+k}^{n-k}.$$

Of particular interest to us are the one-period returns and discount rates for claims to a single cash flow  $D_{t+n}$ . In this case, the one-period returns in all but the last period are

entirely driven by price movements (since  $D_{t+k}$  is zero for all k, except for k=n, the last cash flow at maturity). What makes the return to this security of particular interest is its intimate link to our per-period discount rates for horizon-specific cash flows,  $\bar{r}_t^n$ . To see this, we can rewrite the return to maturity of such a claim as:

$$R_{t,t+n}^{(n)} = \frac{D_{t+n}}{P_t^{(n)}};$$

we can do this since there are no dividends to be reinvested over the life of this security. Taking expectations on both sides, and then rearranging, we obtain:

$$P_t^{(n)} = \frac{E_t[D_{t+n}]}{E_t[R_{t+n}^{(n)}]}.$$

Comparing this equation with equation A.2 in Section A.1, we immediately see that the n-period expected return to maturity of a claim to a single dividend at t + n is exactly the compounded discount rate to be applied to that security:  $E_t[R_{t,t+n}^{(n)}] = (1 + \overline{r}_t^n)^n$ .

Next, we want to link these quantities to the one-period expected return, for which we are able to provide a very intuitive risk-return decomposition. The focus of this paper is on the *average* shape of the term structure of discount rates. Time-variation in discount rates, while important in the asset pricing literature, plays a second-order role in thinking about climate change investments. We therefore derive the link between one-period expected returns  $E_t[R_{t,t+1}^{(n)}]$  and per-period discount rates  $\bar{r}_t^n$  under the assumption that per-period discount rates for a cash flow with a particular maturity are constant over time; relaxing this assumption would complicate the intuition without adding any economically relevant elements to the analysis. If expected returns are constant over time (though they may be different across maturities, such that the term structure of discount rates is not necessarily flat at each point in time), we have

$$E_t[R_{t,t+n}^{(n)}] = \prod_{k=0}^{n-1} E_t[R_{t,t+1}^{(n-k)}],$$

where all the returns are for claims to single cash flows,  $D_{t+k}$ , at different horizons k. The formula shows that in this case, not only the realized but also the expected returns are linked through compounding. Since  $E_t[R_{t,t+n}^{(n)}]$  is directly linked to  $\overline{r}_t^n$  as shown above, we

can then easily substitute and take logs (and recall that  $ln(1 + x) \simeq x$ ), to obtain:

$$\bar{r}_t^n \simeq \frac{1}{n} \sum_{k=1}^n \ln(E_t[R_{t,t+1}^{(k)}]).$$
(A.7)

Therefore, the discount rate for a particular horizon n is simply the average of the one-period expected returns for claims to cash flows at each horizon. A flat term structure of discount rates must then imply a flat term structure of expected one-period returns across maturities; in fact, expected one-period returns and discount rates across maturities would all be equal. In Section A.5.2, we will build on this decomposition to further elaborate on the forces that shape the term structure of discount rates.

#### A.5.2 Decomposing the Term Structure of Expected One-Period Returns

Now that we have clarified the link between one-period returns and per-period discount rates, we focus on the one-period returns of securities with maturity n. We start by using the fundamental asset pricing equation introduced in Section A.1.1 to decompose the expected one-period returns  $R_{t,t+1}^{(n)}$  into a component that reflects time discounting and a component that reflects the riskiness of the underlying cash flow. It follows from:<sup>40</sup>

$$1 = E_t[M_{t,t+1}R_{t,t+1}^{(n)}]$$

that:

$$E_t[R_{t,t+1}^{(n)}] = R_{t,t+1}^f - Cov_t[R_{t,t+1}^{(n)}, M_{t,t+1}]R_{t,t+1}^f,$$

where the first component  $R_{t,t+1}^f = E_t[M_{t,t+1}]^{-1}$  is the one-period risk-free rate that reflects time discounting, and the second component reflects an additional discount compensating the investor for bearing risk (the covariance with the SDF reflects whether this asset primarily pays off in good states of the world that have a low marginal utility of consumption). The risk premium has the opposite sign of the covariance between the stochastic discount factor (SDF) and the one-period return,  $Cov_t[M_{t,t+1}, R_{t,t+1}^{(n)}]$ . This reflects the fact that a claim with a higher return in states of the world in which extra resources are less valuable (i.e., when marginal utility  $M_{t,t+1}$  is low) is less valuable to the investors, and thus has a positive risk premium. Finally, to highlight the fact that only *innovations* in the SDF matter for the purpose of understanding risk premia (rather than

The fundamental asset pricing equation introduced in Section A.1.1 (equation A.1) can be restated as  $P_t^{(n)} = E_t \left[ M_{t,t+1} P_{t+1}^{(n-1)} \right]$ , which implies  $1 = E_t \left[ M_{t,t+1} \frac{P_{t+1}^{(n-1)}}{P_t^{(n)}} \right] = E_t \left[ M_{t,t+1} R_{t,t+1}^{(n)} \right]$ .

its mean, which instead pins down the risk-free rate), we can rewrite excess returns as:

$$E_t[R_{t,t+1}^{(n)}] - R_{t,t+1}^f = -Cov_t[R_{t,t+1}^{(n)}, M_{t,t+1} - E_t[M_{t,t+1}]]R_{t,t+1}^f.$$

As is common in asset pricing theory, we make the additional assumption that log returns  $r_{t,t+1}^{(n)} \equiv \ln(R_{t,t+1}^{(n)})$  as well as the log stochastic discount factor  $m_{t,t+1} \equiv \ln(M_{t,t+1})$  are at least approximately jointly normally distributed, which allows us to simplify our expression for the risk premium to the covariance term alone (see, for example, Campbell and Vuolteenaho, 2004):

$$E_t[R_{t,t+1}^{(n)}] - R_{t,t+1}^f \simeq -Cov_t[r_{t,t+1}^{(n)}, m_{t,t+1} - E_t[m_{t,t+1}]],$$
 (A.8)

The above notation highlights again that only *innovations* in the (log-)SDF,  $m_{t,t+1} - E_t[m_{t,t+1}]$ , matter for expected returns.

To highlight the forces that shape the term structure of expected one-period returns, we will focus on analyzing the set of linear and log-linear consumption-based asset pricing models, in which the stochastic discount factor is a function of consumption growth. This class of models encompasses the vast majority of modern asset pricing models, in particular those employed in climate change analysis, such as power utility models as in Lucas (1978), long-run risk models with Epstein–Zin preferences as in Bansal and Yaron (2004), and rare disaster models as in Barro (2006) and Gabaix (2012). As noted in Dew-Becker and Giglio (2013), these asset pricing models can be nested in the following general representation for the SDF innovations:<sup>41</sup>

$$m_{t,t+1} - E_t[m_{t,t+1}] = -\sum_{k=0}^{\infty} z_k \cdot (E_{t+1} - E_t) \Delta c_{t+1+k},$$
 (A.9)

where  $\Delta c_{t+1} - E_t \Delta c_{t+1}$  (the first term of the sum, i.e. for k = 0) is the shock to current consumption growth, while  $(E_{t+1} - E_t) \Delta c_{t+1+k}$  with k > 0 is *news* about future consumption growth at horizon k, received during the holding period (between t and t + 1).

The terms  $z_k$  depend only on the parameters of the utility function (not on the consumption growth process), and represent risk aversion regarding news about consumption growth at a particular horizon. They can be thought of as horizon-specific risk prices. Substituting equation A.9 into equation A.8, we can write the expected return of any asset

<sup>&</sup>lt;sup>41</sup>More precisely, all of these models produce this representation of the SDF depending only on consumption growth news as long as the variance of consumption growth and higher moments are constant; the decomposition easily generalizes to cases with arbitrary affine processes.

by decomposing it across horizons:

$$E_{t}[R_{t,t+1}^{(n)}] - R_{t,t+1}^{f} \simeq z_{0}Cov_{t}[r_{t,t+1}^{(n)}, \Delta c_{t+1}]$$

$$+ z_{1}Cov_{t}[r_{t,t+1}^{(n)}, (E_{t+1} - E_{t})\Delta c_{t+2}]$$

$$+ z_{2}Cov_{t}[r_{t,t+1}^{(n)}, (E_{t+1} - E_{t})\Delta c_{t+3}]$$

$$+ \dots$$
(A.10)

The above decomposition holds for *any* asset, and therefore holds for all claims to cash flows at one particular point in the future,  $D_{t+n}$ , which jointly characterize the term structure of discount rates. It highlights that the shape of the term structure of expected one-period returns (and thus ultimately of horizon-specific discount rates) can be attributed to the interaction of two forces:

- 1. The term structure of horizon-specific *risk prices*  $z_k$ , i.e., how much agents care about long-term news relative to short-term news. The higher  $z_k$  is for long maturities, the more worried agents are about long-run risks in the economy.
- 2. The term structure of *risk quantities*, i.e. how much news about future consumption growth there is in the economy, and how it affects claims at different maturities. If there is no news about future consumption growth, for example if consumption growth is *iid*, all the news terms  $(E_{t+1} E_t)\Delta c_{t+n}$  and hence all of the respective covariances will be equal to zero. If instead there is long-horizon consumption risk (that is if consumption growth is predictable, for example because cash flows are persistently mean-reverting), then the news terms  $(E_{t+1} E_t)\Delta c_{t+n}$  are non-zero. Moreover, the returns to claims of different horizons  $r_{t,t+1}^{(n)}$  are then differentially exposed to shocks at different horizons.

# A.5.3 Explanations for a Downward-Sloping Term Structure of Discount Rates for Risky Assets – Preferences vs. Cash Flows

We can put the above decomposition to work and ask what mechanisms can generate a downward-sloping term structure of discount rates. As we can see from equation 14 of our model as presented in Section 2,  $z_0 = \gamma$ , but all  $z_k$ 's are zero for k > 0 in equation A.10 for an agent with power utility preferences. Put differently, such an agent is *only* worried about one-period innovations in consumption. For an Epstein–Zin investor by contrast,  $z_0 = \gamma$  like in the power utility case, but  $z_k = (\gamma - \frac{1}{\psi})\theta^k$  for k > 0, where  $\psi$  is the elasticity of intertemporal substitution and  $\theta$  is a parameter close to 1 related to the time

discount factor. Epstein–Zin parameterizations with  $\gamma > \frac{1}{\psi}$ , as in standard calibrations of the long-run risk model, thus imply that agents are worried *both* about immediate consumption growth *and* pure news regarding future consumption. Since claims to long-run cash flows  $D_{t+n}$  are naturally exposed to *all* dividend growth shocks from t to t+n ( $D_{t+n} = D_t \exp[\Delta d_{t+1} + ... + \Delta d_{t+n}]$ ), claims become *more exposed* to long-run shocks as their maturity increases. Accordingly, their risk premium grows with maturity as more and more of the positive covariance terms in equation A.10 are added up with positive weights. Therefore, introducing Epstein–Zin preferences would push the slope of the term structure of discount rates upwards. To match the data on a downward-sloping term structure of discount rates for risky assets, we would require an even stronger mean reversion in cash flows as a consequence of our model presented in Section 2. More generally, we are not aware of a standard representation of preferences that would push towards a downward-sloping term structure of discount rates for risky assets.

