B  Additional extensions

B.1 Convex trading costs

Key to the results in the paper is the assumption that there is some friction that stands in the way of information aggregation in financial markets. In the main text, such frictions are parsimoniously captured by a linear trading cost incurred by traders (together with a wealth constraint that prevents unlimited positions). In this section, we replace the assumption of a linear trading cost by the assumption that trading costs are convex and show that our main insights remain unchanged, namely: (i) debt issuance boosts information aggregation; (ii) if there is margin to improve information aggregation\(^1\), risky debt is always optimal for firms with good prospects \((X^* \leq 0)\), and it is optimal for firms with projects that are not ex-ante profitable \((X^* > 0)\) if trading costs are not too high.

In particular, we assume that if a trader submits an order to buy (or sell, if negative) \(s \in \mathbb{R}\) dollars in stocks, it now incurs a cost \(\tau|s|^\gamma\), where \(\tau > 0\) and \(\gamma > 1\). We remove the order-size limit, \(s \in [-1, 1]\), since the assumed convex trading costs are enough to prevent informed traders from trading infinite amounts (besides, it also makes the analysis more tractable with convex costs). All the remaining assumptions and notation are as in the main text.

Trading profits per dollar traded, excluding trading costs, are given by the same function \(\pi(\kappa)\) as before. Hence, for a given \(\kappa\), a trader \(i\) receiving good news chooses \(s_i\) as to maximize:

\[
s_i \pi(\kappa) - \tau |s_i|^\gamma,
\]

which implies that it chooses \(s_i = \left[ \frac{\pi(\kappa)}{\gamma \tau} \right]^\frac{1}{1-\gamma}\). Similarly, a trader receiving bad news chooses \(s_i = -\left[ \frac{\pi(\kappa)}{\gamma \tau} \right]^\frac{1}{1-\gamma}\). Hence, conditional on the high and low state, aggregate orders from informed

\(^1\)Here, as discussed in more details below, there are is no exogenous limit to trading, so there is always margin for improving information aggregation, differently from the main model with trading limits.
Figure 1: Equilibrium in the trading stage for a given $\overline{X}$ under convex trading costs. The figure illustrates the computation of equilibrium for different parameter cases. The solid line is the right-hand side of (B.1), and the dashed line is the 45-degree line.

Theorem B.1. If $\overline{X} \leq 0$, there is a unique equilibrium in the trading stage, and $\kappa^* > 0$. If $\overline{X} > 0$, there is always an equilibrium with $\kappa^* = 0$, and there exists $\tilde{\tau} > 0$ such that an equilibrium with $\kappa^* > 0$ exists if and only if $\tau \leq \tilde{\tau}$.

Proof. Suppose first that $\overline{X} \leq 0$. Since $\pi(0) = 0$, the RHS of (B.1) is strictly decreasing. Since $\lim_{\kappa \to 0} \pi(\kappa) = 2\lambda - 1$, we have that the equilibrium in the trading stage exists and is unique, as the LHS of (B.1) is the 45-degree line.

Next proposition characterizes the equilibrium in the trading stage under convex trading costs, and Figure 1 illustrates.

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Now suppose $\overline{X} > 0$. First, note that since $\pi(0) = 0$, $\kappa^* = 0$ is always a solution to (B.1). Also, since $\pi(\kappa) > 0$ for all $\kappa > 0$, it is trivial that (B.1) must hold for some $\kappa > 0$ for low enough $\tau$. Now note that $\pi(\kappa)$ is bounded from above by $2\lambda - 1$ (see (11)). Hence, for large enough $\tau$, (B.1) cannot hold for any $\kappa > 0$ since the RHS of (B.1) is decreasing in $\tau$ and $\lim_{\tau \to \infty} \pi(\kappa) / \gamma \tau = 0$ for any $\kappa > 0$. Let $\tilde{\tau}$ be the lowest value of $\tau$ such that (B.1) holds at some $\kappa > 0$. Given the continuity of $\pi(\kappa)$ and the fact that the RHS of (B.1) decreases in $\tau$, (B.1) must also hold at some $\kappa > 0$ for any $\tau < \tilde{\tau}$.

In what follows, we denote by $D^*$ the optimal value of $D$ chosen by the firm. Also, for convenience
and with some abuse of notation, we sometimes write $\pi(\kappa, \bar{X}, D)$, which represents the RHS of (13) for a given $D$ when setting $A = [\bar{X}, \infty]$, and write firm value as $V(\bar{X}, \kappa)$. The next propositions characterize the optimal capital structure under convex trading costs.

**Proposition B.2.** Suppose $\bar{X}^* \leq 0$. Then, the optimal capital structure involves risky debt, $D^* > V_L$.

**Proof.** We first claim that issuing $D = V_L$ yields a weakly larger firm value than any $D < V_L$ (which we prove for any $\bar{X}^*$, for future reference). To see that, first recall that $\bar{X}(D) = \bar{X}^*$ for $D \leq V_L$. Next, for $D < V_L$, any increase in $D$ increases $\pi(\kappa, \bar{X}^*, D)$ (and therefore the RHS of (B.1)) for all $\kappa > 0$. Thus, such an increase in $D$ cannot reduce $\kappa$ in the largest equilibrium, since in a largest equilibrium the RHS of (B.1) cannot cross the 45-degree line from below. Finally, firm value is increasing in $\kappa$ when $\bar{X} = \bar{X}^*$ (by Lemma A.2), which concludes the argument.

Suppose $\bar{X}^* \leq 0$. Since $\pi(\cdot)$ is strictly decreasing in $\kappa$ and $\bar{X}$, any increase in $D$ for $D \geq V_L$ reduces $\bar{X}(D)$ (see (5)) and increases $\kappa$ in equilibrium. The effect of a marginal change in $D$ on firm value is captured by the same expression as in (A.15). Since $\bar{X}(V_L) = \bar{X}^*$ and, by Lemma A.2, $\frac{\partial V}{\partial X} \bigg|_{X=\bar{X}^*} = 0$ and $\frac{\partial V}{\partial \kappa} \bigg|_{X=\bar{X}^*} > 0$, a marginal increase in $D$ at $D = V_L$ strictly increases $\kappa$ and thus $V$, so $D^* > V_L$. 

Proposition B.2 shows that the key insight of Proposition 2 extends to the setting with convex trading costs: a firm with good investment opportunities ($\bar{X}^* \leq 0$) has incentives to issue risky debt to boost information aggregation. In fact, in this extension, the firm always issues some risky debt since it always has some margin to increase information aggregation. The source of this qualitative difference among the two propositions is that in this extension we have not only replaced a linear trading cost by a convex cost, but we have also eliminated the wealth constraints on speculators, so there is no exogenous bound on the potential for information aggregation. In the case of Proposition 2, for low trading costs, there was no benefit in issuing risky debt since by only issuing safe debt the maximum level of information aggregation was already achieved, without the need to distort the managerial use of information. If we were to reinstate wealth constraints, in the case of convex trading costs, the same feature would be present.