As we can see again from equation 14, what we require for a downward-sloping term structure of discount rates for risky assets are declining exposures of claims of different maturity  $r_{t,t+1}^{(n)}$  to the consumption shock  $\Delta c_{t+1}$ , i.e. *risk quantities*. In our setting presented in Section 2, mean reversion in cash flows makes growth in the economy predictable and implies that a climate disaster that strikes today has larger effects on immediate cash flows than on distant cash flows, which exposes short-run returns more than long-run returns to a consumption shock.

#### A.6 Details on the Model

This section presents details on our model. We derive the prices of claims to consumption and rents at different horizons and all results presented in Section 2.

#### A.6.1 Assumptions and Parameter Restrictions

Throughout, we are going to evaluate the term structure of discount rates and expected returns at the ergodic mean of all variables, i.e. evaluated when  $\lambda_t = E[\lambda_t] \equiv \bar{\lambda}$ ,  $x_t = E[x_t] \equiv \bar{x}$ , and  $y_t = E[y_t] \equiv \bar{y}$ . We assume that x and y have mean zero, which implies:

$$\mu_x = -\bar{\lambda}\phi\xi$$
 and  $\mu_y = -\bar{\lambda}\psi\xi$ .

The unconditional mean of  $\lambda_t$  is:

$$\bar{\lambda} = \frac{\mu_{\lambda}}{1 - \alpha - \chi \xi} > 0.$$

The long-run growth rate of consumption is:

$$\mu - \bar{\lambda}\xi > 0.$$

We further assume that consumption and rents have the same long-run growth rates, requiring:

$$\mu_d = \mu + (\eta - 1) \,\bar{\lambda}\xi.$$

Our calibration is discussed in Section 2.2 and summarized in Table A.8.

#### A.6.2 Pricing Claims to Single-Period Cash Flows

In Section A.7.1, we derive the prices of claims to arbitrary cash flows  $Z_{t+1}$  at different horizons. This section presents the results. We start by generalizing the cash flow process to:

$$\Delta z_{t+1} = \mu_z + \pi_z y_t - \eta_z J_{t+1},$$

where  $y_t$  still captures persistent changes in the growth rate of the cash flows and  $J_{t+1}$  is the underlying economic shock. Including separate and flexible loadings on persistent changes in the growth rate of cash flows,  $\pi_z$ , as well as the underlying economic shock,  $\eta_z$ , will allow us to nest the dynamics of all assets and liabilities relevant to our discussion in Section 2. This allows us to solve the model once and parameterize the solution as needed. The solution is recursive and takes the following form:

$$p_{z,t}^{(n)} = a_n^z + b_n^z x_t + e_n^z y_t + f_n^z \left( \lambda_t - \bar{\lambda} \right), \tag{A.11}$$

where  $p_{z,t}^{(n)}$  is the log price-dividend ratio of a claim to the cash flow n periods ahead (in levels, we write  $PD_{z,t}^{(n)}$ ). The full recursive expressions of the coefficients in terms of

primitives are as follows:

$$a_{n}^{z} = \ln \delta - \gamma \mu + \mu_{z} + a_{n-1}^{z} + b_{n-1}^{z} \mu_{x} + e_{n-1}^{z} \mu_{y} + f_{n-1}^{z} \left(\mu_{\lambda} + \bar{\lambda} \left(\alpha - 1\right)\right)$$

$$+ \ln \left[1 + \bar{\lambda} \left(\exp \left\{\left(\gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi\right) \xi\right\} - 1\right)\right]$$

$$b_{n}^{z} = -\gamma + b_{n-1}^{z} \rho + f_{n-1}^{z} \nu$$

$$e_{n}^{z} = e_{n-1}^{z} \omega + \pi_{z}$$

$$f_{n}^{z} = f_{n-1}^{z} \alpha + \frac{\exp \left\{\left(\gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi\right) \xi\right\} - 1}{1 + \bar{\lambda} \left(\exp \left\{\left(\gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi\right) \xi\right\} - 1\right)}.$$

with  $a_0^z = b_0^z = e_0^z = f_0^z = 0$ . The prices of all assets and liabilities we discuss in Section 2 can be derived based on various parameterizations of the above solution.

**Consumption:** To derive claims to consumption, we need  $\Delta z_{t+1} = \Delta c_{t+1}$ , i.e.  $\mu_z = \mu$ ,  $\pi_z = 1$ , and  $\eta_z = 1$ . Also note that we need to replace  $y_t$  with  $x_t$  as a consequence, i.e.  $\mu_y$  with  $\mu_x$ ,  $\omega$  with  $\rho$ , and  $\psi$  with  $\phi$ . The price of a consumption strip claim is then:

$$p_{c,t}^{(n)} = a_n^c + b_n^c x_t + f_n^c \left( \lambda_t - \bar{\lambda} \right). \tag{A.12}$$

Note that we can sum  $b_n^z$  and  $e_n^z$  to get  $b_n^c$  as x and y are the same for a consumption claim. The full recursive expressions of the coefficients in terms of primitives are as follows:  $a_0^c = b_0^c = f_0^c = 0$ , and

$$a_{n}^{c} = \ln \delta + (1 - \gamma) \mu + a_{n-1}^{c} + b_{n-1}^{c} \mu_{x} + f_{n-1}^{c} (\mu_{\lambda} + \bar{\lambda} (\alpha - 1))$$

$$+ \ln \left[ 1 + \bar{\lambda} \left( \exp \left\{ \left( - (1 - \gamma) + b_{n-1}^{c} \phi + f_{n-1}^{c} \chi \right) \xi \right\} - 1 \right) \right]$$

$$b_{n}^{c} = (1 - \gamma) + b_{n-1}^{c} \rho + f_{n-1}^{c} \nu$$

$$f_{n}^{c} = f_{n-1}^{c} \alpha + \frac{\exp \left\{ \left( - (1 - \gamma) + b_{n-1}^{c} \phi + f_{n-1}^{c} \chi \right) \xi \right\} - 1}{1 + \bar{\lambda} \left( \exp \left\{ \left( - (1 - \gamma) + b_{n-1}^{c} \phi + f_{n-1}^{c} \chi \right) \xi \right\} - 1 \right)}.$$

**Risk-Free Bond:** To derive claims to a risk-free bond with maturity n, we need  $Z_{t+n} = 1$  and zero otherwise. We set  $\mu_z = 0$ ,  $\pi_z = 0$ , and  $\eta_z = 0$ . The price of a risk-free strip claim is then:

$$b_{f,t}^{(n)} = a_n^f + b_n^f x_t + f_n^f (\lambda_t - \bar{\lambda}).$$
 (A.13)

Note that y and hence  $e_f$  drops as a result of the above parameterization. The full recursive expressions of the coefficients in terms of primitives are as follows:  $a_0^f = b_0^f = f_0^f = 0$ ,

and

$$a_{n}^{f} = \ln \delta - \gamma \mu + a_{n-1}^{f} + b_{n-1}^{f} \mu_{x} + f_{n-1}^{f} \left( \mu_{\lambda} + \bar{\lambda} \left( \alpha - 1 \right) \right) \\ + \ln \left[ 1 + \bar{\lambda} \left( \exp \left\{ \left( \gamma + b_{n-1}^{f} \phi + f_{n-1}^{f} \chi \right) \xi \right\} - 1 \right) \right] \\ b_{n}^{f} = -\gamma + b_{n-1}^{f} \rho + f_{n-1}^{f} \nu \\ f_{n}^{f} = f_{n-1}^{f} \alpha + \frac{\exp \left\{ \left( \gamma + b_{n-1}^{f} \phi + f_{n-1}^{f} \chi \right) \xi \right\} - 1}{1 + \bar{\lambda} \left[ \exp \left\{ \left( \gamma + b_{n-1}^{f} \phi + f_{n-1}^{f} \chi \right) \xi \right\} - 1 \right]}.$$

**Rents:** To derive claims to rents, we need  $\Delta z_{t+1} = \Delta d_{t+1}$ , i.e.  $\mu_z = \mu_d$ ,  $\pi_z = 1$ , and  $\eta_z = \eta$ . The price of a rent strip claim is then:

$$p_{d,t}^{(n)} = a_n^d + b_n^d x_t + e_n^d y_t + f_n^d \left( \lambda_t - \bar{\lambda} \right). \tag{A.14a}$$

The full recursive expressions of the coefficients in terms of primitives are as follows:  $a_0^d = b_0^d = e_0^d = f_0^d = 0$ , and

$$a_{n}^{d} = \ln \delta - \gamma \mu + \mu_{d} + a_{n-1}^{d} + b_{n-1}^{d} \mu_{x} + e_{n-1}^{d} \mu_{y} + f_{n-1} \left( \mu_{\lambda} + \bar{\lambda} \left( \alpha - 1 \right) \right)$$

$$+ \ln \left[ 1 + \bar{\lambda} \left( \exp \left\{ \left( \gamma - \eta + b_{n-1}^{d} \phi + e_{n-1}^{d} \psi + f_{n-1}^{d} \chi \right) \xi \right\} - 1 \right) \right]$$

$$b_{n}^{d} = -\gamma + b_{n-1}^{d} \rho + f_{n-1}^{d} \nu$$
(A.14c)

$$e_n^d = e_{n-1}^d \omega + 1 \tag{A.14d}$$

$$f_n^d = f_{n-1}^d \alpha + \frac{\exp\left\{\left(\gamma - \eta + b_{n-1}^d \phi + e_{n-1}^d \psi + f_{n-1}^d \chi\right) \xi\right\} - 1}{1 + \bar{\lambda} \left(\exp\left\{\left(\gamma - \eta + b_{n-1}^d \phi + e_{n-1}^d \psi + f_{n-1}^d \chi\right) \xi\right\} - 1\right)}. \quad (A.14e)$$

**Damages:** To derive claims to damages, we need  $\Delta z_{t+1} = \Delta q_{t+1}$ , i.e.  $\mu_z = \mu_q$ ,  $\pi_z = -\pi_q$ , and  $\eta_z = -\eta_q$ . The price of a damage strip claim is then:

$$p_{q,t}^{(n)} = a_n^q + b_n^q x_t + e_n^q y_t + f_n^q \left( \lambda_t - \bar{\lambda} \right). \tag{A.15}$$

The full recursive expressions of the coefficients in terms of primitives are as follows:  $a_0^q = b_0^q = e_0^q = f_0^q = 0$ , and

$$\begin{split} a_{n}^{q} &= \ln \delta - \gamma \mu + \mu_{q} + a_{n-1}^{q} + b_{n-1}^{q} \mu_{x} + e_{n-1}^{q} \mu_{y} + f_{n-1}^{q} \left( \mu_{\lambda} + \bar{\lambda} \left( \alpha - 1 \right) \right) \\ &+ \ln \left[ 1 + \bar{\lambda} \left( \exp \left\{ \left( \gamma + \eta_{q} + b_{n-1}^{q} \phi + e_{n-1}^{q} \psi + f_{n-1}^{q} \chi \right) \xi \right\} - 1 \right) \right] \\ b_{n}^{q} &= -\gamma + b_{n-1}^{q} \rho + f_{n-1}^{q} \nu \\ e_{n}^{q} &= e_{n-1}^{q} \omega - \pi_{q} \\ f_{n}^{q} &= f_{n-1}^{q} \alpha + \frac{\exp \left\{ \left( \gamma + \eta_{q} + b_{n-1}^{q} \phi + e_{n-1}^{q} \psi + f_{n-1}^{q} \chi \right) \xi \right\} - 1}{1 + \bar{\lambda} \left( \exp \left\{ \left( \gamma + \eta_{q} + b_{n-1}^{q} \phi + e_{n-1}^{q} \psi + f_{n-1}^{q} \chi \right) \xi \right\} - 1 \right)}. \end{split}$$