**Proposition B.3.** Suppose $\bar{X}^* > 0$. Then, there exists $\tau^*$ such that:

1. For $\tau < \tau^*$ the optimal capital structure involves risky debt, $D^* > V_L$;

2. For $\tau > \tau^*$, the firm chooses any $D$ that leads to $\kappa^* = 0$, and in particular, any level of safe debt is optimal.

**Proof.** Suppose $\bar{X}^* > 0$. As shown in the first paragraph of the proof of Proposition B.2, $D = V_L$ is always weakly preferred by the firm to any $D < V_L$. Consider $D = V_L$. By Proposition B.1,
there exists $\bar{\tau}$ such that for any $\tau \leq \bar{\tau}$ there is an equilibrium with $\kappa > 0$. Consider $\tau \leq \bar{\tau}$. Since $\lim_{\kappa \to \infty} \pi \left( \kappa, X^*, D \right) = 0$, increasing $D$ at $D = V_L$ must strictly increase $\kappa$ in the largest equilibrium in that case. Hence, since $X(V_L) = X^*$ and, by Lemma A.2, $\frac{\partial V}{\partial \lambda} \bigg|_{X=X^*} = 0$ and $\frac{\partial V}{\partial \kappa} \bigg|_{X=X^*} > 0$, a marginal increase in $D$ at $D = V_L$ strictly increases $\kappa$ and thus $V$, so $D^* > V_L$. The firm sets $D$ as to solve the problem: $\max_{D > V_L} V(X^*(D), \kappa')$ subject to $\kappa' > 0$ being the largest root of $\kappa' = \frac{2\alpha^2(2\lambda - 1)^2}{\sigma^2} \left[ \frac{\gamma(X^*(D),D)}{\gamma^2} \right]^{\frac{2}{\gamma^2}}$. For future reference, denote by $D'$ the solution to this problem, by $\kappa'^*$ the (largest) equilibrium value of $\kappa$ when $D = D'$, and let $V' \equiv V(X^*(D'), \kappa'^*)$.

Now consider $\tau > \bar{\tau}$. At $D = V_L$, the unique equilibrium features $\kappa = 0$. A sufficient increase in $D$ leads to a (largest) equilibrium with $\kappa > 0$ since $\lim_{D \to V_0} \pi \left( \kappa, X(D) \right) = -\infty$ and $\lim_{D \to V_0} \pi \left( \kappa, X(D), D \right) = 2\lambda - 1 > 0$ for all $\kappa$. Analogously to what happens in the case of Proposition 3, the firm then compares $V'$ (the maximum possible firm value with a positive $\kappa$, as previously defined) with $V_0$ to determine the optimal $D$.

We now show that $V'$ is decreasing in $\tau$. By the envelope theorem, we have that $\frac{\partial V'}{\partial \tau} = \frac{\partial V}{\partial \kappa} (X^*(D'), \kappa'^*)$. Since $X^*(D)$ does not directly depend on $\tau$, $\frac{\partial V}{\partial \kappa} = \frac{\partial V}{\partial \kappa} \frac{\partial \kappa}{\partial \tau}$. Suppose by contradiction that $\frac{\partial V}{\partial \kappa} (X^*(D'), \kappa'^*) \leq 0$. Then the firm could marginally reduce $D$, increasing $X^*(D)$ and (weakly) reducing $\kappa$ in the largest equilibrium, which would strictly increase firm value, a contradiction. We then have $\frac{\partial V}{\partial \kappa} (X^*(D'), \kappa'^*) > 0$. Now note that $\frac{\partial \kappa}{\partial \tau}$ must be negative in any largest equilibrium with interior $\kappa$. This is easy to see by rewriting (B.1) as

$$\left( \gamma \tau \right)^{2/(\gamma - 1)} \kappa = \frac{2\alpha^2 (2\lambda - 1)^2}{\sigma^2} \pi \left( \kappa \right)^{2/(\gamma - 1)}.$$  \hspace{1cm} (B.2)

Since $\pi \left( 0 \right) = 0$ and $\lim_{\kappa \to \infty} \pi \left( \kappa \right) = 0$, if there is a single positive root to (B.2), increasing $\tau$ leads to a largest equilibrium with $\kappa = 0$. If there is more than one root to (B.2), at the largest one the RHS crosses the LHS of (B.2) from above, and therefore increasing $\tau$ reduces $\kappa$ in the largest equilibrium. This concludes the proof that $\frac{\partial V'}{\partial \tau} = \frac{\partial V}{\partial \kappa} (X^*(D'), \kappa'^*) < 0$. Finally, recall that for $\tau = \bar{\tau}$, $V(X^*(D'), \kappa'^*) > V_0$, since $D^* = D'$ in that case. Also note that, as $\tau \to \infty$, $\kappa'^* \to 0$ and $V(X^*(D'), \kappa'^*) \to \frac{V_D + V_L}{2} < V_0$. Therefore, there exists $\tau^* > \bar{\tau}$ such that $D'$ is optimal if and only if $\tau < \tau^*$. If $\tau > \tau^*$, any $D$ that leads to a largest equilibrium with $\kappa = 0$ is optimal.

In the case of $X^* > 0$, the optimal capital structure result is also qualitatively very similar to the case of linear trading costs (Proposition 3). The main difference, again, arises from the elimination of trading limits. If the potential for information aggregation was bounded, for sufficiently low $\tau$, naturally there would be another region where safe debt would be optimal, as is the case in Proposition 3.

We conclude the exposition of the model with convex trading costs with a discussion of how debt affects the probability of bad and good investments. Using the fact that when $D \geq V_L$ trading profits per dollar traded can be written as $\pi \left( \kappa \right) = (2\lambda - 1) \left[ 1 - \Phi \left( \frac{\sqrt{2\kappa}}{\sqrt{\lambda V}} \right) \right] = (2\lambda - 1) \Pr \left( a = 1 \big| \theta = \theta_L \right)$
and the equilibrium condition (B.1), we can write:

\[ \Pr (a = 1|\theta = \theta_L) = \frac{\gamma^\tau}{(2\lambda - 1)^\gamma} \left( \frac{\kappa \sigma^2}{2\alpha^2} \right)^{\gamma - 1}. \]

Hence, any increase in risky debt that leads to an increase in \( \kappa^* \) will end up increasing the probability of a bad investment. Intuitively, we can decompose the effect of risky debt on the probability of bad investments in two components:

\[
\frac{d\Pr (a = 1|\theta = \theta_L)}{dD} = \left( \frac{\partial \Pr (a = 1|\theta = \theta_L)}{\partial X} \frac{dX}{dD} \right)_{\text{Risk-shifting (direct effect)}} + \left( \frac{\partial \Pr (a = 1|\theta = \theta_L)}{\partial \kappa^*} \frac{d\kappa^*}{dD} \right)_{\text{Stock market feedback (indirect effect)}}.
\]

When the firm is issuing risky debt, the first effect is always positive, and represents the usual risk-shifting channel: fixing the information set, the manager is more likely to invest in any state. The indirect effect captures how changes in debt affect the information obtained by the manager. In the model with linear trading costs, the indirect effect was strong enough to fully offset the direct effect, and the probability of investment in the low state remained constant after an increase in risky debt (starting from an interior equilibrium). In the model with convex trading costs, the indirect effect will not fully offset the direct effect, since \( \Pr (a = 1|\theta = \theta_L) \) actually increases after an increase in risky debt that leads to an increase in \( \kappa^* \). Intuitively, convex costs make \( \kappa^* \) less responsive to increases in traders’ incentives to trade, since part of the benefits of placing higher orders is offset by an increase in the marginal cost of trading. The opposite conclusion could be reached if we had some concavity in the cost function, say, because of some returns to scale in trading, which could make \( \kappa^* \) even more responsive than in the linear case.