#### A.6.3 Per-Period Discount Rates

We derive per-period discount rates for claims to arbitrary cash flows  $Z_{t+1}$  at different horizons again and parameterize those accordingly to derive implied discount rates for all assets and liabilities discussed in Section 2. Remember from Section A.5.1 that perperiod discount rates  $\bar{r}_{z,t}^n$  are implicitly defined by:

$$P_{z,t}^{(n)} = \frac{E_t [Z_{t+n}]}{(1 + \overline{r}_{z,t}^n)^n}.$$

In Section A.7.2, we show that expected cash flows can be expressed as:

$$E_{t}[Z_{t+n}] = Z_{t} \exp\left\{n\mu_{z} + \pi_{z} \frac{1 - \omega^{n}}{1 - \omega} y_{t} + \pi_{z} \mu_{y} \sum_{s=0}^{n-1} \frac{1 - \omega^{s}}{1 - \omega}\right\} A_{z,n,t},$$
 (A.16)

where the second and third term inside the curly brackets are related to persistent changes in the growth rate of the economy, and  $A_{z,n,t}$  is a term that captures the history of (path-dependent) jump events. Formally,  $A_{z,n,t}$  is defined as:

$$A_{z,n,t} \equiv E_t \left[ \exp \left\{ \sum_{i=0}^{n-1} J_{t+n-i} \left( \pi_z \psi \frac{1-\omega^i}{1-\omega} - \eta_z \right) \right\} \right]. \tag{A.17}$$

These expectations can be computed in closed form for short horizons, but are best computed numerically for longer horizons. We outline a numerical solution algorithm in Section A.7.3. Rearranging and substituting for  $E_t[Z_{t+n}]$  using equation A.16, we get:

$$(1 + \overline{r}_{z,t}^n)^n = \frac{\exp\left\{n\mu_z + \pi_z\mu_y \sum_{s=0}^{n-1} \frac{1-\omega^s}{1-\omega} + \pi_z \frac{1-\omega^n}{1-\omega}y_t\right\} A_{z,n,t}}{\exp\left\{p_{z,t}^{(n)}\right\}}.$$

Taking logs and approximating  $\ln(1+x) \simeq x$ , we get:

$$\overline{r}_{z,t}^{n} \simeq \frac{1}{n} \left[ n\mu_z + \pi_z \mu_y \sum_{s=0}^{n-1} \frac{1-\omega^s}{1-\omega} + \pi_z \frac{1-\omega^n}{1-\omega} y_t + \ln A_{z,n,t} - p_{z,t}^{(n)} \right].$$

Substituting for  $p_{z,t}^{(n)}$  using equation A.11, we get:

$$\overline{r}_{z,t}^{n} \simeq \mu_{z} + \frac{1}{n} \left[ \pi_{z} \mu_{y} \sum_{s=0}^{n-1} \frac{1 - \omega^{s}}{1 - \omega} + \pi_{z} \frac{1 - \omega^{n}}{1 - \omega} y_{t} + \ln A_{z,n,t} \right] - \frac{1}{n} \left[ a_{n}^{z} + b_{n}^{z} x_{t} + e_{n}^{z} y_{t} + f_{n}^{z} \left( \lambda_{t} - \bar{\lambda} \right) \right], \tag{A.18}$$

with  $a_n^z$ ,  $b_n^z$ ,  $e_n^z$ , and  $f_n^z$  defined as in equation A.11, and  $A_{z,n,t}$  defined as above.

**Consumption:** To derive per-period discount rates for consumption, we need  $\Delta z_{t+1} = \Delta c_{t+1}$ , i.e.  $\mu_z = \mu$ ,  $\pi_z = 1$ , and  $\eta_z = 1$ . Also note that long-run dynamics in consumption are driven by x instead of y, and so we also need to replace  $y_t$  with  $x_t$  as a consequence, i.e.  $\mu_y$  with  $\mu_x$ ,  $\omega$  with  $\rho$ , and  $\psi$  with  $\phi$ . The per-period discount rate is then:

$$\overline{r}_{c,t}^{n} \simeq \mu + \frac{1}{n} \left[ \mu_{x} \sum_{s=0}^{n-1} \frac{1-\rho^{s}}{1-\rho} + \frac{1-\rho^{n}}{1-\rho} x_{t} + \ln A_{c,n,t} - \left( a_{n}^{c} + b_{n}^{c} x_{t} + f_{n}^{c} \left( \lambda_{t} - \bar{\lambda} \right) \right) \right],$$

with  $a_n^c$ ,  $b_n^c$ , and  $f_n^c$  defined as in equation A.12 (note that we can sum  $b_n^z$  and  $e_n^z$  to get  $b_n^c$  as x and y are the same for a consumption claim), and  $A_{c,n,t}$  defined as:

$$A_{c,n,t} = E_t \left[ \exp \left\{ \sum_{i=0}^{n-1} J_{t+n-i} \left( \phi \frac{1-\rho^i}{1-\rho} - 1 \right) \right\} \right].$$

**Risk-free rate:** The risk-free rate in the economy is given by:

$$R_{t,n}^f = 1/B_t^{(n)},$$

and linked to the risk-free discount rate by  $R_{t,n}^f = \left(1 + r_{t,n}^f\right)^n$ . Approximating  $\ln\left(1 + x\right) \simeq x$ , we have:

$$r_{t,n}^f \simeq \frac{1}{n} \ln R_{t,n}^f = -\frac{1}{n} \ln B_t^{(n)} = -\frac{1}{n} b_{f,t}^{(n)} = -\frac{1}{n} \left( a_n^f + b_n^f x_t + f_n^f \left( \lambda_t - \bar{\lambda} \right) \right),$$

where  $a_n^f$ ,  $b_n^f$ , and  $f_n^f$  are defined as in equation A.13. The risk-free rate increases in  $x_t$  and decreases in the severity and probability of disasters.

**Rents:** To derive per-period discount rates for rents, we need  $\Delta z_{t+1} = \Delta d_{t+1}$ , i.e.  $\mu_z = \mu_d$ ,  $\pi_z = 1$ , and  $\eta_z = \eta$ . The per-period discount rate is then:

$$\overline{r}_{d,t}^{n} \simeq \mu_{d} + \frac{1}{n} \left[ \mu_{y} \sum_{s=0}^{n-1} \frac{1-\omega^{s}}{1-\omega} + \frac{1-\omega^{n}}{1-\omega} y_{t} + \ln A_{d,n,t} - \left( a_{n}^{d} + b_{n}^{d} x_{t} + e_{n}^{d} y_{t} + f_{n}^{d} \left( \lambda_{t} - \bar{\lambda} \right) \right) \right].$$

where  $a_n^d$ ,  $b_n^d$ ,  $e_n^d$ , and  $f_n^d$  are defined as in equations A.14b to A.14e, and  $A_{d,n,t}$  is defined as:

$$A_{d,n,t} \equiv E_t \left[ \exp \left\{ \sum_{i=0}^{n-1} J_{t+n-i} \left( \psi \frac{1-\omega^i}{1-\omega} - \eta \right) \right\} \right].$$

**Damages:** To derive claims to damages, we need  $\Delta z_{t+1} = \Delta q_{t+1}$ , i.e.  $\mu_z = \mu_q$ ,  $\pi_z = -\pi_q$ , and  $\eta_z = -\eta_q$ . The per-period discount rate is then:

$$\overline{r}_{q,t}^{n} \simeq \mu_{q} + \frac{1}{n} \left[ -\pi_{q} \mu_{y} \sum_{s=0}^{n-1} \frac{1 - \omega^{s}}{1 - \omega} - \pi_{q} \frac{1 - \omega^{n}}{1 - \omega} y_{t} + \ln A_{q,n,t} \right] - \frac{1}{n} \left[ a_{n}^{q} + b_{n}^{q} x_{t} + e_{n}^{q} y_{t} + f_{n}^{q} \left( \lambda_{t} - \bar{\lambda} \right) \right], \tag{A.19}$$

where  $a_n^q$ ,  $b_n^q$ ,  $e_n^q$ , and  $f_n^q$  are defined as in equation A.15, and  $A_{q,n,t}$  is defined as:

$$A_{q,n,t} \equiv E_t \left[ \exp \left\{ \sum_{i=0}^{n-1} J_{t+n-i} \left( -\pi_q \psi \frac{1-\omega^i}{1-\omega} + \eta_q \right) \right\} \right].$$

## A.6.4 Expected Returns and Return Decomposition for Rent Strips

In this section, we derive expressions for expected returns to rent strips and decompose rent strip returns following the methodology outlined in Section A.5.2.

#### A.6.4.1 Expected Returns on Rent Strips

The return on a rent strip is given by:

$$R_{d,t,t+1}^{(n)} = \frac{P_{d,t+1}^{(n-1)}}{P_{d,t}^{(n)}}.$$

Consequently, the log return on the strip is simply:

$$r_{d,t,t+1}^{(n)} = \ln P_{d,t+1}^{(n-1)} - \ln P_{d,t}^{(n)} = p_{d,t+1}^{(n-1)} - p_{d,t}^{(n)} + \Delta d_{t+1}$$

for n > 1, and  $\Delta d_{t+1} - p_{d,t}^{(1)}$  for n = 1 (for the first return, we just set  $p_{d,t}^{(0)} = 0$ ; note that we have to adjust for  $\Delta d_{t+1}$  because we denote by p the log price-dividend ratio, not just the log price). Substituting for the log price-dividend ratio and for dividend growth, we get:

$$r_{d,t,t+1}^{(n)} = \left[ a_{n-1}^d + b_{n-1}^d x_{t+1} + e_{n-1}^d y_{t+1} + f_{n-1}^d \left( \lambda_{t+1} - \bar{\lambda} \right) \right]$$

$$- \left[ a_n^d + b_n^d x_t + e_n^d y_t + f_n^d \left( \lambda_t - \bar{\lambda} \right) \right] + \left[ \mu_d + y_t - \eta J_{t+1} \right].$$

Substituting for  $x_{t+1}$ ,  $y_{t+1}$ , and  $\lambda_{t+1}$  and collecting shock terms, we get:

$$r_{d,t,t+1}^{(n)} = \left[ a_{n-1}^d + b_{n-1}^d \left( \mu_x + \rho x_t \right) + e_{n-1}^d \left( \mu_y + \omega y_t \right) + f_{n-1}^d \left( \mu_\lambda + \alpha \lambda_t + \nu x_t - \bar{\lambda} \right) \right]$$

$$- \left[ a_n^d + b_n^d x_t + e_n^d y_t + f_n^d \left( \lambda_t - \bar{\lambda} \right) \right] + \left[ \mu_d + y_t \right]$$

$$+ \left( b_{n-1}^d \phi + e_{n-1}^d \psi + f_{n-1}^d \chi - \eta \right) J_{t+1}.$$
(A.20)