Despite \( \Pr (a = 1|\theta = \theta_L) \) increasing with risky debt, our results that the optimal capital structure often requires the issuance of risky debt still holds, as we have shown in Propositions B.2 and B.3. This is so because the increase in the probability of good investments being undertaken, \( \Pr (a = 1|\theta = \theta_H) \), still has a higher impact on firm value than the increase in \( \Pr (a = 1|\theta = \theta_L) \), starting from \( D = V_L \) and an interior \( \kappa^* \).\(^2\) Intuitively, if the firm issues no risky debt (say, \( D = V_L \)) and that yields some \( \kappa^* > 0 \), the marginal cost of distorting the use of information is zero since the information obtained is being used as efficiently as possible (\( X(V_L) = X^* \)). The marginal benefit of obtaining extra information is positive in this situation, and therefore the firm will always find it optimal to issue some risky debt.

\(^2\)The fact that \( \Pr (a = 1|\theta = \theta_H) \) goes up after an increase in \( D \), starting from \( D = V_L \) and an interior \( \kappa^* \), is a corollary of the fact that \( \Pr (a = 1|\theta = \theta_L) \) increases (as discussed in the text) and \( V \) increases (as shown in the proofs of Propositions B.2 and B.3).
B.2 Debt overhang

In the main text, we characterize the optimal capital structure in a setting where, fixing market informativeness, risky debt leads to a problem of risk-shifting: our equity-value maximizer manager sometimes undertakes investment projects that a firm-value maximizer decision maker would not undertake. In other words, if the firm issues a sufficiently large amount of debt, our manager sometimes invests in projects that have negative NPV. However, as discussed in the debt overhang literature (Myers, 1977; Diamond and He, 2014), risky debt may also lead to an opposite problem: if a firm has too much debt, a manager may forego investment opportunities that have positive NPV.

Davis and Gondhi (2021) argue that the nature of the agency conflict created by debt (risk-shifting or debt overhang) can affect some predictions of models with stock market feedback. In this section we ask whether the nature of the agency conflict faced by firms matter for our results regarding the optimal capital structure. We argue that it does not, as explained below.

In our setting with a binary state, whether risky debt leads to risk-shifting or debt overhang depends on whether the investment opportunity increases or decreases the volatility of the firm cash flow. In our main setting, undertaking the investment opportunity increases the variance of firm’s cash flows. We now modify our assumptions about the firm’s asset side, so that the investment opportunity reduces the volatility of the firm’s cash flow, thus creating the possibility of debt overhang, using the terminology of Davis and Gondhi (2021). In particular, we now assume that the firm has assets in place that pay a state contingent cash flow $V$ given by

$$V = \begin{cases} V_H & \text{if } \theta = \theta_H, \\ V_L & \text{if } \theta = \theta_L, \end{cases} \quad (B.3)$$

where $V_H > V_L > 0$. The firm has access to an investment opportunity that pays a cash flow given by

$$Z = \begin{cases} Z_H & \text{if } \theta = \theta_H, \\ Z_L & \text{if } \theta = \theta_L, \end{cases}$$

where $Z_H < 0$ and $Z_L > 0$, meaning that investment is desirable only if the state is low. The firm’s final cash flow is then given by $V + Z$ if the firm invests, and $V$ otherwise. The payoff of the investment opportunity is therefore negatively correlated with the payoff of assets in place, and so, importantly, investing leads to a reduction in the volatility of the firm’s cash flow. One can think of such an investment project as an opportunity to diversify the firm’s operations.

For simplicity, we focus on the extreme case of debt overhang, assuming that investment eliminates all volatility from the firm cash flow, and therefore $V_H + Z_H = V_L + Z_L \equiv V_0 > 0$. This implies that firm cash flow is given by $V_0$ if the firm invests and is given by $V$ in (B.3) if the firm does not invest. Note that this model with debt overhang is isomorphic to the model with risk-shifting, and for our
results to apply, we only need to relabel what “taking the risky action” means. In our main model, taking the risky action meant investing, while now taking the risky action means not investing (since investment is a hedge). Hence, all of our previous results continue to hold if we refer to “investing” as “not investing” and vice versa, and redefine \(a = 1\) as not investing and \(a = 0\) as investing. In particular, the optimal capital structure (presented in Propositions 2 and 3) is the same. Note that now the manager refrains from investing whenever she observes a high enough aggregate order (but this means that she still “takes the risky action” for a large enough \(X\)).

B.3 General security design

In the model of Section 2, the firm was allowed to issue debt and equity securities only. This assumption is motivated by the fact that those are the most common types of securities that companies issue. However, this is not key for our results. In this section, we generalize our main model, allowing the firm to split its cash flow between equity (the residual-claimant security) and another general security in any arbitrary way. As will be shown, the firm has no incentives to issue securities different from debt and equity.

We define a general security as a triple \((Y_H, Y_L, Y_0)\), where \(Y_H \in [0, V_H]\), \(Y_L \in [0, V_L]\) and \(Y_0 \in [0, V_0]\) denotes how much this asset pays if the firm’s cash flow is \(V_H\), \(V_L\) and \(V_0\), respectively. Note that we are assuming limited liability (assets cannot make negative payments), as in Allen and Gale (1988). Equity holders are residual claimants, and hence stock dividends are \(R_H = V_H - Y_H\), \(R_L = V_L - Y_L\) and \(R_0 = V_0 - Y_0\). Debt is then a special case of a general security. The model is as in Section 2, except that now the firm is allowed to issue any general security at \(t = 0\). Before showing that any optimum can be implemented via debt, we prove the following auxiliary result.

**Lemma B.1.** Consider the problem of a firm that can choose any general security at \(t = 0\). If this problem has a solution, then there is always an optimal security that leads to \(R_H > R_L\).

**Proof.** We perform a change of variables and think of the problem as that of a firm that chooses \(R_L \in [0, V_L]\), \(R_H \in [0, V_H]\) and \(R_0 \in [0, V_0]\). For a given choice \((R_L, R_H, R_0)\), the manager optimally invests whenever

\[
\mu(X) R_H + (1 - \mu(X)) R_L \geq R_0.
\]

Following the text, \(A \subseteq \mathbb{R}\) represents the set of orders that satisfy the equation above, given a choice \((R_L, R_H, R_0)\). We also define the set \(\mathcal{O} = \{z \in [0, 1] : \mu^{-1}(z) \in A\}\). That is, \(\mathcal{O}\) represents the posterior probabilities assigned to the high state under which the manager invests. Define the sets \(\mathcal{N} = \left[0, \frac{V_0 - V_L}{V_H - V_L}\right)\) and \(\mathcal{E} = \left[\frac{V_0 - V_L}{V_H - V_L}, 1\right]\) and write \(\mathcal{O} = \mathcal{O}_E \cup \mathcal{O}_N\), where \(\mathcal{O}_E = \mathcal{O} \cap \mathcal{E}\) and \(\mathcal{O}_N = \mathcal{O} \cap \mathcal{N}\). That is, \(\mathcal{O}_E\) represents the set of posteriors under which the manager invests and it is ex-post efficient, and \(\mathcal{O}_N\) represents the posteriors under which ex-post inefficient investment happens. For a given \(\kappa\), the unconditional distribution of \(X\) only depends on \(\kappa\) (see (A.12)) and hence the distribution of the
Applying the change of variables with such deviation (Lemma A.2). If it yields $R^*$.

**Case 3:** $R_0^* \leq R_H^*$. Suppose first that $R_0^* \leq R_H^*$, in which case the manager always invests. If $\overline{X}^* > 0$ the firm would be better off by setting $R_H = R_L < R_0$, so that the manager never invests. If $\overline{X}^* \leq 0$, then expected firm value would weakly increase if the firm chose $R_H = V_H$, $R_L = V_L$ and $R_0 = V_0$ (which we refer hereafter as ex-post firm value maximization). To see that, notice that if ex-post firm value maximization yields $\kappa > 0$ in equilibrium, the firm payoff strictly increases with such deviation (Lemma A.2). If it yields $\kappa = 0$, then the firm invests with probability one, and such a deviation does not affect firm value. Hence, assuming $R_L^* > R_H^*$ and $R_0^* \leq R_H^*$ either yields a contradiction, or yields that there is a maximum with $R_H > R_L$.

**Case 2:** $R_0^* \geq R_H^*$. Now assume that $R_0^* \geq R_H^*$. In that case, the probability of the manager investing is zero. If $\overline{X}^* > 0$, this is weakly dominated by ex-post firm value maximization—as before, deviating to ex-post firm value maximization increases firm value if it leads to $\kappa > 0$ (Lemma A.2) and does not affect it if it leads to $\kappa = 0$. If $\overline{X}^* \leq 0$, the firm would be weakly (strictly, if $\overline{X}^* < 0$) better off by always investing (setting any $R_H > R_L > R_0$, for instance). Hence, assuming $R_L^* > R_H^*$ and $R_0^* \geq R_H^*$ either yields a contradiction, or yields that there is a maximum with $R_H > R_L$.

**Case 3:** $R_L^* > R_0^* > R_H^*$. Now consider the remaining case where $R_L^* > R_0^* > R_H^*$. We define $\chi = \frac{R_L - R_0}{R_L - R_H}$. The manager now invests whenever it assigns a sufficiently low probability to $\theta = \theta_H$:

$$
\mu(X) \leq \frac{R_L - R_0}{R_L - R_H^*} \equiv \chi^* \in (0, 1).
$$

Note that under the assumed optimal choice, the set $\mathcal{O}$ is $[0, \chi^*]$. One can see, by equation (10), that, conditional on trading, agents that receive high (low) signals will sell (buy), since $R_L > R_H$, and trading profits per dollar and before trading costs are given by $-\pi(\kappa)$, where $\pi(\kappa)$ is given by (11). Applying the change of variables $h = \mu(u)$ to the integral in (11), whenever $R_L > R_H$, trading profits per dollar (ignoring trading costs) can be written as

$$
- \pi(\kappa) = (2\lambda - 1) \int_0^{\chi^*} \frac{R_L - R_H}{R_L + h(R_H - R_L)} \frac{1}{1 - h} \frac{1}{\sqrt{2\kappa}} \phi \left( \frac{\mu^{-1}(h) + \kappa}{\sqrt{2\kappa}} \right) dh. \quad (B.5)
$$
The assumption that \((R_L^*, R_H^*, R_0^*)\) is optimal requires that \(\chi^* > \frac{V_H - V_L}{V_H - V_L}\), otherwise the firm would almost always invest in states where investment is ex-post inefficient (the first integral in (B.4) would be zero) and therefore such strategy would be dominated by choosing \(R_H = R_L < R_0\), so that the manager never invests. Hence, hereafter we assume that \(\chi^* > \frac{V_H - V_L}{V_H - V_L}\). Notice that it implies \(O_N = N\). Let \(\kappa^*\) be the equilibrium \(\kappa\) implied by \((R_L^*, R_H^*, R_0^*)\).

Suppose first that \(\kappa^* = \pi\), which implies \(-\pi(\kappa^*) \geq \tau\). In that case, we argue that the firm can profitably reduce \(R_0\) in a small amount \(d_1\). Such deviation would increase \(\chi\), thus increasing trading profits \(-\pi(\kappa)\) and maintaining the implied \(\kappa\) at \(\pi\). Moreover, it would increase firm value, since it enlarges the interval of integration \(O_E\) in (B.4), and does not change \(O_N\). Now suppose \(\kappa^* = 0\), which implies \(-\pi(\kappa^*) \leq \tau\). In that case, it must be that ex-post firm value maximization \((R_H = V_H, \ R_L = V_L \text{ and } R_0 = V_0)\) yields a firm value at least as large as \((R_L^*, R_H^*, R_0^*)\), since either it leads to \(\kappa^* > 0\) (and firm value is increasing in \(\kappa\) under ex-post firm value maximization, Lemma A.2), or it also leads to \(\kappa = 0\), in which case it is an ex-ante optimal choice when no information is received.

Finally, suppose \(\kappa^* \in (0, \pi)\), which implies \(-\pi(\kappa^*) = \tau\). Suppose the firm deviates by reducing \(R_0\) in \(d_0 > 0\) units, with \(d_0\) satisfying \(R_L^* > R_0^* - d_0 > R_H^*\). Such a deviation would increase the cutoff \(\chi^*\), hence increasing trading profits and \(\kappa\) in the largest equilibrium. Then, for a \(d_0\) sufficiently small, the firm can undo the change in \(\kappa\) by reducing \(R_L\), since both \(\chi\) and the integrand in (B.5) increase with \(R_L\). Let \(d_L > 0\) be the size of the reduction in \(R_L\) required for \(\kappa\) not to change. Denote by \(d_\chi\) the implied change in \(\chi\) after the mentioned decreases in \(R_0\) and \(R_L\). Then, using (B.5) and \(-\pi(\kappa^*) = \tau\), a necessary condition for the equilibrium \(\kappa\) not to change is:

\[
\int_{0}^{\chi^*} \frac{R_L^* - R_H^*}{R_L^* + h(R_H^* - R_L^*)} \frac{1}{1 - h \sqrt{2\kappa^*}} \phi \left( \frac{\mu^{-1}(h) + \kappa^*}{\sqrt{2\kappa^*}} \right) dh = \int_{0}^{\chi^* + d_\chi} \frac{R_L^* - d_L - R_H^*}{R_L^* - d_L + h(R_H^* - R_L^* + d_L)} \frac{1}{1 - h \sqrt{2\kappa^*}} \phi \left( \frac{\mu^{-1}(h) + \kappa^*}{\sqrt{2\kappa^*}} \right) dh.
\]

Note that for a given \(h\), the integrand in the second integral is smaller than in the first. It must then be that \(d_\chi > 0\). But then, the mentioned changes in \(R_0\) and \(R_L\) increase firm value, since they do not alter \(\kappa\) and enlarge the interval of integration \(O_E\) in (B.4) without affecting \(O_N\), contradicting that \((R_L^*, R_H^*, R_0^*)\) is optimal. This concludes the proof that there is always an optimum with \(R_H \geq R_L\). It remains to show that there is an optimum with \(R_H > R_L\).