For the expected return of the rent strip, we have:

$$E_{t}\left[R_{d,t,t+1}^{(n)}\right] = E_{t}\left[\exp\left\{r_{d,t,t+1}^{(n)}\right\}\right]$$

$$= \exp\left\{\left[a_{n-1}^{d} + b_{n-1}^{d}\left(\mu_{x} + \rho x_{t}\right) + e_{n-1}^{d}\left(\mu_{y} + \omega y_{t}\right) + f_{n-1}^{d}\left(\mu_{\lambda} + \alpha \lambda_{t} + \nu x_{t} - \bar{\lambda}\right)\right]\right\}$$

$$\times \exp\left\{-\left[a_{n}^{d} + b_{n}^{d} x_{t} + e_{n}^{d} y_{t} + f_{n}^{d}\left(\lambda_{t} - \bar{\lambda}\right)\right] + \left[\mu_{d} + y_{t}\right]\right\}$$

$$\times \left[1 + \lambda_{t}\left(\exp\left\{\left(b_{n-1}^{d} \phi + e_{n-1}^{d} \psi + f_{n-1}^{d} \chi - \eta\right) \xi\right\} - 1\right)\right],$$
(A.21)

where the last line follows from  $J_{t+1}$  only taking value  $\xi \in (0,1)$  with probability  $\lambda_t$  and zero otherwise, and therefore:

$$E_{t} \left[ \exp \left\{ \left( b_{n-1}^{d} \phi + e_{n-1}^{d} \psi + f_{n-1}^{d} \chi - \eta \right) J_{t+1} \right\} \right]$$

$$= (1 - \lambda_{t}) + \lambda_{t} \exp \left\{ \left( b_{n-1}^{d} \phi + e_{n-1}^{d} \psi + f_{n-1}^{d} \chi - \eta \right) \xi \right\}.$$

#### A.6.4.2 Return Decomposition for Rent Strips

As discussed in Section A.5, in our model with power utility, we have  $z_0 = \gamma$  and  $z_k = 0$  for k > 0, and thus equation A.10 simplifies to:

$$E_t[R_{d,t,t+1}^{(n)}] - R_{t,t+1}^f \simeq \gamma Cov_t \left[ r_{d,t,t+1}^{(n)}, \Delta c_{t+1} \right].$$

Substituting for consumption growth  $\Delta c_{t+1} = \mu + x_t - J_{t+1}$  and the log strip return from equation A.20, and dropping constant terms, we get:

$$\gamma Cov_t \left[ r_{d,t,t+1}^{(n)}, \Delta c_{t+1} \right] = \gamma Cov_t \left[ \left( b_{n-1}^d \phi + e_{n-1}^d \psi + f_{n-1}^d \chi - \eta \right) J_{t+1}, -J_{t+1} \right].$$

Since  $Var_t[J_{t+1}] = \xi^2 \lambda_t (1 - \lambda_t)$ , we obtain:

$$\gamma Cov_{t}\left[r_{d,t,t+1}^{(n)},\Delta c_{t+1}\right] = \gamma\left[\eta - \phi b_{n-1}^{d} - \psi e_{n-1}^{d} - \chi f_{n-1}^{d}\right]\xi^{2}\lambda_{t}\left(1 - \lambda_{t}\right),$$

and therefore:

$$E_{t}[R_{d,t,t+1}^{(n)}] - R_{t,t+1}^{f} \simeq \gamma \operatorname{Cov}_{t}[r_{d,t,t+1}^{(n)}, \Delta c_{t+1}]$$

$$= \gamma \left[ \eta - \psi e_{n-1}^{d} - \phi b_{n-1}^{d} - \chi f_{n-1}^{d} \right] \xi^{2} \lambda_{t} (1 - \lambda_{t}). \tag{A.22}$$

where  $b_{n-1}^d$ ,  $e_{n-1}^d$ , and  $f_{n-1}^d$  are defined in equations A.14c to A.14e.

#### A.6.5 Price-Rent Ratio Semi-Elasticity to Disaster Probability

The price-rent ratio of the freehold is simply the sum of the price-rent ratios of strips across all maturities:

$$PD_{d,t}^{fh} = \sum_{n=1}^{\infty} PD_{d,t}^{(n)} = \sum_{n=1}^{\infty} \exp\left\{p_{d,t}^{(n)}\right\}.$$

After substituting for the log price-rent ratio for the strips from equation A.14a, we can see that the semi-elasticity of the price-dividend ratio with respect to the disaster probability  $\lambda_t$  is:

$$\frac{\partial p_{d,t}}{\partial \lambda_t} = \frac{1}{PD_{d,t}^{fh}} \sum_{n=1}^{\infty} PD_{d,t}^{(n)} f_n^d.$$

It tells us by how much the price dividend-ratio moves (in percent) if the probability of a disaster increases by one percentage point. Note that this semi-elasticity is affected by two opposing forces: As the disaster probability increases, the risk-free rate falls for precautionary motives, while the risk premium increases. To isolate the effect of an increase in disaster probabilities (without a disaster having occurred) on the risk premium, we can look at properties that are differentially exposed to the disaster risk. In particular, the difference between the semi-elasticity of two properties, one with high and one with low loadings on climate risk ( $\eta_d^H > \eta_d^L$ ) is:

$$\frac{\partial \left( p_{d,t}^{H} - p_{d,t}^{L} \right)}{\partial \lambda_{t}} = \frac{1}{PD_{d,t}^{H}} \sum_{n=1}^{\infty} PD_{d,t}^{H(n)} f_{d}^{n,H} - \frac{1}{PD_{d,t}^{L}} \sum_{n=1}^{\infty} PD_{d,t}^{L(n)} f_{d}^{n,L}.$$

Intuitively, we expect this number to be negative – all else equal, the risk premium of a property with higher disaster-risk exposure should increase by more than the risk premium of a property with lower disaster-risk exposure as disaster risk increases (and hence the price of a property with higher disaster-risk exposure should decrease relative to the price of a property with lower disaster-risk exposure).

#### A.6.6 Expected Returns and Risk Premia for the Freehold

The return on the freehold is:

$$E_t \left[ R_{d,t,t+1}^{fh} \right] = \sum_{n=1}^{\infty} \left[ \frac{PD_{d,t}^n}{PD_{d,t}^{fh}} E_t \left[ R_{d,t,t+1}^{(n)} \right] \right].$$

The risk premium for the freehold then is:

$$E_{t}\left[R_{d,t,t+1}^{fh}-R_{t,t+1}^{f}\right] = \sum_{n=1}^{\infty} \left[\frac{PD_{d,t}^{(n)}}{PD_{d,t}^{fh}}E_{t}\left[R_{d,t,t+1}^{(n)}-R_{t,t+1}^{f}\right]\right].$$

### A.7 Solving the Model

#### A.7.1 Cash Flow Strip Prices

We are going to derive prices of arbitrary cash flows  $Z_{t+1}$  here, where

$$\Delta z_{t+1} = \mu_z + \pi_z y_t - \eta_z J_{t+1},$$

and where  $y_t$  captures persistent changes in the growth rate of the cash flows and  $J_{t+1}$  is the underlying economic shock. Including separate and flexible loadings on persistent changes in the growth rate of cash flows,  $\pi_z$ , as well as the underlying economic shock,  $\eta_z$ , will allow us to nest the dynamics of all assets and liabilities relevant to our discussion

in Section 2. This allows us to solve the model once and parameterize the solution as needed. The solution is recursive:

**For maturity 1:** We can price a claim to next period's cash flow (i.e., the first cash flow strip) as:

$$P_{z,t}^{(1)} = E_t [M_{t+1} Z_{t+1}],$$

or:

$$\frac{P_{z,t}^{(1)}}{Z_t} = E_t \left[ M_{t+1} \frac{Z_{t+1}}{Z_t} \right].$$

Rewriting in logs and substituting for the log stochastic discount factor, we have:

$$\exp\left\{p_{z,t}^{(1)}\right\} = E_t \left[\exp\left\{\ln \delta - \gamma \Delta c_{t+1} + \Delta z_{t+1}\right\}\right],$$

and substituting for the consumption and cash flow growth rates, and collecting terms, we have:

$$= E_t \left[ \exp \left\{ \ln \delta - \gamma \left( \mu + x_t \right) + \mu_z + \pi_z y_t + (\gamma - \eta_z) J_{t+1} \right\} \right].$$

We can now pull time-*t* information out of the expectation:

$$=\exp\left\{\ln\delta-\gamma\left(\mu+x_{t}\right)+\mu_{z}+\pi_{z}y_{t}\right\}E_{t}\left[\exp\left\{\left(\gamma-\eta_{z}\right)J_{t+1}\right\}\right],$$

and recall that  $J_{t+1}$  only takes value  $\xi$  with probability  $\lambda_t$  and zero otherwise, and therefore:

$$E_t \left[ \exp \left\{ \left( \gamma - \eta_z \right) J_{t+1} \right\} \right] = \left( 1 - \lambda_t \right) + \lambda_t \exp \left\{ \left( \gamma - \eta_z \right) \xi \right\}.$$

After taking logs, we get:

$$p_{z,t}^{(1)} = \ln \delta - \gamma \left(\mu + x_t\right) + \mu_z + \pi_z y_t + \ln \left[1 + \lambda_t \left(\exp \left\{\left(\gamma - \eta_z\right) \xi\right\} - 1\right)\right].$$

Via Taylor expansion of the last term around the unconditional mean of  $\lambda_t$ ,  $\bar{\lambda}$ , we obtain:

$$\ln\left[1 + \lambda_t \left(\exp\left\{\left(\gamma - \eta_z\right)\xi\right\} - 1\right)\right]$$

$$\simeq \ln\left[1+\bar{\lambda}\left(\exp\left\{\left(\gamma-\eta_{z}\right)\xi\right\}-1\right)\right]+\frac{\exp\left\{\left(\gamma-\eta_{z}\right)\xi\right\}-1}{1+\bar{\lambda}\left(\exp\left\{\left(\gamma-\eta_{z}\right)\xi\right\}-1\right)}\left(\lambda_{t}-\bar{\lambda}\right).$$

Thus:

$$p_{z,t}^{(1)} = \ln \delta - \gamma \left(\mu + x_t\right) + \mu_z + \pi_z y_t$$

$$+\ln\left[1+\bar{\lambda}\left(\exp\left\{\left(\gamma-\eta_{z}\right)\xi\right\}-1\right)\right]+\frac{\exp\left\{\left(\gamma-\eta_{z}\right)\xi\right\}-1}{1+\bar{\lambda}\left(\exp\left\{\left(\gamma-\eta_{z}\right)\xi\right\}-1\right)}\left(\lambda_{t}-\bar{\lambda}\right),$$

or:

$$p_{z,t}^{(1)} = a_1^z + b_1^z x_t + e_1^z y_t + f_1^z \left( \lambda_t - \bar{\lambda} \right), \tag{A.23}$$

with

$$a_{1}^{z} = \ln \delta - \gamma \mu + \mu_{z} + \ln \left[ 1 + \bar{\lambda} \left( \exp \left\{ (\gamma - \eta_{z}) \, \xi \right\} - 1 \right) \right]$$

$$b_{1}^{z} = -\gamma$$

$$e_{1}^{z} = \pi_{z}$$

$$f_{1}^{z} = \frac{\exp \left\{ (\gamma - \eta_{z}) \, \xi \right\} - 1}{1 + \bar{\lambda} \left( \exp \left\{ (\gamma - \eta_{z}) \, \xi \right\} - 1 \right)}.$$