Suppose that \(R_H = R_L\) is optimal. Then, trading profits are always zero, and in equilibrium \(\kappa = 0\). But then, if the firm deviates to ex-post firm value maximization \((R_H = V_H, R_L = V_L \text{ and } R_0 = V_0)\), either it leads to \(\kappa > 0\) and firm value increases (since, as argued before, firm value is increasing in \(\kappa\) with ex-post firm value maximization) or it still leads to \(\kappa = 0\), in which case ex-post firm value maximization is ex-ante optimal, since no information is received. Hence, \(R_H = V_H > R_L = V_L\) is also optimal. \(\square\)
Then, if an optimum exists, the firm can achieve it by issuing debt.

**Proposition B.4.** Consider the problem of a firm that can choose any general security at $t = 0$. Then, if an optimum exists, the firm can achieve it by issuing debt.

**Proof.** As we did in the proof of Lemma B.1, we state the problem of the firm as that of choosing $R_L \in [0, V_L]$, $R_H \in [0, V_H]$ and $R_0 \in [0, V_0]$. Suppose an optimum exists. Since we know by Lemma B.1 that there is always a solution with $R_H > R_L$, hereafter we impose that restriction on the choice of the firm. Then, the manager will undertake investment whenever

$$X \geq \ln \left( \frac{R_0 - R_L}{R_H - R_0} \right) = X^*.$$

Note that by appropriately setting $R_0 \in (R_L, R_H)$ the firm can achieve any $X \in \mathbb{R}$: if $R_0$ approaches $R_H$ from below, $X \to -\infty$; if $R_0$ approaches $R_L$ from above, $X \to -\infty$. Now notice that $R_0$ does not enter directly on $V$ or $\pi(\kappa)$, it only affects those payoffs through its effect on $X$. Hence, we hereafter think of the problem of the firm as that of choosing $R_L \in [0, V_L]$, $R_H \in [0, V_H]$ and $X \in \mathbb{R}$, subject to $R_H > R_L$. By Lemma 3, we know that there is always an optimum with $X \leq X^*$, since that lemma was shown for any arbitrary $R_H$, $R_L$ and $R_0$, with $R_H \geq R_L$. Hence, we hereafter assume that the firm chooses $X \leq X^*$.

We now claim that there is always an optimal with $R_L = 0$. Suppose there is an optimal $(R_L, R_H, X)$, with $R_L > 0$. First, suppose $X = X^*$. Then, by Lemma A.2, firm value is strictly increasing in $\kappa$. Moreover, decreasing $R_L$ increases trading profits $\pi(\kappa)$, which weakly increases $\kappa$ in the largest equilibrium. Hence, deviating to $R_L = 0$ must weakly increase firm value, and so there is an optimum with $R_L = 0$. Now assume $X < X^*$ and let $V(X, \kappa)$ be the firm value for a given $X$ and $\kappa$. Suppose again that the firm deviates to $R_L = 0$, which increases trading profits, and hence weakly increases $\kappa$. If $\kappa$ remains unchanged, then such deviation also leads to an optimum. Now suppose $\kappa$ increases in $\Delta \kappa > 0$ and denote by $\kappa^*$ the equilibrium value of $\kappa$ before the deviation. If firm value falls after the deviation, then it must be that $X \left( e^{X} - e^{X^*} \right) + \kappa \left( e^{X} + e^{X^*} \right) < 0$ for some $\kappa \in (\kappa^*, \kappa^* + \Delta \kappa)$ (see equation (A.6)), which implies that $\frac{\partial V(X, \kappa^*)}{\partial \kappa} < 0$. But then we reach a contradiction: If the firm deviates from the proposed optimal choice by slightly increasing $X$, that would increase firm value, since such deviation would not increase $\kappa$ (trading profits $\pi(\kappa)$ decrease with $X$) and the increase in $X$ would directly increase firm value by (A.7). This concludes the proof that there is always an optimum with $R_L = 0$.

Now notice that when $R_L = 0$, trading profits $\pi(\kappa)$ do not depend on $R_H$. Hence, for a given $X$ and $R_L = 0$, any $R_H > 0$ leads to the same firm value. Therefore, consider a solution $(R_L, R_H, X) = (0, R_H^*, X^*)$ with $X^* \leq X^*$ to our problem, and denote the maximum firm value obtained by $V^*$. Now consider the value of a firm that, in the model of Section 2, issues $D \in [V_L, V_0]$
such that $X^*(D) = \bar{X}^\dagger$. Such level of debt implies $R_L = 0$, $R_H > 0$ and $\bar{X} = \bar{X}^\dagger$, and hence it yields a firm value equal to $\mathcal{V}^\dagger$, since with $R_L = 0$ any $R_H > 0$ leads to the same firm value. Therefore, the firm can always get to an optimum by issuing debt.

\section*{B.4 Publicly traded debt}

In this section we adapt our main model to allow for two different secondary security markets.

Suppose the firm raises debt by issuing corporate bonds that are later traded in secondary financial markets. Speculators then have two distinct venues to trade on their information: they may speculate on the expected return to shareholders, but also on the default risk of the firm.

We adjust our setting in the following way. For a given level of debt $D$, let $R_B(\theta, a) \equiv \min\{D, v(\theta, a)\}$ denote the return to a bond holder in the final period, as a function of the state and the investment decision. We normalize the quantity of bonds to 1, so $R_B(\theta, a)$ is the ex-post value of bonds, and denote by $P_B$ the bond price. Each speculator can trade $s_i$ dollars in stocks and $b_i$ dollars in bonds, subject to the resources constraint that $|s_i| + |b_i| \leq 1$. Moreover, noise traders place a random order $\tilde{n}_B$ (in dollars) for bonds, which is normally distributed with zero mean and variance $\sigma^2_B$ (independent from noise traders’ orders in the stock market). For each dollar traded in bonds, speculators pay a trading cost $\tau_B$. For simplicity, let $\sigma_B = \sigma$ and $\tau_B = \tau$—i.e., there is the same amount of noise and the same trading costs in both markets. After receiving signal $m_i$, speculator $i$ solves the following problem:

$$\max_{s_i, b_i} s_i E \left[ \frac{R(\theta, a)}{P} - 1 \right] m_i + b_i E \left[ \frac{R_B(\theta, a)}{P_B} - 1 \right] m_i - \tau (|s_i| + |b_i|)$$

s.t. $|s_i| + |b_i| \leq 1$.

The aggregate order in the bond market is $\bar{X}_B = \tilde{n}_B + \int_0^\alpha b_i \, di$. For a given strategy profile for informed speculators $\{s_i(m_H), s_i(m_L), b_i(m_H), b_i(m_L)\}_{i \in [0, 1]}$, their aggregate order for stocks is as previously defined, and their aggregate order for bonds is $B_H = \int_0^\alpha \left[ \lambda b_i(m_H) + (1 - \lambda) b_i(m_L) \right] \, di$ in the high state and $B_L = \alpha \left[ \lambda b_i(m_L) + (1 - \lambda) b_i(m_H) \right]$ in the low state. We can then redefine variables $X$ and $\kappa$ as:

$$X \equiv \frac{1}{\sigma^2} \left[ -\frac{1}{2} \left( S_H^2 - S_L^2 \right) - \frac{1}{2} \left( B_H^2 - B_L^2 \right) + (S_H - S_L) \bar{X} + (B_H - B_L) \bar{X}_B \right],$$

$$\kappa = \frac{1}{2\sigma^2} (S_H - S_L)^2 + \frac{1}{2\sigma^2} (B_H - B_L)^2. \quad (B.6)$$

Importantly, $X$ is a sufficient statistic for activity in both markets, and as before, speculators’ trading profits only depend on other speculators’ strategies through $\kappa$. We assume that the market maker and the manager can observe the aggregate orders in both markets ($\bar{X}$ and $\bar{X}_B$) to form their
posterior beliefs, which are given by (3). Lemma 1 continues to hold, and trading profits per dollar traded in the stock market are \( \pi(\kappa) \), as given by (11).\(^3\)

Bonds are priced in an analogous way to stocks in financial markets. After observing \( \tilde{X} \) and \( \tilde{X}_B \), or analogously, \( X \), the market maker sets \( P_B(X) = E[R_B(\theta, a) | X] \). Trading profits in the bond market are then computed in an equivalent manner as profits in the stock market, only replacing \( R_L \) by \( R^B_L \equiv R_B(\theta_L, 1) \) and \( \Delta_R \) by \( \Delta^B_R \equiv R^B_H - R^B_L \), with \( R^B_H \equiv R_B(\theta_H, 1) \), in equation (11). For a given \( \kappa \) and set \( A \) such that investment is undertaken for \( X \in A \), we can write the trading profits per dollar in the stock and bond markets, respectively, as \( \pi(\kappa) = \int_A G^S(u) \frac{1}{\sqrt{2\pi}} \phi \left( \frac{u+\kappa}{\sqrt{2\pi}} \right) du \) and \( \pi^B(\kappa) = \int_A G^B(u) \frac{1}{\sqrt{2\pi}} \phi \left( \frac{u+\kappa}{\sqrt{2\pi}} \right) du \), where

\[
G^S(u) = \frac{\mu(u)\Delta_R}{R_L + \mu(u)\Delta_R} \quad \text{and} \quad G^B(u) = \frac{\mu(u)\Delta^B_R}{R^B_L + \mu(u)\Delta^B_R}.
\]

Notice that if \( D \leq V_L \), \( R^B_L = R^B_H = D \), and \( \Delta^B_R = 0 \). If \( D \in (V_L, V_H) \), \( R^B_L = V_L \), \( R^B_H = D \), so \( \Delta^B_R = D - V_L \). Finally, if \( D \geq V_H \), bond holders become the residual claimants of the company (as if bonds become stocks). We then have:

\[
G^S(u) = \begin{cases} 
\frac{\mu(u)(V_H - V_L)}{V_L - D + \mu(u)(V_H - V_L)} & \text{for } D \leq V_L, \\
1 & \text{for } D \in (V_L, V_H), \\
0 & \text{for } D \geq V_H,
\end{cases} \quad \text{and} \quad G^B(u) = \begin{cases} 
0 & \text{for } D \leq V_L, \\
\frac{\mu(u)(D - V_L)}{V_L + \mu(u)(D - V_L)} & \text{for } D \in (V_L, V_H), \\
\frac{\mu(u)(V_H - V_L)}{V_L + \mu(u)(V_H - V_L)} & \text{for } D \geq V_H.
\end{cases}
\]

It follows from the expressions above that, for any set \( A \) and for any \( \kappa \), trading profits per dollar in the stock market are always strictly larger than trading profits for bonds—except when debt is so high that bonds become stock (such values of debt would never be optimal for the firm).\(^4\)

Therefore, the secondary market for bonds in this setting is endogenously illiquid: speculators always choose \( b_i(m_H) = b_i(m_L) = 0 \). The results presented in Section 3.2 remain unchanged when we allow bonds to be (potentially) publicly traded. Finally, note that the fact that \( \kappa \) is a convex function of \( \Delta S = S_H - S_L \) and \( \Delta B = B_H - B_L \) implies that market informativeness \( \kappa \) is larger when informed speculators concentrate their orders in one security.\(^5\)

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\(^3\)The proof is omitted since it is identical to the proof of Lemma 1 if one uses the new definitions of \( X, \kappa \) and redefines \( n \) as \( n = \frac{1}{\sqrt{2\pi}} (S_H - S_L) \tilde{n} + \frac{1}{\sqrt{2\pi}} (B_H - B_L) \tilde{n}_B \).

\(^4\)Recall that any \( D \geq V_0 \) leads to a cutoff \( \overline{X} = -\infty \). Hence, when bonds are publicly traded, if \( D \geq V_H \) the manager always invests (she does not react to market activity). If \( \overline{X}^* > 0 \), this is clearly not optimal, as setting \( D = V_L \) leads to \( \overline{X} = \overline{X}^* \) and \( \kappa \geq 0 \), and achieves a strictly larger firm value. If \( \overline{X}^* \leq 0 \), firm value when \( D \geq V_H \) is the same as in a situation with \( \kappa = 0 \) and \( \overline{X} = \overline{X}^* \) (in which the firm always invests). But since \( \frac{\partial V}{\partial \kappa} \big|_{\overline{X}=\overline{X}^*} > 0 \) (see (A.6)), setting \( D = V_L \) is strictly better, as it leads to \( \overline{X} = \overline{X}^* \) and \( \kappa > 0 \).

\(^5\)Suppose a mass \( \alpha^* \) of speculators trades stocks up to their limits (buying after good news and selling after bad news), and no speculator trades bonds. Then, using (B.6), \( \kappa \) is equal to \( \kappa^* = \frac{2(\alpha^*(2\lambda-1))^2}{(1-\ell)^2 + \ell^2} \). Now suppose a fraction \( \ell \in (0, 1) \) of those speculators decide to trade bonds up to their limits (buying after good news and selling after bad news) instead. Then, \( \kappa \) is given by \( \kappa^* \left((1-\ell)^2 + \ell^2\right) \), and hence market informativeness \( \kappa \) is reduced.
B.5 Short-selling restrictions

Suppose that short-selling is not allowed and informed speculators have no initial position on the stock. In this case, speculators cannot submit negative orders, i.e., $s_i \in [0, 1]$. The model with short-selling restrictions can be solved in an analogous manner to the main model. By equation (10), it is easy to see that negatively informed traders will not trade in equilibrium ($s_i (m_L) = 0$ for every $i$). For positively informed speculators, trading profits per dollar bought are given by (11). The maximum possible level of informativeness (reached when $s_i (m_H) = 1$ for every $i$) is then $\overline{\pi}^{ss} \equiv \frac{\alpha^2 (2\lambda - 1)^2}{2\varphi^2} = \frac{\pi}{4}$. All results previously derived apply, only replacing $\overline{\pi}$ by $\overline{\pi}^{ss}$. Hence, imposing short-selling restrictions has an effect equivalent to reducing $\alpha$ (and then $\overline{\pi}$).