**For arbitrary maturity** n: For arbitrary maturities, we conjecture that all  $p_{z,t}^{(n)}$  will follow the recursion:

$$p_{z,t}^{(n)} = a_n^z + b_n^z x_t + e_z^d y_t + f_n^z \left( \lambda_t - \bar{\lambda} \right),$$

where  $a_1^z$ ,  $b_1^z$ ,  $e_1^z$ , and  $f_1^z$  are defined as above. We can price a claim to an n-period cash flow (i.e., the n-th dividend strip) as:

$$P_{z,t}^{(n)} = E_t \left[ M_{t+1} P_{z,t+1}^{(n-1)} \right],$$

or:

$$\frac{P_{z,t}^{(n)}}{Z_t} = E_t \left[ M_{t+1} \frac{P_{z,t+1}^{(n-1)}}{Z_{t+1}} \frac{Z_{t+1}}{Z_t} \right].$$

Rewriting in logs and substituting for the log stochastic discount factor, we have:

$$\exp\left\{p_{z,t}^{(n)}\right\} = E_t\left[\exp\left\{\ln\delta - \gamma\Delta c_{t+1} + \Delta z_{t+1} + p_{z,t+1}^{(n-1)}\right\}\right],$$

and substituting for the consumption and cash flow growth rates and the log pricedividend ratio, and collecting terms, we have:

$$= \exp \left\{ \ln \delta - \gamma \left( \mu + x_t \right) + \mu_z + \pi_z y_t \right\}$$

$$\times E_{t} \left[ \exp \left\{ \left( \gamma - \eta_{z} \right) J_{t+1} + a_{n-1}^{z} + b_{n-1}^{z} x_{t+1} + e_{n-1}^{z} y_{t+1} + f_{n-1}^{z} \left( \lambda_{t+1} - \bar{\lambda} \right) \right\} \right].$$

Substituting for  $x_{t+1}$ ,  $y_{t+1}$  and  $\lambda_{t+1}$ , and collecting a few terms and all shocks, we get:

$$= \exp \left\{ \ln \delta - \gamma \mu + \mu_z + a_{n-1}^z + b_{n-1}^z \mu_x + e_{n-1}^z \mu_y + f_{n-1}^z \left( \mu_{\lambda} + \bar{\lambda} \left( \alpha - 1 \right) \right) \right\}$$

$$\times \exp \left\{ \left( -\gamma + b_{n-1}^{z} \rho + f_{n-1}^{z} \nu \right) x_{t} + \left( \pi_{z} + e_{n-1}^{z} \omega \right) y_{t} + f_{n-1}^{z} \alpha \left( \lambda_{t} - \bar{\lambda} \right) \right\}$$

$$\times E_{t} \left[ \exp \left\{ \left( \gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi \right) J_{t+1} \right\} \right].$$

Recall that  $J_{t+1}$  only takes value  $\xi$  with probability  $\lambda_t$  and zero otherwise, and therefore:

$$E_{t} \left[ \exp \left\{ \left( \gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi \right) J_{t+1} \right\} \right]$$

$$= (1 - \lambda_{t}) + \lambda_{t} \exp \left\{ \left( \gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi \right) \xi \right\}.$$

After taking logs, we get:

$$p_{z,t}^{(n)} = \ln \delta - \gamma \mu + \mu_z + a_{n-1}^z + b_{n-1}^z \mu_x + e_{n-1}^z \mu_y + f_{n-1}^z \left( \mu_\lambda + \bar{\lambda} \left( \alpha - 1 \right) \right)$$

$$+ \left( -\gamma + b_{n-1}^z \rho + f_{n-1}^z \nu \right) x_t + \left( \pi_z + e_{n-1}^z \omega \right) y_t + f_{n-1}^z \alpha \left( \lambda_t - \bar{\lambda} \right)$$

$$+ \ln \left[ 1 + \lambda_t \left( \exp \left\{ \left( \gamma - \eta_z + b_{n-1}^z \phi + e_{n-1}^z \psi + f_{n-1}^z \chi \right) \xi \right\} - 1 \right) \right].$$

Via Taylor expansion of the last term around the unconditional mean of  $\lambda_t$ ,  $\bar{\lambda}$ , we obtain:

$$\ln \left[ 1 + \lambda_t \left( \exp \left\{ \left( \gamma - \eta_z + b_{n-1}^z \phi + e_{n-1}^z \psi + f_{n-1}^z \chi \right) \xi \right\} - 1 \right) \right] 
\simeq \ln \left[ 1 + \bar{\lambda} \left( \exp \left\{ \left( \gamma - \eta_z + b_{n-1}^z \phi + e_{n-1}^z \psi + f_{n-1}^z \chi \right) \xi \right\} - 1 \right) \right] 
+ \frac{\exp \left\{ \left( \gamma - \eta_z + b_{n-1}^z \phi + e_{n-1}^z \psi + f_{n-1}^z \chi \right) \xi \right\} - 1}{1 + \bar{\lambda} \left( \exp \left\{ \left( \gamma - \eta_z + b_{n-1}^z \phi + e_{n-1}^z \psi + f_{n-1}^z \chi \right) \xi \right\} - 1 \right)} \left( \lambda_t - \bar{\lambda} \right).$$

Thus:

$$\begin{split} p_{z,t}^{(n)} &= \ln \delta - \gamma \mu + \mu_z + a_{n-1}^z + b_{n-1}^z \mu_x + e_{n-1}^z \mu_y + f_{n-1}^z \left( \mu_\lambda + \bar{\lambda} \left( \alpha - 1 \right) \right) \\ &+ \ln \left[ 1 + \bar{\lambda} \left( \exp \left\{ \left( \gamma - \eta_z + b_{n-1}^z \phi + e_{n-1}^z \psi + f_{n-1}^z \chi \right) \xi \right\} - 1 \right) \right] \\ &+ \left( -\gamma + b_{n-1}^z \rho + f_{n-1}^z \nu \right) x_t + \left( \pi_z + e_{n-1}^z \omega \right) y_t + f_{n-1}^z \alpha \left( \lambda_t - \bar{\lambda} \right) \\ &+ \frac{\exp \left\{ \left( \gamma - \eta_z + b_{n-1}^z \phi + e_{n-1}^z \psi + f_{n-1}^z \chi \right) \xi \right\} - 1}{1 + \bar{\lambda} \left( \exp \left\{ \left( \gamma - \eta_z + b_{n-1}^z \phi + e_{n-1}^z \psi + f_{n-1}^z \chi \right) \xi \right\} - 1 \right)} \left( \lambda_t - \bar{\lambda} \right). \end{split}$$

Matching coefficients, we get the following expression for the log price-dividend ratio:

$$p_{zt}^{(n)} = a_n^z + b_n^z x_t + e_n^z y_t + f_n^z \left( \lambda_t - \bar{\lambda} \right), \tag{A.24}$$

with:

$$a_{n}^{z} = \ln \delta - \gamma \mu + \mu_{z} + a_{n-1}^{z} + b_{n-1}^{z} \mu_{x} + e_{n-1}^{z} \mu_{y} + f_{n-1}^{z} \left(\mu_{\lambda} + \bar{\lambda} \left(\alpha - 1\right)\right) + \ln \left[1 + \bar{\lambda} \left(\exp \left\{\left(\gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi\right) \xi\right\} - 1\right)\right]$$

$$b_{n}^{z} = -\gamma + b_{n-1}^{z} \rho + f_{n-1}^{z} \nu$$

$$e_{n}^{z} = e_{n-1}^{z} \omega + \pi_{z}$$

$$f_{n}^{z} = f_{n-1}^{z} \alpha + \frac{\exp \left\{\left(\gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi\right) \xi\right\} - 1}{1 + \bar{\lambda} \left(\exp \left\{\left(\gamma - \eta_{z} + b_{n-1}^{z} \phi + e_{n-1}^{z} \psi + f_{n-1}^{z} \chi\right) \xi\right\} - 1\right)}$$

and  $a_0^z = b_0^z = e_0^z = f_0^z = 0$ .

#### A.7.2 Expected Cash Flows

The expected cash flow from a strip of arbitrary maturity n is:

$$E_t[Z_{t+n}] = Z_t E_t \left[ \exp \left\{ \Delta z_{t+1} + ... + \Delta z_{t+n} \right\} \right].$$

To compute  $E_t$  [exp { $\Delta z_{t+1} + ... + \Delta z_{t+n}$ }], we iterate forward:

**For** n = 1: For n = 1, we have:

$$E_t \left[ \exp \left\{ \Delta z_{t+1} \right\} \right] = E_t \left[ \exp \left\{ \mu_z + \pi_z y_t - \eta_z J_{t+1} \right\} \right]$$
$$= \exp \left\{ \mu_z + \pi_z y_t \right\} E_t \left[ \exp \left\{ -\eta_z J_{t+1} \right\} \right]$$

Recall that  $J_{t+1}$  only takes value  $\xi$  with probability  $\lambda_t$  and zero otherwise, and therefore:

$$E_t [Z_{t+1}] = Z_t \exp \left\{ \mu_z + \pi_z y_t \right\} \left[ (1 - \lambda_t) + \lambda_t \exp \left\{ -\eta_z \xi \right\} \right].$$

**For** n = 2: Iterating forward to n = 2, we have:

$$E_t \left[ \exp \left\{ \Delta z_{t+1} + \Delta z_{t+2} \right\} \right] = E_t \left[ \exp \left\{ \left( \mu_z + \pi_z y_t - \eta_z J_{t+1} \right) + \left( \mu_z + \pi_z y_{t+1} - \eta_z J_{t+2} \right) \right\} \right],$$

substituting for  $y_{t+1}$  and collecting terms, we get:

= 
$$\exp \{2\mu_z + \pi_z (1 + \omega) y_t + \pi_z \mu_y\} E_t [\exp \{[\pi_z \psi - \eta_z] J_{t+1} - \eta_z J_{t+2}\}].$$

Note that the jump events are not independent unless  $\lambda_t$  is constant and define:

$$A_{z,2,t} \equiv E_t \left[ \exp \left\{ \left[ \pi_z \psi - \eta_z \right] J_{t+1} - \eta_z J_{t+2} \right\} \right].$$

Thus:

$$E_t[Z_{t+2}] = Z_t \exp\{2\mu_z + \pi_z(1+\omega)y_t + \pi_z\mu_y\} A_{z,2,t}.$$

**For** n = 3: Iterating forward to n = 3, we have:

$$E_t \left[ \exp \left\{ \Delta z_{t+1} + \Delta z_{t+2} + \Delta z_{t+3} \right\} \right]$$

= 
$$E_t \left[ \exp \left\{ (\mu_z + \pi_z y_t - \eta_z J_{t+1}) + (\mu_z + \pi_z y_{t+1} - \eta_z J_{t+2}) + (\mu_z + \pi_z y_{t+2} - \eta_z J_{t+3}) \right\} \right]$$
, substituting for  $y_{t+1}$  and  $y_{t+2}$  iteratively and collecting terms, we get:

$$= \exp \left\{ 3\mu_z + \pi_z \left( 1 + \omega + \omega^2 \right) y_t + \pi_z \left[ 1 + (1 + \omega) \right] \mu_y \right\}$$

$$\times E_t \left[ \exp \left\{ \left[ \pi_z \left( 1 + \omega \right) \psi - \eta_z \right] I_{t+1} + \left[ \pi_z \psi - \eta_z \right] I_{t+2} - \eta_z I_{t+3} \right\} \right].$$

As before, define:

$$A_{z,3,t} \equiv E_t \left[ \exp \left\{ \left[ \pi_z (1 + \omega) \psi - \eta_z \right] J_{t+1} + \left[ \pi_z \psi - \eta_z \right] J_{t+2} - \eta_z J_{t+3} \right\} \right].$$

Thus:

$$E_t[Z_{t+3}] = Z_t \exp \left\{ 3\mu_z + \pi_z \left( 1 + \omega + \omega^2 \right) y_t + \pi_z \left[ 1 + (1 + \omega) \right] \mu_y \right\} A_{z,3,t}.$$

**For arbitrary n:** We conclude that cash flow growth is given by:

$$E_{t}[Z_{t+n}] = Z_{t} \exp\left\{n\mu_{z} + \pi_{z} \frac{1 - \omega^{n}}{1 - \omega} y_{t} + \pi_{z} \mu_{y} \sum_{s=0}^{n-1} \frac{1 - \omega^{s}}{1 - \omega}\right\} A_{z,n,t},$$
 (A.25a)

with

$$A_{z,n,t} \equiv E_t \left[ \exp \left\{ \sum_{i=0}^{n-1} J_{t+n-i} \left( \pi_z \psi \frac{1-\omega^i}{1-\omega} - \eta_z \right) \right\} \right]. \tag{A.25b}$$

These expectations can be computed in closed form for short horizons, but are best computed numerically for longer horizons. We outline a numerical solution algorithm in Section A.7.3.