B.6 Ex-ante optimal investment policy

In this section, we further characterize optimal investment strategies in the model of Section 4.1, fixing an exogenous $D \in [0, V_H)$. Throughout this section, we denote an ex-ante optimal cutoff by $\overline{X}_c^*$. If $D \geq V_H$, regardless of the investment strategy, $\pi(\kappa) = 0$ for any $\kappa$ and trivially $\overline{X}_c^* = \overline{X}^*$ is optimal. The next propositions make reference to the following bounds:

\[
\begin{align*}
\tilde{\tau}_1 &\equiv (2\lambda - 1) \int_{\overline{X}^*}^{\infty} \frac{\mu(u) \Delta_R}{\mu(u) \Delta_R + 1} \sqrt{2\kappa} \phi \left( \frac{u + \overline{\pi}}{\sqrt{2\kappa}} \right) du, \\
\tilde{\tau}_2 &\equiv \max_{\kappa \in [0, \overline{\pi}]} (2\lambda - 1) \int_{\overline{X}^*}^{\infty} \frac{\mu(u) \Delta_R}{\mu(u) \Delta_R + 1} \sqrt{2\kappa} \phi \left( \frac{u + \kappa}{\sqrt{2\kappa}} \right) du, \\
\tilde{\tau}_3 &\equiv (2\lambda - 1) \frac{\Delta_R}{\mu(u) \Delta_R + 1}.
\end{align*}
\]

Proposition B.5. Suppose investment is profitable under the prior ($\overline{X}^* \leq 0$). Then, for $\tau \leq \tilde{\tau}_1$ and $\tau \geq \tilde{\tau}_3$, there is no gain in commitment, $\overline{X}_c^* = \overline{X}^*$. For $\tau \in (\tilde{\tau}_1, \tilde{\tau}_3)$, there are gains in commitment, $\overline{X}_c^* < \overline{X}^*$.

Proof. Denote by $\mathcal{V}(\overline{X}, \kappa)$ the ex-ante value of the firm for some investment cutoff $\overline{X}$ and some $\kappa$, as given by equation (A.8). Denote by $\kappa^* (\overline{X})$ the resulting level of informativeness when $\overline{X}$ is played, that is, the value of $\kappa$ in the largest equilibrium. Throughout the proof, we use the results in Lemma A.2. We first show that if commitment increases firm value, then $\overline{X}_c^* < \overline{X}^*$ (we show this for any $\overline{X}^* \in \mathbb{R}$, for future reference). Note that decreasing $\overline{X}$ shifts the curve $\pi(\kappa)$ up and must weakly increase $\kappa^* (\overline{X})$. Since $\frac{\partial \mathcal{V}}{\partial \kappa} \Big|_{\overline{X} = \kappa} > 0$ and $\frac{\partial \mathcal{V}}{\partial \overline{X}} < 0$ for any $\overline{X} > \overline{X}^*$ and $\kappa > 0$, then

\[
\mathcal{V}(\overline{X}', \kappa^* (\overline{X}')) \leq \mathcal{V}(\kappa^* (\overline{X}'), \overline{X}^*) \leq \mathcal{V}(\overline{X}^*, \kappa^* (\overline{X}'))
\]

for any $\overline{X}' > \overline{X}^*$, which means that $\overline{X}^*$ weakly dominates any cutoff $\overline{X}' > \overline{X}^*$. Hence, if commitment increases firm value, it must be that $\overline{X}_c^* < \overline{X}^*$.

Hereafter, consider $\overline{X}^* \leq 0$. Note that the effect of a marginal change in $\overline{X}$ on the ex-ante firm
value is given by:

\[
\frac{dV}{dX} = \frac{\partial V}{\partial X} + \frac{\partial V}{\partial \kappa^*} \frac{\partial \kappa^*}{\partial X}.
\]

Consider \( \tau \leq \tilde{\tau}_1 \), where \( \tilde{\tau}_1 \) is the value of \( \pi (\tilde{\pi}) \) for \( X = X^* \), as defined in (B.7). From Proposition 1, the unique trading stage equilibrium features \( \kappa^* = \tilde{\pi} \) for any \( X \leq X^* \). Committing to some \( X < X^* \) would reduce firm value, relative to \( X = X^* \), since \( \partial V / \partial X > 0 \) and \( \partial \kappa^* / \partial X = 0 \) for any \( X < X^* \) (since \( \kappa^* (X^*) = \tilde{\pi} \)). Hence, \( X_c^* = X^* \) is ex-ante optimal. Now consider \( \tau \in (\tilde{\tau}_1, \tilde{\tau}_3) \), where \( \tilde{\tau}_3 \) is the value of \( \pi (0) \) for any \( X \leq 0 \), as given in (B.8) (its computation follows from Lemma A.1). Given Proposition 1, \( \kappa^* (X^*) \in (0, \tilde{\tau}_3) \). Hence, at \( X = X^* \) decreasing \( X \) increases firm value, since \( \frac{\partial V}{\partial X} = 0 \), \( \frac{\partial V}{\partial \kappa} > 0 \), and \( \frac{\partial \kappa^*}{\partial X} < 0 \) (since \( \pi (\cdot) \) is decreasing). Since \( V (X, \kappa^* (X)) \) is continuous in \( X \) for \( X \in (-\infty, X^*) \) and \( \lim_{X \to -\infty} V (X, \kappa^* (X)) = \frac{V_L + V_H}{2} < V (X^*, 0) \), we must have \( X_c^* \in (-\infty, X^*) \). Finally, if \( \tau \geq \tilde{\tau}_3 \), at \( X = X^* \) speculators have no incentives to trade, \( \kappa^* (X^*) = 0 \), and \( V = \frac{V_L + V_H}{2} \). There is no reduction in \( X \) that could lead to an equilibrium with interior \( \kappa \), since \( \pi (0) = \tilde{\tau}_3 \leq \tau \) for all \( X \leq 0 \) (from Lemma A.1) and \( \pi (\cdot) \) is strictly decreasing in \( \kappa \). Therefore, for \( \tau \geq \tilde{\tau}_3 \), \( X^* \) is optimal.