#### A.7.3 Solution method for $A_{z,n,t}$

In this section, we describe our numerical solution method for  $A_{z,n,t}$ . Remember that  $A_{z,n,t}$  is defined as:

$$A_{z,n,t} \equiv E_t \left[ \exp \left\{ \sum_{i=0}^{n-1} J_{t+n-i} \left( \pi_z \psi \frac{1-\omega^i}{1-\omega} - \eta_z \right) \right\} \right],$$

and captures the history of (path-dependent) jump events that go into cash flow growth. More generically, we want to compute:

$$A_{z,n,t} = E_t \left[ \exp \left\{ \beta_1^z J_{t+1} + \beta_2^z J_{t+2} + \dots + \beta_n^z J_{t+n} \right\} \right]$$

for arbitrary constants  $\beta_1^z...\beta_n^z$  and arbitrary maturity n, where  $J_{t+1}$  is a variable that, conditional on time-t information (particularly  $\lambda_t$ ), has value  $\xi$  with probability  $\lambda_t$  and zero otherwise, and where

$$\lambda_{t+1} = \mu_{\lambda} + \alpha \lambda_t + \nu x_t + \chi J_{t+1},$$

with

$$x_{t+1} = \mu_x + \rho x_t + \phi J_{t+1}.$$

To solve for  $A_{z,n,t}$  numerically, we start from the last period n. By the law of iterated expectations, we have:

$$E_{t} \left[ \exp \left\{ \beta_{1}^{z} J_{t+1} + \beta_{2}^{z} J_{t+2} + \dots + \beta_{n}^{z} J_{t+n} \right\} \right]$$

$$= E_{t} \left[ E_{t+n-1} \left[ \exp \left\{ \beta_{1}^{z} J_{t+1} + \beta_{2}^{z} J_{t+2} + \dots + \beta_{n}^{z} J_{t+n} \right\} \right] \right]$$

$$= E_{t} \left[ \exp \left\{ \beta_{1}^{z} J_{t+1} + \beta_{2}^{z} J_{t+2} + \dots + \beta_{n-1}^{z} J_{t+n-1} \right\} E_{t+n-1} \left[ \exp \left\{ \beta_{n}^{z} J_{t+n} \right\} \right] \right].$$

Note that the jumps  $J_{t+1}$  to  $J_{t+n-1}$  are all known by time t + n - 1, so we can pull them outside the internal expectation. Next, we can solve for the internal expectation:

$$E_{t+n-1}\left[\exp\left\{\beta_{n}^{z}J_{t+n}\right\}\right] = (1 - \lambda_{t+n-1}) + \lambda_{t+n-1}\exp\left\{\beta_{n}^{z}\xi\right\}.$$

We can think of this as a sort of binomial tree where jump probabilities change over time – each period either a jump is realized or not, and that outcome also changes  $\lambda$  for the next node. Note that the above expression is only a function of  $\lambda$ ; call it  $f_{n-1}(\lambda, x)$  for

consistency with later steps:

$$f_{n-1}(\lambda, x) \equiv (1 - \lambda) + \lambda \exp \{\beta_n^z \xi\}.$$

We compute  $f_{n-1}(\lambda, x)$  on a grid of possible values for  $\lambda$  and x and store it for later use. Updating our expression for  $A_{z,n,t}$ , we have:

$$A_{z,n,t} = E_t \left[ \exp \left\{ \beta_1^z J_{t+1} + \beta_2^z J_{t+2} + \dots + \beta_{n-1}^z J_{t+n-1} \right\} f_{n-1} \left( \lambda_{t+n-1}, x_{t+n-1} \right) \right].$$

Next, we iterate back, and condition on the information set at t + n - 2:

$$E_t \left[ E_{t+n-2} \left[ \exp \left\{ \beta_1^z J_{t+1} + \beta_2^z J_{t+2} + \dots + \beta_{n-1}^z J_{t+n-1} \right\} f_{n-1} \left( \lambda_{t+n-1}, x_{t+n-1} \right) \right] \right] =$$

$$E_{t} \left[ \exp \left\{ \beta_{1}^{z} J_{t+1} + \dots + \beta_{n-2}^{z} J_{t+n-2} \right\} E_{t+n-2} \left[ \exp \left\{ \beta_{n-1}^{z} J_{t+n-1} \right\} f_{n-1} \left( \lambda_{t+n-1}, x_{t+n-1} \right) \right] \right].$$

Note again that the internal expectation is only a function of  $\lambda_{t+n-2}$  and  $x_{t+n-2}$ . In particular:

$$E_{t+n-2} \left[ \exp \left\{ \beta_{n-1}^{z} J_{t+n-1} \right\} f_{n-1} \left( \lambda_{t+n-1}, x_{t+n-1} \right) \right] =$$

$$E_{t+n-2} \left[ \exp \left\{ \beta_{n-1}^{z} J_{t+n-1} \right\} f_{n-1} \left( \mu_{\lambda} + \alpha \lambda_{t+n-2} + \nu x_{t+n-2} + \chi J_{t+n-1}, \mu_{x} + \rho x_{t+n-2} + \phi J_{t+n-1} \right) \right]$$

$$= \left( 1 - \lambda_{t+n-2} \right) \left[ f_{n-1} \left( \mu_{\lambda} + \alpha \lambda_{t+n-2} + \nu x_{t+n-2}, \mu_{x} + \rho x_{t+n-2} \right) \right]$$

$$+ \lambda_{t+n-2} \left[ \exp \left\{ \beta_{n-1}^{z} \xi \right\} f_{n-1} \left( \mu_{\lambda} + \alpha \lambda_{t+n-2} + \nu x_{t+n-2} + \chi \xi, \mu_{x} + \rho x_{t+n-2} + \phi \xi \right) \right],$$

where the last expression reflects the fact that  $J_{t+n-1}$  is  $\xi$  with probability  $\lambda_{t+n-2}$  and zero otherwise. As before, call this function  $f_{n-2}(\lambda, x)$ :

$$f_{n-2}(\lambda, x) \equiv (1 - \lambda) \left[ f_{n-1} \left( \mu_{\lambda} + \alpha \lambda + \nu x, \mu_{x} + \rho x \right) \right]$$
$$+ \lambda \left[ \exp \left\{ \beta_{n-1}^{z} \xi \right\} f_{n-1} \left( \mu_{\lambda} + \alpha \lambda + \nu x + \chi \xi, \mu_{x} + \rho x + \phi \xi \right) \right].$$

We compute  $f_{n-2}(\lambda, x)$  on a grid of possible values for  $\lambda$  and x again and store it for later use. Note that we are using the function  $f_{n-1}$  that we had computed in the previous iteration to compute  $f_{n-2}$ .

Next, we iterate back once more, and condition on the information set at t + n - 3:

$$A_{z,n,t} = E_t \left[ \exp \left\{ \beta_1^z J_{t+1} + \beta_2^z J_{t+2} + \dots + \beta_{n-2}^z J_{t+n-2} \right\} f_{n-2} \left( \lambda_{t+n-2}, x_{t+n-2} \right) \right] =$$

$$E_t \left[ \exp \left\{ \beta_1^z J_{t+1} + \dots + \beta_{n-3}^z J_{t+n-3} \right\} E_{t+n-3} \left[ \exp \left\{ \beta_{n-2}^z J_{t+n-2} \right\} f_{n-2} \left( \lambda_{t+n-2}, x_{t+n-2} \right) \right] \right].$$

We keep iterating this way until we condition on the information set t, which only depends on  $\lambda_t$  (and  $x_t$ ).

### **Appendix References**

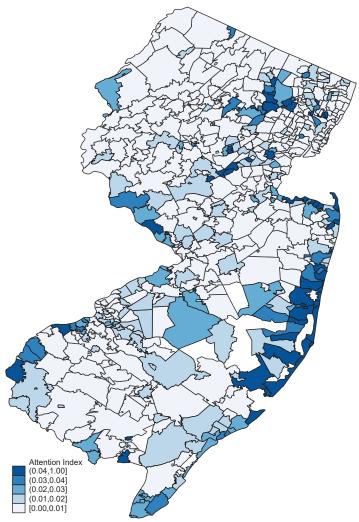
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## **Appendix Figures**

Figure A.1: Heatmap of Climate Attention Index in New Jersey

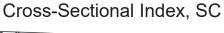
Cross-Sectional Index, NJ

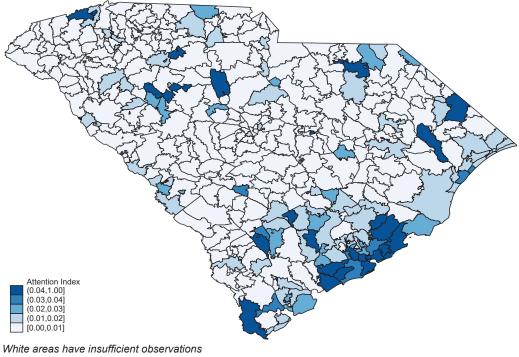


White areas have insufficient observations

Note: Figure shows a heatmap of our "Climate Attention Index" in New Jersey at the ZIP-code level. The "Climate Attention Index" is defined as the fraction of for-sale listings whose description includes climate-related text for the period from 2008Q1 to 2017Q2.