Proposition B.5 states that, if \( X^* \leq 0 \) and the firm can commit to an investment strategy ex ante, it commits to a cutoff \( X_c^* < X^* \) if trading costs are in an intermediate range. For very low or high trading costs, there is no gain in deviating from the ex-post optimal investment strategy. The intuition is the following: If trading costs are sufficiently large, speculators would optimally choose not to trade regardless of their beliefs about other speculators’ behavior, so distorting the use of information ex post to try to improve information aggregation does not pay off. If trading costs are sufficiently low, speculators trade on information as much as possible if the firm plays the ex-post optimal cutoff \( X^* \) (the first best is achieved). Hence, committing to a different cutoff does not pay off either, since maximum information aggregation is already achieved without the need to distort the ex-post use of information.

If instead trading costs are in an intermediate range, the firm optimally commits to a lower investment cutoff. This means that projects with \( \text{ex-post} \) negative NPV—i.e., projects appraised as having negative NPV after the observation of market activity—will be undertaken with positive probability. If the firm would set \( X_c^* = X^* \), this would lead to an equilibrium with interior \( \kappa^* \), in which there would be some margin to improve information aggregation ex ante by investing in a broader set of states (with \( X \) to the left of \( X^* \)). Moreover, at \( X_c^* = X^* \) the marginal cost of distorting the ex-post use of information is zero. That is, if the manager is making the very best use of information ex post—only investing for \( X \geq X^* \)—committing to a marginally lower investment cutoff ex ante compensates, due to the improvement in information aggregation (even though information is somewhat misused).

The next proposition details the optimal investment rule under commitment when risky investment
would not be undertaken under the prior.

**Proposition B.6.** Suppose investment is not profitable under the prior \((X^* > 0)\). Then:

1. If \(\tau \leq \bar{\tau}_1\) or \(\tau \geq \bar{\tau}_3\), there is no gain in commitment: \(X^*_c = X^*\);

2. If \(\tau \in (\bar{\tau}_1, \bar{\tau}_2]\), there are gains in commitment: \(X^*_c < X^*\);

3. If \(\tau \in (\bar{\tau}_2, \bar{\tau}_3)\) whenever there are gains in commitment, \(X^*_c < X^*\).

**Proof.** Consider \(X' > 0\). Denote by \(V(X, \kappa)\) the ex-ante value of the firm for some cutoff \(X\) and some \(\kappa\), as given by equation (A.8), and denote by \(\kappa^*(X)\) the (largest) equilibrium level of \(\kappa\) when \(X\) is played. As already shown in the proof of Proposition B.5, any cutoff \(X > X^*\) is weakly dominated by \(X = X^*\), so we can restrict attention to \(X \leq X^*\) when choosing a cutoff to maximize ex-ante firm value.

We first claim that this problem has a solution. Note that \(\kappa^*(X)\) is (at least) left-continuous and weakly decreasing in \(X\) given that trading profits are increasing in \(X\) for \(\kappa > 0\). Let \(g(X) = V(X, \kappa^*(X))\). Note that \(g(X)\) is also left-continuous and is bounded by \(\bar{V}_0 + V_U\) (the firm value under perfect information). Suppose by contradiction that the problem does not have a solution. Then, it must be that, for some \(X' < X^*\): (i) \(g(\cdot)\) has a discontinuity at \(X'\); (ii) \(\lim_{X \to X^*} g(X) > g(X')\); (iii) there exists \(\delta > 0\) such that \(g'(X' + \epsilon) < 0\) for all \(\epsilon \in (0, \delta)\); and (iv) \(\sup_{X \leq X^*} g(X) = \lim_{X \to X^*} g(X)\). Let \(\Delta \kappa = \kappa^*(X') - \lim_{X \to X^*} \kappa^*(X)\). Note that (i) implies \(\Delta \kappa > 0\). Using Lemma A.2, for any \(X < X^*\) such that \(g'(X)\) is well defined we have:

\[
g'(X) = \frac{\partial V(X, \kappa^*(X))}{\partial X} + \frac{\partial V(X, \kappa^*(X))}{\partial \kappa} \frac{d \kappa^*(X)}{dX} \bigg|_{X \leq 0}.
\]

Hence, (iii) implies that \(\frac{\partial V(X, \kappa^*(X))}{\partial \kappa} \geq 0\) (otherwise for any \(\epsilon > 0\) arbitrarily close to zero we would have \(g'(X' + \epsilon) > 0\)). Using (ii), we have \(g(X') = \lim_{X \to X^*} g(X) = \int_{\kappa^*(X')}^{\kappa^*(X)} \frac{\partial V(X, \kappa)}{\partial \kappa} d\kappa < 0\). But then, \(\frac{\partial V(X, \kappa)}{\partial \kappa} < 0\) for some \(\kappa \in (\kappa^*(X') - \Delta \kappa, \kappa^*(X'))\), which contradicts that \(\frac{\partial V(X, \kappa^*(X)) - \Delta \kappa)}{\partial \kappa} \geq 0\). Hence, this problem has a solution, and now we characterize it.

First consider \(\tau \leq \bar{\tau}_1\). By the definition of \(\bar{\tau}_1\), setting \(X = X^*\) leads to the first best (under Assumption 2), so \(X^*_c = X^*\) is ex-ante optimal. Now consider \(\tau \in (\bar{\tau}_1, \bar{\tau}_2]\) (whenever \(\bar{\tau}_1 < \bar{\tau}_2\)). In this case, \(\kappa^*(X^*) \in (0, \pi)\). Starting at \(X^*\), marginally decreasing \(X\) shifts \(\pi(\kappa)\) up and strictly increases \(\kappa^*(X)\), and from Lemma A.2, \(\frac{\partial V}{\partial X} |_{X = X^*} = 0\) and \(\frac{\partial V}{\partial \kappa} |_{X = X^*} > 0\). Hence, using (B.9), \(\frac{\partial V}{\partial X} |_{X = X^*} < 0\), and thus the optimal cutoff is some \(X^*_c < X^*\). Next, consider \(\tau \in (\bar{\tau}_2, \bar{\tau}_3)\), where \(\bar{\tau}_3\) is the value of \(\pi(0)\) when \(X \leq 0\), as given in (B.8). In this case, \(X^*_c = X^*\) is optimal if \(V_0 > \max_{X \leq X^*} V(X, \kappa^*(X))\).
and \( \bar{X}^* \in \arg \max_{X \in \mathcal{X}^*} \mathcal{V}(\bar{X}, \kappa^*(\bar{X})) \) is optimal otherwise. Finally, if \( \tau \geq \tilde{\tau}_3 \), since \( \kappa^*(\bar{X}) = 0 \) for any \( \bar{X} \), there is no gain in committing to a cutoff different from \( \bar{X}^* \).

Proposition B.6 is quite similar to Proposition B.5, with the only difference that, for an intermediate range of trading costs, committing to a lower cutoff may not pay off even though it would increase information aggregation. This is because the distortion in terms of the ex-post misuse of information by the manager would be too large compared to the informational gain achieved. In the case of Proposition B.5, instead, commitment was always good, as long as there was margin to increase information aggregation. In both cases, naturally, if trading costs are very large and speculators are not willing to trade on information regardless of their beliefs about their peers’ behavior, then simply playing according to \( \bar{X}^* \) is optimal, since there is no scope for improving market informativeness.

References