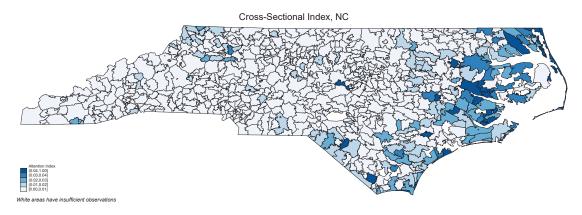
Figure A.2: Heatmap of Climate Attention Index in South Carolina





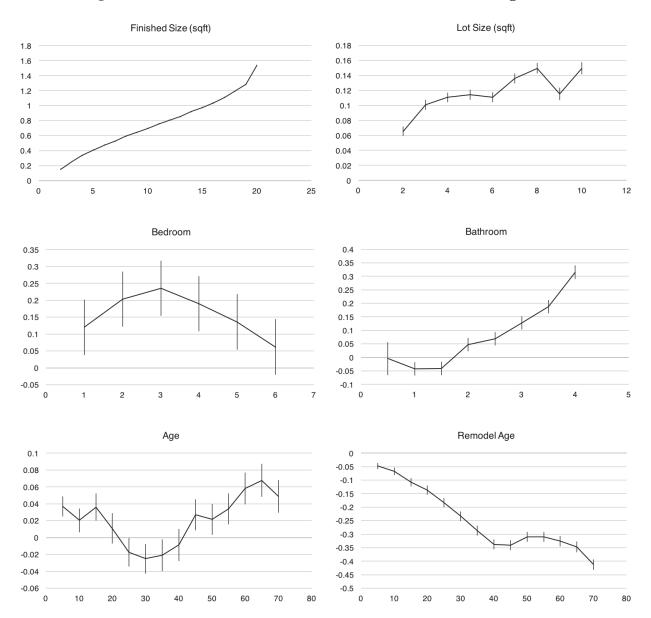
**Note:** Figure shows a heatmap of our "Climate Attention Index" in South Carolina at the ZIP-code level. The "Climate Attention Index" is defined as the fraction of for-sale listings whose description includes climate-related text for the period from 2008Q1 to 2017Q2.

Figure A.3: Heatmap of Climate Attention Index in North Carolina



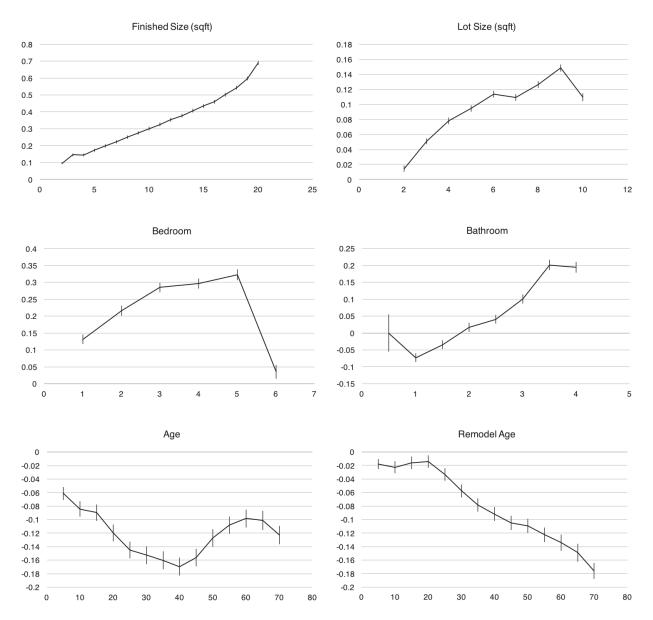
**Note:** Figure shows a heatmap of our "Climate Attention Index" in North Carolina at the ZIP-code level. The "Climate Attention Index" is defined as the fraction of for-sale listings whose description includes climate-related text for the period from 2008Q1 to 2017Q2.

Figure A.4: Hedonic Coefficients in Transaction Regression



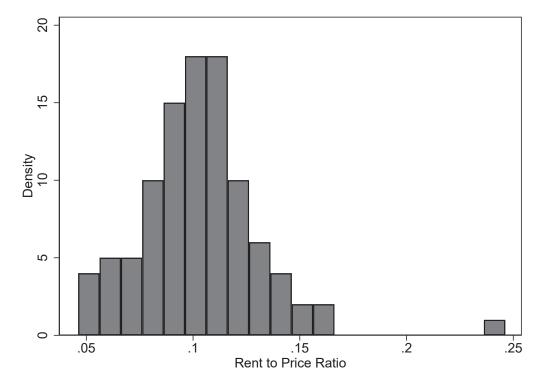
**Note:** Figures show coefficients on hedonic controls from regression 1. The dependent variable is the log price paid. Starting from the top left, the different panels show the coefficients on (i) indicators for ventiles of property size, (ii) indicators for deciles of lot size, (iii) indicators for the number of bedrooms, (iv) indicators for the number of bathrooms, (v) indicators on property age, and (vi) indicators on the time since the last major remodeling of the property. The regression includes other control variables and fixed effects as in Column 1 of Panel A, Table 1. The bars show 95% confidence intervals for standard errors clustered at the ZIP-code-quarter level.

Figure A.5: Hedonic Coefficients in Rental Regression



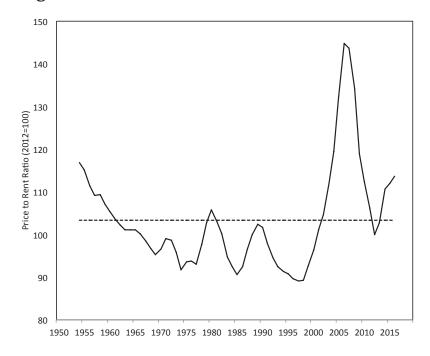
**Note:** Figures show coefficients on hedonic controls from regression 1. The dependent variable is the log of the rental listing price. Starting from the top left, the different panels show the coefficients on (i) indicators for ventiles of property size, (ii) indicators for deciles of lot size, (iii) indicators for the number of bedrooms, (iv) indicators for the number of bathrooms, (v) indicators on property age, and (vi) indicators on the time since the last major remodeling of the property. The regression includes other control variables and fixed effects as in column 1 of Panel B, Table 1. The bars show 95% confidence intervals for standard errors clustered at the ZIP-code-quarter level.

Figure A.6: Cross-Sectional Distribution of the Rent-to-Price Ratio in the U.S.



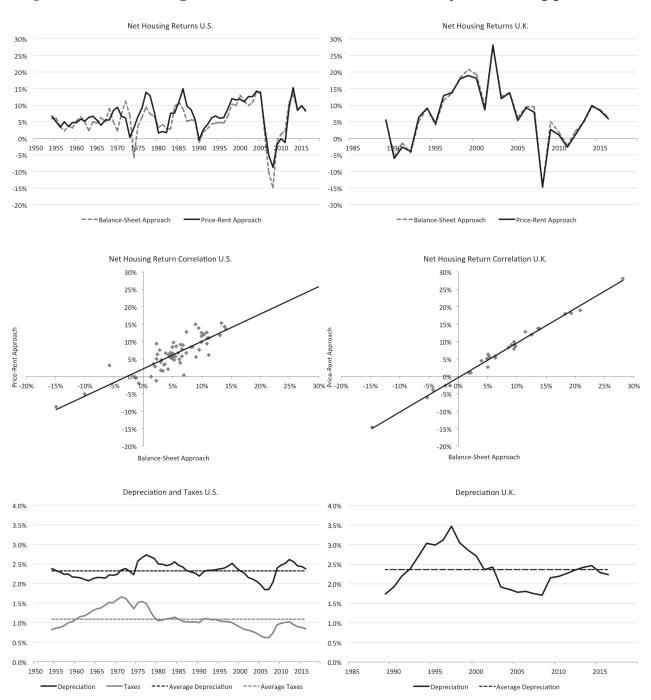
**Note:** The figure shows the distribution of the rent-to-price ratio for the 100 largest MSAs in the U.S. in 2012 as constructed by Trulia, which observes a large set of both for-sale and for-rent listings. It is constructed using a metro-level hedonic regression of log price on property attributes, ZIP-code fixed effects, and a dummy for whether the unit is for rent. The rent-to-price ratio is constructed by taking the exponent of the coefficient on this dummy variable.





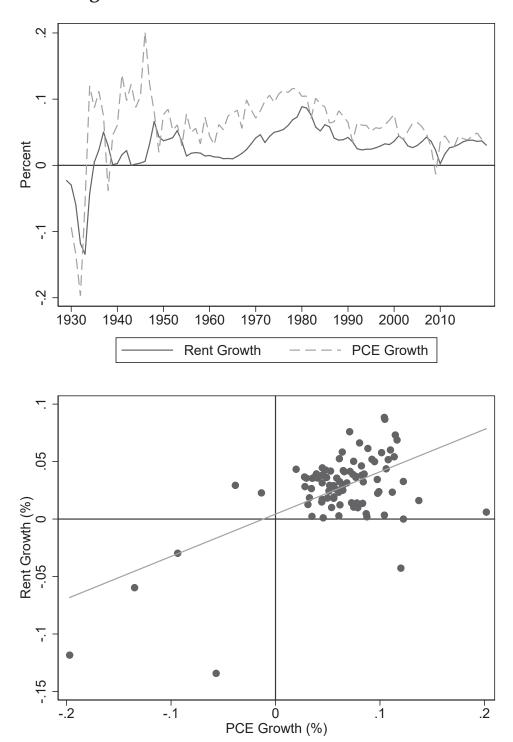
**Note:** The figure shows the time series of the price-rent ratio in the U.S., constructed as the ratio of the Case-Shiller House Price Index and a rental price index that is constructed as discussed in Section A.4.1. The index ratio is normalized to 100 in 2012.

Figure A.8: Housing Return Estimates – Consistency Across Approaches



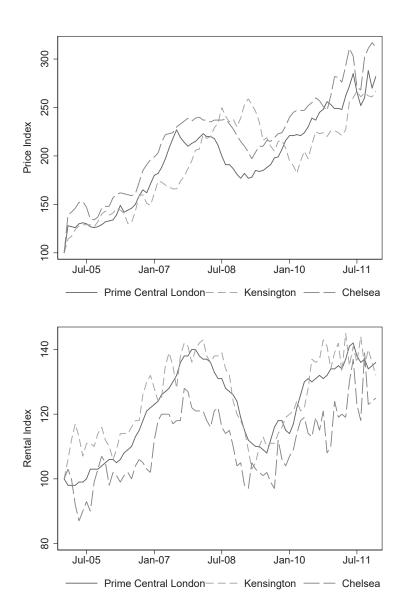
**Note:** Figures show the net housing returns for the balance-sheet and the price-rent approach for the U.S. and the U.K. (top row), the correlation between net housing returns from the balance-sheet and the price-rent approach for the U.S. and the U.K. (middle row), and housing depreciation (gross of maintenance) and tax yields from the balance-sheet approach for the U.S. and the U.K. (bottom row; there are no property taxes in the U.K.). The U.S. results are based on specifications (2) and (9) in Table 4. The U.K. results are based on specifications (12) and (15) in the same table.

**Figure A.9:** Rent Growth vs. PCE Growth in the U.S.



**Note:** The figure shows the annual growth rates of the "Consumer Price Index for All Urban Consumers: Rent of Primary Residence" (FRED ID: CUUR0000SEHA) and "Personal Consumption Expenditure" (FRED ID: PCECA) since 1929.

**Figure A.10:** House Prices and Rents in Prime Central London Areas during the 2007-09 Financial Crisis



**Note:** The figure shows the time series of house prices and rents for Prime Central London, Kensington, and Chelsea for the period January 2005 to January 2012. The series are monthly and available from John D Wood & Co. at http://www.johndwood.co.uk/content/indices/london-property-prices/, last accessed February 2014.

# **Appendix Tables**

**Table A.1:** Dictionary for Climate Attention Index

Text Type	Text
Single Words	'storm','storms','superstorm','hurricane','hurricanes', 'fema', 'tornado', 'tornadoes','floodplain'
Pairs	('flood','risk'),('flood','insurance'),('flood','ins'),('flood','plain'),('flood','risk'), ('flood','damage'),('flood','zone'),('flood','zones'),('flood','protection'), ('flood','safe'),('hurricane','zone'),('hurricane','shutter'), ('hurricane','shutters'),('hurricane','shelter'),('hurricane','shelters'), ('hurricane','protection'),('hurricane','safe'),('hurricane','impact'), ('hurricane','curtains'), ('sea','level'),('storm','zone'),('storm','zones'), ('storm', 'window'),('storm', 'windows'),('storm','door'),('storm','doors'), ('storm','water'),('storm','protection'),('storm','safe'),('tornado','shutter'), ('tornado','shutters'),('tornado','shelters')
Hurricane Names	'keith','allison','iris','michelle','isidore','lili','fabian','isabel','juan','charley', 'frances','ivan','jeanne','dennis','katrina','rita','stan','wilma','dean','felix', 'noel','gustav','ike','paloma','igor','tomas','irene','sandy','ingrid','erika', 'joaquin','matthew','otto'

**Note:** The table shows the dictionary used to construct the "Climate Attention Index".

Table A.2: Top Climate Words in Florida

Words	Number of Listings Containing the Word	Frequency
hurricane(s)	465,308	3.309%
hurricane shutter(s)	241,812	1.720%
storm(s)	114,893	0.817%
hurricane impact	66,485	0.473%
flood insurance	57,737	0.411%
flood zone(s)	45,696	0.325%
hurricane protection	18,285	0.130%
storm door(s)	13,286	0.094%
storm window(s)	7,692	0.055%
storm protection	5,644	0.040%
sea level	3,808	0.027%
FEMA	3,448	0.025%
hurricane safe	1,798	0.013%
flood plain	1,684	0.012%
storm water	971	0.007%
hurricane shelter(s)	603	0.004%
storm safe	491	0.003%
tornado(es)	457	0.003%
flood risk	373	0.003%
flood damage	235	0.002%
hurricane zone(s)	178	0.001%
hurricane curtains	171	0.001%
flood protection	117	0.001%
tornado shelter(s)	74	0.001%
storm zone(s)	30	0.000%
flood safe	9	0.000%
tornado shutter(s)	1	0.000%
Total number of listings	14,059,936	

**Note:** The table shows the most commonly occurring words signaling increased attention paid to climate change in the state of Florida.

Table A.3: Top Climate Words in New Jersey

Words	Number of Listings Containing the Word	Frequency
storm(s)	20,702	0.602%
flood insurance	15,342	0.446%
flood zone(s)	14,354	0.417%
storm door(s)	10,020	0.291%
hurricane(s)	5,842	0.170%
FEMA	5,253	0.153%
storm window(s)	2,316	0.067%
flood risk	1,395	0.041%
flood damage	834	0.024%
flood plain	678	0.020%
superstorm	529	0.015%
storm water	369	0.011%
sea level	326	0.009%
hurricane shutter(s)	213	0.006%
hurricane impact	68	0.002%
storm protection	27	0.001%
flood protection	25	0.001%
storm zone(s)	17	0.000%
hurricane protection	9	0.000%
storm safe	9	0.000%
flood safe	3	0.000%
hurricane zone(s)	2	0.000%
Total number of listings	3,441,094	

**Note:** The table shows the most commonly occurring words signaling increased attention paid to climate change in the state of New Jersey.

Table A.4: Top Climate Words in North Carolina

Words	Number of Listings Containing the Word	Frequency
storm(s)	18,160	0.376%
flood zone(s)	11,788	0.244%
storm door(s)	11,075	0.229%
flood insurance	8,587	0.178%
hurricane(s)	5,232	0.108%
storm window(s)	4,161	0.086%
flood plain	4,215	0.087%
hurricane shutter(s)	2,639	0.055%
sea level	1,169	0.024%
FEMA	635	0.013%
storm water	385	0.008%
tornado(es)	211	0.004%
storm protection	107	0.002%
hurricane protection	74	0.002%
hurricane impact	74	0.002%
flood damage	55	0.001%
tornado shelter(s)	42	0.001%
flood risk	32	0.001%
flood protection	22	0.000%
hurricane shelter(s)	22	0.000%
storm safe	10	0.000%
hurricane safe	8	0.000%
hurricane zone(s)	6	0.000%
storm zone(s)	1	0.000%
Total number of listings	4,827,756	

**Note:** The table shows the most commonly occurring words signaling increased attention paid to climate change in the state of North Carolina.

Table A.5: Top Climate Words in South Carolina

Words	Number of Listings Containing the Word	Frequency
storm(s)	11,354	0.472%
hurricane(s)	7,243	0.301%
storm door(s)	6,406	0.266%
flood insurance	5,340	0.222%
flood zone(s)	3,848	0.160%
hurricane shutter(s)	2,531	0.105%
storm window(s)	2,305	0.096%
flood plain	614	0.026%
sea level	422	0.018%
hurricane protection	343	0.014%
FEMA	300	0.012%
hurricane impact	182	0.008%
tornado(es)	175	0.007%
flood damage	165	0.007%
storm water	160	0.007%
hurricane zone(s)	103	0.004%
storm protection	101	0.004%
tornado shelter(s)	97	0.004%
hurricane shelter(s)	29	0.001%
hurricane safe	21	0.001%
flood risk	19	0.001%
storm safe	18	0.001%
flood safe	8	0.000%
flood protection	3	0.000%
Total number of listings	2,406,832	

**Note:** The table shows the most commonly occurring words signaling increased attention paid to climate change in the state of South Carolina.

**Table A.6:** Rent-to-Price Ratio Singapore - 2012

	(1)	(2)	(3)	(4)
For-Rent Dummy	-3.095*** (0.044)	-3.131*** (0.019)	-3.123*** (0.014)	-3.107*** (0.025)
Fixed Effects	Quarter × Postal Code	Quarter × Postal Code	$\begin{array}{c} {\rm Month} \times \\ {\rm Postal}  {\rm Code} \end{array}$	$\begin{array}{c} {\rm Month} \times \\ {\rm Postal} \ {\rm Code} \times \\ {\rm Bedrooms} \end{array}$
Controls		✓	✓	✓
Implied Rent-to-Price Ratio	4.5%	4.4%	4.4%	4.5%
R-squared N	0.804 106,145	0.873 105,189	0.872 105,189	0.872 105,189

**Note:** This table shows results from regression (A.6). The rent-to-price ratio is constructed by taking the exponent of the coefficient on this dummy variable. The dependent variable is the price (for-sale price or annualized for-rent price) for properties listed on iProperty.com in Singapore in 2012. Fixed effects are included as indicated. In columns 2 to 4, we also control for characteristics of the property: we include dummy variables for the type of the property (condo, house, etc.), indicators for the number of bedrooms and bathrooms, property age, property size (by adding dummy variables for 50 equal-sized buckets), information on the kitchen (ceramic, granite, etc.), which floor the property is on, and the tenure type for leaseholds. Standard errors are clustered at the level of the fixed effect. Significance levels are as follows: \* (p<0.10), \*\* (p<0.05), \*\*\* (p<0.01).

Table A.7: House Prices, Banking Crises, Rare Disasters

	House Price Index Time Period	Banking Crises	Rare Disasters
Australia	1880 - 2013	1893, 1989	1918, 1932, 1944
Belgium	1975 - 2012	2008	
Canada	1975 - 2012		
Denmark	1975 - 2012	1987	
Finland	1975 - 2012	1991	1993
France	1840 - 2010	1882, 1889, 1907, 1930, 2008	1871, 1915, 1943
Germany	1975 - 2012	2008	
Italy	1975 - 2012	1990, 2008	
Japan	1975 - 2012	1992	
Netherlands	1649 - 2009	1893, 1907, 1921, 1939, 2008	1893, 1918, 1944
New Zealand	1975 - 2012	1987	
Norway	1819 - 2013	1899, 1922, 1931, 1988	1918, 1921, 1944
Singapore	1975 - 2012	1982	
South Africa	1975 - 2012	1977, 1989	
South Korea	1975 - 2012	1985, 1997	1998
Spain	1975 - 2012	1978, 2008	
Sweden	1952 - 2013	1991, 2008	
Switzerland	1937 - 2012	2008	1945
U.K.	1952 - 2013	1974, 1984, 1991, 2007	
U.S.	1890 - 2012	1893, 1907, 1929, 1984, 2007	1921, 1933

**Note:** The table shows the time series of house price indices used in the first column. The second and third columns report dates of banking crises or rare consumption disasters for each country in the time period provided in the first column. Banking crisis dates for all countries, except Singapore, Belgium, Finland, New Zealand, South Korea, and South Africa, are from Schularick and Taylor (2012). Banking crisis dates for the countries not covered by Schularick and Taylor (2012) are from Reinhart and Rogoff (2009). Rare disaster dates indicate the year of the trough in consumption during a consumption disaster as reported by Barro and Ursua (2008).

**Table A.8:** Parameters of the Calibrated Model

Calibrated Variables		Value
δ	Time discount rate	0.99
$\gamma$	Risk aversion	10
μ	Average consumption growth	2%
ρ	Consumption growth persistence	0.85
φ	Consumption growth after disaster	0.025
η	Exposure of rents to disaster	3
$\dot{\omega}$	Rent growth persistence	0.915
ψ	Rent growth after disaster	0.24
$\dot{\bar{\lambda}}$	Unconditional mean of disaster probability	3%
α	Disaster probability persistence	0.75
ν	Relation between disaster probability and consumption growth	0.1
χ	Exposure of disaster probability to disaster	0.05
ξ	Consumption drop after disaster	21%

**Note:** The table summarizes the calibration of the model in Section 2. The time discount rate  $\delta$ , risk aversion  $\gamma$ , drop in consumption following a disaster  $\xi$ , exposure of risky cash flows to the climate shock  $\eta$ , and average consumption growth in the absence of a disaster  $\mu$  are set following the standard asset pricing literature. All other parameters are calibrated to match some of our new moments estimated in Section 1. The remaining parameters of the consumption process are chosen to generate a recovery in consumption growth after disasters ( $\phi > 0$ ) and persistent growth rates ( $\rho > 0$ ). The magnitude of these parameters targets a term structure of real interest rates that is slightly upward-sloping with a level of around 1.0%. The remaining parameters of the rent process are chosen to generate a recovery in rent growth after disasters ( $\psi > 0$ ) and persistent rent growth ( $\omega > 0$ ). The magnitudes of these parameters are chosen to match the shape and the level of the observed term structure of discount rates in the housing market as described in Section 1. The steady-state conditional probability of disasters,  $\bar{\lambda}$  is set based on estimates in Barro (2006), and the remaining parameters for the  $\lambda$ -process are chosen to obtain economically reasonable interactions between the real economy and the disaster probability, while at the same time matching the term structure of the risk-free rate (which is directly affected by the disaster probability dynamics through the precautionary savings channel). In particular, the disaster probability is persistent ( $\alpha$ ), increases after a jump  $(\chi)$ , and increases when expected consumption growth is above its trend  $(\nu)$ . x and y are assumed to have mean zero, which implies:  $\mu_x = -\bar{\lambda}\phi\xi$  and  $\mu_y = -\bar{\lambda}\psi\xi$ . The unconditional mean of  $\lambda$  pins down  $\mu_{\lambda}$  as  $\bar{\lambda} = \frac{\mu_{\lambda}}{1-\alpha-\chi\bar{\zeta}} > 0$ . Consumption and rents are assumed to have the same long-run growth rates, requiring:  $\mu_d = \mu + (\eta - 1) \bar{\lambda} \xi$ . Further details of the calibration are discussed in Section 2.2. Parameter restrictions are discussed in Section A.6.1.